CONTINUOUS CURRENT
ENGINEERING

AN INTRODUCTORY COURSE OF

CONTINUOUS CURRENT ENGINEERING

BY

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PREFACE TO FIRST EDITION.

The present work is introductory in the sense that only elementary methods of treatment are made use of throughout, and that no attempt is made to cover the entire field of continuous current engineering. The aim of the author has been to present the reader with a simple and by no means exhaustive account of the component parts of a continuous current lighting and power plant—dynamos, motors, secondary cells, measuring instruments, etc. The selection and arrangement of these component parts to form a connected system are subjects lying outside the scope of this work.

Although introductory in the sense explained above, the work is not intended for absolute beginners, and an elementary knowledge of magnetism and electricity is assumed on the part of the reader.

The author's thanks are due to the various firms who kindly supplied him with information relating to their manufactures, especially to Messrs. Crompton & Co., Ltd., Messrs. Morris and Lister, and Messrs. C. A. Parsons & Co.

The illustrations have all been specially prepared for the work, and the author desires to acknowledge his great indebtedness to his friend, Mr. A. P. Thurston, B.Sc., for valuable help in the preparation of a number of drawings.

A. H.

London, N.W.
September, 1907.

PREFACE TO THIRD EDITION.

THE changes which have taken place since the publication of the second have necessitated a somewhat drastic revision of this work for a third edition. Much new matter has been added, and certain sections have been practically re-written, while others, dealing with types of machines and apparatus which have now become obsolete, have been omitted. It is the author's hope that in its new guise the book will have lost none of its usefulness.

A. H.

HAMPSHIRE,
September, 1927.

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§ 1. Electro-magnetic C. G. S. System of Units.

The practical system of electrical units is based on the C. G. S. electro-magnetic system. It will be convenient to recall briefly the definitions of the more important electrical units in this latter system.

The *absolute or C. G. S. unit of current* is a current which when flowing in a conductor in the form of a circle of 1 cm. radius, produces a magnetic force of 2π units at the centre of the circle.

When a unit current has been allowed to flow in a circuit for one second, a positive *unit electric charge* or *quantity*, or *unit quantity of electricity*, is said to have been transferred across every cross-section of the circuit, or to have passed round the circuit, in the direction of the current.

The *difference of electric potential*, generally denoted by "p.d.," between two points is the work done in transferring unit electric quantity from the one point to the other. If in passing from a point A to a point B we find that work has to be done against the electric forces during the transfer of the (positive) unit quantity, B is said to be at a higher potential than A; if work is done by

the electric forces (corresponding to a gain of work), A is at a higher potential than B. From the above definitions of "higher" and "lower" potential it follows that a positive charge always tends to move from places of higher to places of lower potential.

If instead of transferring a unit charge from one point to another we carry it completely round a closed curve, thus returning to the starting-point, then the total work done during the journey is defined to be the *electro-motive force*, or *e.m.f.*, around the closed curve or circuit.

Since p.d. and e.m.f. are essentially of the same nature, it is obvious that they are measured in terms of the same unit (erg/unit quantity). They are frequently used indiscriminately, and there are writers who speak of "the e.m.f. between two points." Besides being incorrect, this somewhat slovenly practice is liable to give rise to confusion. The e.m.f. of a dynamo or motor, for example, is by no means identical with its terminal p.d.

From the definitions of p.d. and quantity it follows that the *power* in any part of a circuit in which the current is i and across which the p.d. is v is given by the *product of p.d. into current*, i.e., by $v\ i$. For since i units of electric quantity are transferred per second from one end of the portion of the circuit considered to the other, and since during the transfer v units of work are done on each unit quantity, the work done per second, which is the power, is equal to $v\ i$. If instead of considering a portion only of the circuit we take the entire circuit, it is evident, from what has just been said, that the *total power* is the *product of the e.m.f. into the current.*

The flow of a current through a conductor is always found to be accompanied by an evolution of heat. Heat being a form of energy, the rate of heat production is expressible in ergs per second, i.e., in terms of the power unit. Experiment shows that in a given conductor maintained in a constant physical state (as regards temperature or stress) the rate of heat generation is proportional to the *square* of the current. If, therefore, i denote the current, the rate of heat production (in power units) may be written in the form $r\ i^2$, where r is a constant depending on the material, size and shape of the conductor, and on its physical state as regards temperature and stress; so long as these remain unaltered, *r does not change with the current.* This latter result is known as *Ohm's law*, and r is defined to be the *resistance* of

the conductor. Thus a conductor is of unit resistance if the rate at which heat is generated in it by unit current is equal to the unit of power.

If the p.d. across the ends of a conductor of resistance r be v, and if a current i be then found to flow through it, it follows, since the entire power is assumed to be spent in heat production, that $v\,i = r\,i^2$, or that $r = \frac{v}{i}$. Hence the resistance of a conductor may be defined as the ratio of the p.d. across its ends to the current flowing through it, and Ohm's law asserts that this ratio, for a constant physical state of the conductor, is independent of the current.

Under ordinary working conditions, no steps are taken to maintain the temperature of conductors constant, and hence, *owing to the rise of temperature* with increase of current, the resistance generally increases with increase of current. This increase of resistance is frequently utilised for the purpose of determining the temperature rise.

The *resistivity* of a given material, in C. G. S. units, is the resistance between opposite faces of a cube whose edge is 1 cm., the flow of current taking place parallel to the edges which are perpendicular to the two faces.

The *capacity* of a condenser is the charge corresponding to unit p.d. across its terminals.

From the above definitions we can immediately deduce some important results.

§ 2. Force on Conductor in Magnetic Field.

Considering the definition of unit current, we see that since the force exerted by the circle of wire on the unit pole supposed to be placed at its centre is equal and opposite to the force exerted by the unit pole on the circle, this force amounts to 2π dynes (the current in the circle of wire being assumed to be one C. G. S. unit). Hence each cm. length of the circle experiences a force of 1 dyne if a unit pole is placed at its centre. This force, it must be noted, is perpendicular to the wire at every point, and perpendicular to the (unit) magnetic field produced at every point of the wire by the unit pole. Since the only effect due to the pole is the production of a radial magnetic field at the wire, it follows that

if the circle of wire be opened out so as to form a straight conductor, and if at the same time we substitute for the unit pole some arrangement of magnets or coils such that at every point of the straight wire the magnetic field will still be perpendicular to the wire and of unit intensity, the force acting on every cm. length of the wire will remain unaltered—i.e., it will amount to 1 dyne. The direction of this force is perpendicular to the wire and to the magnetic field. If the field intensity or current be increased in any ratio, the force will be increased in the same ratio (since a current is regarded as proportional to its magnetic effect). If, therefore, we have a wire l cms. long, conveying a current of i C. G. S. units, and placed with its length at right angles to a magnetic field of intensity H, the total force acting on the wire is $l\,\mathrm{H}\,i$ dynes. The direction of the force is easily determined by Fleming's rule.

If the wire just considered be displaced through a distance d in the direction of the force, the work done will be $l\,\mathrm{H}\,i.d = i.l\,d\,\mathrm{H}$. But since $l\,d$ is the area swept out by the wire during the motion, and H the number of unit magnetic lines per unit of this area, it follows that $l\,d\,\mathrm{H}$ is the number of lines cut by the wire during the motion. Hence if a circuit, or part of a circuit, be made to cut a number of magnetic lines, the work done during the process is equal to the product of the current into the number of lines cut.

§ 3. Theorem concerning M.M.F. around Closed Curve.

Consider any closed curve which is linked with a circuit conveying a current of i C. G. S. units. Let a unit magnetic pole be carried round the closed curve. Since the unit pole gives rise to 4π unit lines, 4π lines will cut the circuit during the motion. Hence, by what has been said above, the work done in carrying the unit pole around the closed curve is $i.\ 4\pi$, or $4\pi i$. Now the work done in carrying a unit pole around any closed curve is defined to be the *magneto-motive force*, or m.m.f., around

The thumb, forefinger, and middle finger of the *left* hand being held at right angles to each other, the hand is placed so that the *f*orefinger is along the direction of the magnetic *f*ield, and the middle finger along the direction of the current (i). The thumb will then indicate the direction along which the wire tends to *m*ove, *i.e.*, the direction of the force acting on the wire.

that curve. The result just obtained may therefore be stated as follows :—

The m.m.f. around any closed curve which is linked with an electric circuit conveying a current of i C. G. S. units is equal to $4\pi i$.

This result is of immense importance, theoretical and practical, and forms one of the great fundamental principles of electro-magnetism. The most convincing proof of it is furnished by the fact that the consequences deduced from it have invariably been found to agree with the results of experiments.

A very simple application of this principle enables us to calculate the magnetic force at a given distance from a very long straight wire carrying a current of i C. G. S. units. Experiment shows that the field due to such a current consists of circular lines of force having their centres on the axis of the wire. Consider the m.m.f. around a circular line of force of radius $= r$ cms. By symmetry, the value of H is constant along such a line, and since H is, by definition, the force acting on a unit pole, the work done in carrying the unit pole around the circle is $\mathrm{H} \times 2\pi r$. But by the fundamental principle established above, the m.m.f. around the circle is also $4\pi i$. Thus $\mathrm{H}.\ 2\pi r = 4\pi i$, or $\mathrm{H} = \frac{2i}{r}$. The magnetic force is thus shown to vary inversely as the distance from the wire.

§ 4. Force between two Parallel Wires.

The above expression for H enables us to determine the attraction or repulsion between two parallel straight wires conveying currents of i and i' C. G. S. units. At every point of the second wire there is a magnetic field—due to the current in the first wire—of intensity $\frac{2i}{r}$. Hence every cm. length of the second wire experiences a force of $\frac{2i}{r}.i'$ or $\frac{2ii'}{r}$ dynes (§ 2). The force is therefore proportional to the product of the two currents, and inversely proportional to the distance apart of the wires. If we suppose each current to be unity, and the distance between the wires to be 1 cm., we see that the force per cm. length of either wire is 2 dynes.

§ 5. Principle of Electro-magnetic Induction.

In 1831, Faraday made the important discovery that an e.m.f. could be generated or "induced" in a circuit by causing it to cut magnetic lines so as to produce changes in the total number of lines, or total magnetic flux, linked with the circuit. Let us imagine part of the circuit to consist of a straight wire capable of gliding with its ends on two conducting rails, so that as the motion takes place the wire cuts the lines of a magnetic field of intensity H which we shall suppose to be at right angles to the moving conductor and to its direction of motion. Let at a given instant the velocity be v cms. per second, the e.m.f. e, and the current i. Then the electrical power is $e\,i$. Neglecting friction, the mechanical power required to maintain the motion is force $\times$ velocity $= \mathrm{H}\,l\,i\,v$, since by § 2 the force acting on the conductor is $\mathrm{H}\,l\,i$. Now by supposition the entire mechanical power is transformed into electrical power, and we must have $e\,i = \mathrm{H}\,l\,i\,v$, or $e = \mathrm{H}\,l\,v$. But $\mathrm{H}\,l\,v$ is the number of lines cut per second by the conductor. We thus have the following result:—

The e.m.f. induced in a conductor which moves across a magnetic field is equal to the rate at which it is cutting magnetic lines.

Since the lines cut are added to (or subtracted from, according to the direction of motion) the closed circuit of which the conductor forms part, the above result may also be expressed in the following alternative form:—

The e.m.f. induced in any closed circuit is numerically equal to the rate at which the number of magnetic lines linked with the circuit is changing. The direction of this e.m.f. is always such as to oppose the change which gives rise to it (Lenz's law).

§ 6. Electric Discharge due to Change of Flux.

Consider a circuit of resistance r, with which there is linked a magnetic flux Φ. Let this flux be rapidly withdrawn during a short interval of time t. The mean e.m.f. induced in the circuit is Φ/t, and hence the mean current is $\Phi/(t\,r)$. Since this current lasts t seconds, the quantity which passes around the circuit is Φ/r. Thus by withdrawing Φ magnetic lines from a circuit of resistance r we cause the discharge of Φ/r units of quantity around the circuit.

§ 6a. Condition Governing the Transformation of Electrical Energy into Forms Other than Heat.

In the case of a wire or other simple conductor carrying a current, the whole of the electrical energy supplied to it becomes converted into heat, the rate of conversion being given by $r\,i^2$, which in this case is identical with $v\,i$ (§ 1). If, however, the conductor is the seat of an e.m.f. e opposing the current (or a counter-e.m.f.), then part of the p.d. v across the ends of the conductor is utilised to balance or neutralise the counter e.m.f., so that only the difference $v-e$ is available for maintaining the current through the resistance of the conductor. The current now becomes $(v-e)/r$, so that we have $v-e = r\,i$, and $v\,i = e\,i + r\,i^2$. It will be seen that in this case the total power $v\,i$ supplied to the conductor is made up of two parts—$r\,i^2$, which represents the rate of energy dissipation into heat, and $e\,i$, which measures the rate of conversion of electrical energy into *some form other than heat*, such as chemical or mechanical energy.

Conversely, if it is desired to transform electrical energy into some form other than heat, *the presence of a counter e.m.f. is essential*. A secondary battery while being charged and an electric motor when running afford well-known illustrations of this general principle.

§ 7. Practical System of Electric Units.

Many of the C. G. S. units considered are not very convenient for practical use, being either too small or too large. This fact led to the adoption of the following practical system of units:—

The practical unit of current is the *ampere*, and is equal to one-tenth or 10^{-1} of the C. G. S. unit.

The practical unit of quantity (ampere-second) is the *coulomb*, and is equal to 10^{-1} of the C. G. S. unit.

The practical unit of p.d. and e.m.f. is the *volt*, equal to 10^8 C. G. S. units.

The practical unit of power (volt-ampere) is the *watt*, equal to 10^7 ($= 10^8 \times 10^{-1}$) C. G. S. units.

The practical unit of electrical energy (watt-second) is the *joule*, equal to 10^7 C. G. S. units.

The practical unit of resistance (volt/ampere) is the *ohm*, equal to 10^9 ($= 10^8/10^{-1}$) C. G. S. units.

The practical unit of capacity (coulomb/volt) is the *farad*, equal to 10^{-9} ($= 10^{-1}/10^8$) C. G. S. units.

Even the above practical units are frequently either too large or too small in connection with many applications. The prefix *milli-* is in some cases used to denote the thousandth part of the unit to which it is prefixed. Thus, 1 milli-ampere $= 10^{-3}$ ampere; 1 milli-volt $= 10^{-3}$ volt. The prefix *kilo-* denotes a unit which is a thousand times the unit to which it is prefixed. Thus, 1 kilovolt $= 10^3$ volts; 1 kilowatt $= 10^3$ watts. The prefix *micro-* means "one millionth of," while the prefix *mega-* or *meg-* means "one million times." Thus, 1 micro-ampere $= 10^{-6}$ ampere; 1 microhm $= 10^{-6}$ ohm; 1 megohm $= 10^6$ ohms.

A unit of quantity very frequently used in practice is the *ampere-hour.* This is equal to 60 ampere-minutes, or to $60 \times 60 = 3{,}600$ ampere-seconds or coulombs.

The output of generators is generally expressed in kilowatts.

The commercial unit of electrical *energy*, formerly known as the *Board of Trade Unit*, but now generally referred to as a "unit" simply, is represented by a *kilowatt-hour*, and hence is equal to $10^3 \times 3{,}600$ joules or watt-seconds.

The unit of power up to the present generally used by mechanical engineers is the *horse-power* (denoted by h.p.), which represents 550 ft. lbs./sec. Since 1 ft. $= 30{\cdot}48$ cms., and 1 lb. weight $= 453{\cdot}6$ grammes weight $= 453{\cdot}6 \times 981$ dynes, we have 1 ft. lb. $= 13{\cdot}56 \times 10^6$ ergs. Hence we find that 1 h.p. $= 746 \times 10^7$ ergs/sec. $= 746$ watts.

§ 8. Absolute Measurements, International Units and Concrete Standards.

Simple as the system of units outlined above may appear to be, the construction of concrete physical standards embodying the above definitions has presented formidable experimental difficulties. The problem of providing such standards has engaged the attention of a Committee of the British Association for a long period of years. Originally appointed in 1861, the Electrical Standards Committee was dissolved in 1870, but was reappointed in 1880, and continued its labours as late as 1912.

The definitions of the electrical units being based on those of the three fundamental units of length, mass and time, it follows that it should be possible to determine the value of an electrical quantity by measuring one or more of the following three

quantities—a length, a mass and a time. The measurement of an electrical quantity by this method is known as an *absolute measurement.**

Of the three electrical units of current, e.m.f. and resistance, two only—viz., those of current and resistance—have been determined in absolute measure. No absolute determination of the electro-magnetic unit of e.m.f. has yet been made.

By means of the most recent types of apparatus employed for the purpose, it is now possible to determine the value of a resistance in absolute measure with an accuracy of about ·002 per cent.; and to measure a current in absolute measure with an accuracy of about ·005 per cent.

Two types of resistance standards are in use. That most commonly employed takes the form of a coil of wire suitably mounted. The other—which is to be found only in certain laboratories—consists of a long glass tube containing mercury, and provided with spherical end bulbs which contain the electrodes. The mercury standards at the National Physical Laboratory have been shown to be reliable to within ·001 per cent.

At the International Conference on Electrical Units and Standards held in London in 1908 it was decided to adopt the following definitions of the units of resistance, current and p.d., the units so defined to be known as the International Electrical Units.

The *International Ohm* is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14·4521 grammes in mass, of constant cross-sectional area and length equal to 106·3 cm.

The *International Ampere* is the unvarying electric current which, when passed through a solution of silver nitrate in water, deposits silver at the rate of ·001118 gramme per second.†

* For a concise and interesting account of the history of absolute measurements, the reader may be referred to Dr. Glazebrook's Kelvin Lecture in the *Journal of the Institution of Electrical Engineers*, vol. 50, p. 560 (1913).

† In measuring currents by means of the silver voltameter, the electrolyte should be prepared by dissolving 15 to 20 parts by weight of silver nitrate in 100 parts of distilled water. The solution must only be used once, and only for so long that not more than 30 per cent. of the silver contained in the solution is deposited. The anode should be of silver, the cathode of platinum. The current density at the anode should not exceed $\frac{1}{5}$ ampere per sq. cm., and that at the cathode $\frac{1}{50}$ amp. per sq. cm.

The *International Volt* is the p.d. which will produce a current of 1 international ampere in a conductor whose resistance is 1 international ohm.

At the National Physical Laboratory, the units of resistance, current and p.d. are represented by the following concrete standards * :—

	Form of Standard.	Limit of Agreement with International Unit.
Resistance	Coil of insulated wire	·01 per cent.
Current	Ampere balance †	·1 ,,
P.d.	Electrostatic voltmeter ‡	·1 ,,

Examples.

1. A current of five amperes is found to generate in five minutes heat equivalent to 90,000 joules when flowing through a certain conductor. Find the resistance of the conductor, in ohms.

2. A cable whose resistance is ·02 ohm conveys a current of 100 amperes for two hours, a current of 200 amperes for three hours, and a current of 50 amperes for eight hours. If the cost of generating a unit of electrical energy is 1*d.*, what is the money value of the energy lost in the cable?

3. A current of 250 amperes is to be supplied to a distant point from a generating station, and it is desired that the drop along the cables should not exceed 20 volts. Find the horse-power wasted in the cables.

4. A closed curve is divided into two unequal portions, the length of the first being 10 inches and that of the second one-eighth of an inch. The magnetic force is everywhere along the curve, having a value of 30 C. G. S. units along the first portion, and a value of 10,000 along the second, its direction being in the same sense around the curve in each case. Find the total current, in *amperes*, which must be linked with the closed curve.

5. Two wires, conveying currents of 100 and 80 amperes, are placed parallel to each other at a distance apart of 4 inches. Find the force, in lbs. weight, per yard of either wire.

6. The total resistance of a circuit is 2,000 ohms, and there is a magnetic flux of $2{\cdot}56 \times 10^6$ C. G. S. units linked with the circuit. The flux is suddenly reduced to $1{\cdot}2 \times 10^6$ units. Find the quantity, in micro-coulombs, which will pass round the circuit.

7. A conductor 18 inches long carries a current of 50 amperes, and is placed with its length perpendicular to a magnetic field of intensity 6,000 C. G. S. units. Find the pull, in lbs. weight, acting on the conductor.

8. Find the equivalent of 10 horse-power-hours in joules.

9. A cylindrical conductor of 1 cm. radius conveys a current of 300 amperes. Find the magnetic flux due to this current through a rectangular area lying in a radial plane of the conductor, two sides of the rectangle, each

* These standards were formerly (up to 1919) in the custody of the Board of Trade, and were kept in the Board of Trade Standardising Laboratory, which has now been closed.

† See § 39.

‡ See § 38.

10 cm. long, being parallel to the conductor, and at distances of 5 and 20 cm. from its axis.

10. A rectangular frame of wire, 2 in. × 1 in., is placed with its long sides vertical and its plane making an angle of 45° with a uniform horizontal field of intensity 200. Find the torque, in gramme-cm., exerted on the frame when a current of 20 amperes is sent through it.

11. Three long straight parallel conductors are arranged so that in cross-section they appear at the corners of an equilateral triangle ABC whose side is 2 feet long. The conductors A and B convey *outgoing* currents of 70·7 and 25·9 amperes respectively, while the conductor C conveys an *incoming* current of 96·6 amperes. Find the magnetic force at the centre of the equilateral triangle, and the angle which it makes with the side AB.

CHAPTER II.

§ 9. Electro-magnets—§ 10. Magnetic induction—§ 11. Magnetic force—§ 12. Relation of magnetic force to magnetic induction—§ 13. Demagnetising force—§ 14. Retentiveness—§ 15. Ageing of magnets—§ 16. Use of specimen in form of closed ring—§ 17. Calculation of H from ampere-turns—§ 18. Units of magnetic force—§ 19. Variation of H over cross-section of ring—§ 20. Determination of B—§ 21. Standard solenoid—§ 22. Calibration of ballistic galvanometer—§ 23. Precautions to be observed in ballistic ring test—§ 24. Numerical data for materials used in dynamo construction—§ 25. Hysteresis loop—§ 26. Determination of hysteresis loop—§ 27. Calculation of energy dissipated during a magnetic cycle—§ 28. Steinmetz's law.—§ 28a. Kennelly's law—Examples.

§ 9. Electro-magnets.

It is a well-known fact that the introduction of a core of soft iron into a coil carrying a current enormously increases the magnetic effect due to the coil. Such a combination of coil and core forms an *electro-magnet*, and the very intense magnetic fields used in dynamos, motors, &c., are produced by means of electro-magnets. It will readily be seen that the problem of pre-determining, more or less accurately, the field intensities corresponding to given arrangements of electro-magnets, or the converse problem of designing an electro-magnet to produce a field of given intensity over a given region, is of fundamental importance to the electrical engineer. In the present chapter we propose to consider the more important properties of the magnetic materials (all varieties of iron) used in electrical engineering, and to explain the principles underlying the design of electro-magnets.

§10. Magnetic Induction.

When a mass of magnetic material is introduced into a magnetic field, it becomes magnetised, and the degree of magnetisation at any point of the mass may be estimated by determining the number of magnetic lines which pass across every sq. cm. of a narrow gap formed in the neighbourhood of the point considered, the direction of the walls bounding the gap

being so chosen that the lines crossing the gap are at every point exactly perpendicular to the walls. The number of lines per sq. cm. of such an (imaginary, infinitely narrow) gap—or, what amounts to the same thing, the force, in dynes, which would be exerted on a unit magnetic pole if introduced into the gap—is defined to be *the magnetic induction*, or the *induction* simply, at the point considered. The greater portion of the force which would be exerted on a unit pole placed in the gap is in most cases due to the development of magnetic polarity over the walls of the gap when the gap is cut; the direction of the cut being, in accordance with what has been mentioned above, so chosen as to develop the strongest possible polarity on the walls. The magnetic induction at any given point inside a mass of magnetic material may therefore be taken as a measure of the magnetic condition of the material at that point. Magnetic induction is generally denoted by the letter B, and it obviously has a definite direction (corresponding to the direction of the force which would act on a unit north pole if placed in the imaginary gap) as well as magnitude — i.e., it is a vector quantity.

§ 11. Magnetic Force.

Although, as stated above, in most cases the greater part of the force exerted on the unit pole in the gap is due to the presence of magnetic polarity on the walls, a portion of it—generally an extremely small fraction—is independent of this magnetic polarity, i.e., independent of the magnetic state of the material in the immediate neighbourhood of the point considered. It is, in fact, the presence of this latter component of the total force acting on the unit pole that determines (among other things) the value of the induction. This component is defined to be *the magnetic force* at the point in question. We could find its value if the magnetic polarity over the walls of our gap were destroyed (everything else, however—such as magnetic polarities in other regions, if present—remaining undisturbed). This it is impossible to do, but we can obtain a precisely equivalent result by supposing our original gap closed up, and instead of it a cavity formed around the point considered whose walls are everywhere *parallel* to the direction of the lines of force

inside the cavity; the cross-section of the cavity being supposed to be extremely (infinitely) small. It is clear that inside such a cavity the value of the force which would be exerted on a unit pole is independent of the magnetic condition of the material in the immediate neighbourhood, since there is now no magnetic polarity on the walls; and that this value is identical with that which would have been obtained in the first case (the gap with maximum polarity on its walls) had it been possible to wipe out the magnetic polarity on the walls without interfering with anything else. The force exerted on a unit pole when placed in a cavity of the second kind—a cavity whose walls nowhere exhibit magnetic polarity—is, as already stated, defined to be *the magnetic force* at the given point, and is generally denoted by H.

§12. Relation of Magnetic Force to Magnetic Induction.

As students frequently (almost invariably) find a good deal of difficulty in grasping the difference between H and B, it may be well to point out the essence of this difference. The relation connecting H and B is one of cause and effect. The application of a given H at any point of a magnetic material produces a B at that point whose value depends on the nature of the material. B measures the local magnetic state of the material, H being independent of this local state.

It is obvious that in the case of air or any other non-magnetic material, B has the same numerical value as H.

The relation connecting B and H is a complicated one, and the exact determination of this relation when we start with the specimen in a non-magnetic condition and steadily increase H is one of the most important of magnetic tests. The results of such a test are generally exhibited graphically, by plotting H horizontally and B vertically. The curve so obtained is known as the *B-H curve* for the given material.

A known value of H may be produced by sending a current through a coil of wire of suitable shape—the shape being selected so as to render it possible to calculate H by an application of the great theorem regarding m.m.f. (§ 3).

§ 13. Demagnetising Force.

Imagine a uniform magnetic field of known intensity H over a certain region (such a field might, for example, be produced in the interior of a long solenoid—§ 21), and suppose a mass of magnetic material to be introduced into this field. The mass becomes magnetised, and will in general develop magnetic polarity over certain regions of its surface. Now the magnetic poles so formed will react on the original field which was the primary cause of the magnetisation, and will modify the original value of H. The *actual* value of H at any point inside the material will therefore be different from its original value, and will in general no longer be uniform, but will vary from point to point, being the resultant of the original uniform field and the field due to the polar surfaces developed by the specimen. If, for example, we consider a specimen in the form of a cylinder placed with its axis along the direction of the (originally uniform) field, it will acquire magnetic polarity over its end surfaces (the polarity will not be entirely confined to the plane end surfaces, but will also extend along the cylindrical surface), as indicated by the letters N and S in Fig. 1. The direction of the original field is from right to left, as shown by the vector H. If now we consider the middle point of the cylinder, then, in order to determine the actual value of the magnetic force at this point, we must, in accordance with the explanations given in § 10, imagine the cylinder pierced with a long cylindrical hole (of infinitely small cross-section), as shown by the dotted lines in Fig. 1, there being no magnetic polarity over the surface of such a hole. If we imagine a unit north pole placed at the middle of the hole, then it is obvious that if the original uniform field could be wiped out without destroying the magnetisation of the specimen, the unit pole would be acted on by a definite force H′, due to the polarity of the cylinder, having a direction from left to right—i.e., a direction *opposed* to that of the originally existing field. It is evident, therefore, that the actually existing magnetic

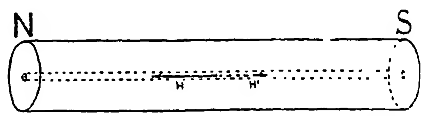


FIG. 1.—To illustrate demagnetising effect of poles in permanent magnet.

force at the middle point of the cylinder is H—H′. We may conveniently speak of H′ as the *demagnetising force*.

§ 14. Retentiveness.

As is well known, a bar of steel if introduced into a magnetic field is capable of retaining a very large amount of its magnetisation after the withdrawal of the original field which caused the magnetisation. This property of steel is spoken of as its *retentiveness*. But although the magnetism is retained, there is a field inside the bar, due to its polarity, which, as we have just seen, tends to destroy the existing magnetisation. Every magnet, in fact, which exhibits magnetic polarity over any portions of its surface tends to demagnetise itself. If the demagnetising force be considerable, then the slightest mechanical disturbance—a slight jar or blow—will cause the magnet to lose a considerable amount of its magnetisation. It is therefore evident that no magnet can be rendered practically permanent—i.e., capable of maintaining its original magnetisation in spite of mechanical or other disturbances—unless the demagnetising force is made sufficiently small. Now for a given value of the magnetic induction, the average demagnetising force along the magnet will decrease with decreasing cross-sectional area and with increasing length of the magnet; for by decreasing the transverse dimensions we decrease (the induction remaining constant) the pole-strength; and by increasing the distance apart of the two poles, we decrease the average magnetic force between them. Hence *a magnet which is intended to be permanent should have a length great in comparison with its cross-section.*

§ 15. Ageing of Magnets.

Permanent magnets are largely employed nowadays in numerous measuring instruments (ammeters, voltmeters, &c.), whose reliability depends on the constancy of the magnets. It is found that although a steel magnet will take a high degree of magnetisation, yet on withdrawing the magnetising force the magnetisation will be gradually weakened. Such a magnet will not be suitable for purposes where *constancy* is an important consideration. The magnetisation disappears at a fairly rapid rate at

first, then more and more slowly. As the magnetisation decreases, the demagnetising force also decreases, and so the magnetism becomes more stable. It will be evident from this that in order to secure constancy in a magnet the final or working magnetisation should not exceed a certain limit. The limit to which it is safe to go depends on the magnetic quality of the steel. In the case of good tungsten steel magnets,* the maximum value of B, which occurs at the middle cross-section, may reach 5,000. As it would obviously be inconvenient to wait until the magnetisation became steady in course of time, an artificial process of *ageing* is resorted to by instrument makers. The magnet is first magnetised very strongly,† and is then deprived of its unstable magnetisation by *ageing*.

The most satisfactory method of ageing magnets consists in raising the temperature of the magnet to 100° C. (by putting it into boiling water), and maintaining it at this temperature for about ten hours. Another variety of ageing consists in subjecting the magnet to a series of shocks or blows, as for example by allowing it to drop on the floor from a suitable height a sufficient number of times.

After the magnet has been aged, its magnetism is further slightly reduced by the application of an alternating magnetic force of about 15 C. G. S. units; this force may be gradually reduced to a zero value as the reversals are taking place. The object of this treatment is to render the magnet proof against any permanent change of magnetism which would otherwise occur if the magnet were exposed to stray magnetic fields.

It should be noted that stable magnetism could *not* be secured by simply imparting to the steel a weak initial magnetisation; such magnetism would entirely lack the stability secured by the process of ageing. The effect of a given demagnetising force on a magnet is, in fact, entirely different according to the way in which the

* The best permanent magnets are made of steel containing about ·6 per cent. of carbon and from 5 to 6 per cent. of the metal tungsten. Madame Curie, who carried out an important series of investigations on permanent magnets in 1897 (see *Comptes Rendus*, December 27, 1897), found that a still greater improvement could be effected by the use of molybdenum.

† The coil used in magnetising the magnet should be capable of producing a field of about 500 in the case of tungsten steel magnets, and of about 1,000 in the case of cobalt steel magnets. See paper by S. Evershed, *Journal of the Institution of Electrical Engineers*, vol. 63, p. 791 (1925).

given state of magnetisation has been reached (this is due to *hysteresis:* see § 29).

§ 16. Use of Specimen in Form of Closed Ring.

We have seen that when a given H is applied to a specimen of magnetic material, the polarity in general acquired by the specimen reacts on the original field, thereby altering the value of H. A known original value of H may be applied by the aid of a coil of suitable shape. But the demagnetising force brought into play by the magnetic polarity of the specimen cannot be accurately calculated except in a very few cases,* with the result that the actual H inside the magnetised specimen is unknown. Thus accurate determinations of the relation connecting B and H become impossible unless we adopt some method of wiping out the demagnetising force. The only way of doing this is to prevent the magnetised specimen from developing magnetic poles anywhere along its surface. One very simple and effective method of securing this result is to use the specimen in the form of a closed ring, and this is accordingly the method adopted in cases where the highest possible accuracy is desired. The ring is magnetised in a circular direction by means of a coil which is uniformly distributed around it, and since everything is symmetrical, the lines of magnetic induction inside the ring will follow circular paths concentric with the ring, and will nowhere leave it, so that there will be no formation of magnetic poles at any point on the surface of the ring, and hence no demagnetising force. The value of H at any point in the interior of the ring will be identical with that which would exist were the ring entirely removed from the interior of the ring-shaped coil.

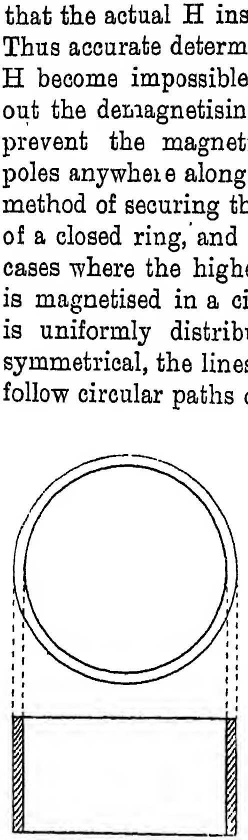


FIG. 2.—Ring for ballistic tests.

The ring used in the determination of a B-H curve may conveniently have an external diameter of about 8 inches, a radial depth of from $\frac{3}{16}$ inch to $\frac{1}{4}$ inch, and a breadth parallel to the axis of about $1\frac{1}{2}$ inches. Such a ring is shown in Fig. 2. The ring having been insulated, an "exploring" or

* Those of ellipsoids of revolution.

"secondary" coil of some 200 turns of thin silk-covered copper wire (about No. 30 S.W.G.) is wound around part of the ring (this coil need not be uniformly distributed around the entire ring). A uniformly distributed exciting coil, arranged in one or more layers, is next wound round the ring (this coil may conveniently consist of two layers of No. 16 S.W.G., or four layers of No. 18 S.W.G.).

§ 17. Calculation of H from Ampere-turns.

The value of H corresponding to any current is easily determined by the aid of the great theorem regarding m.m.f. (§ 3). If we consider any circular line of force inside the coil as the closed curve around which the m.m.f. is to be determined, then it follows from symmetry that H has the same value at every point of this circular curve, and if the radius of the circle be r cms., the m.m.f. around it is H $2\pi r$. Now if the coil conveys a current of i C. G. S. units, and contains S_1 turns, the total C. G. S. current linked with the circular line of force is $S_1 i$. Hence (§ 3),

$$H \cdot 2\pi r = 4\pi\, S_1\, i,$$

or
$$H = \frac{2\, S_1\, i}{r}.$$

If the current be i' amperes, then $i = \frac{i'}{10}$, and

$$H = \frac{2\, S_1\, i'}{10r}.$$

The product of the turns and amperes in the coil is spoken of as the *ampere-turns* of the coil. Since the m.m.f. ($4\pi\, S_1\, i = \frac{4\pi}{10} S_1\, i'$) is proportional to the ampere-turns, the *ampere-turn* is frequently used as a practical unit of m.m.f., and we clearly have

$$1 \text{ C. G. S. unit of m.m.f.} = \frac{10}{4\pi} \text{ or } \cdot 7955 \text{ ampere-turn,}$$

As an approximation sufficient for many practical purposes, 1 C. G. S. unit of m.m.f. may be taken to be equal to ·8 of an ampere-turn. Thus a given m.m.f. expressed in ampere-turns is equal to ·8 times the same m.m.f. expressed in C. G. S. units.

and 1 ampere-turn $= \frac{4\pi}{10}$ or 1·257 C. G. S. units of m.m.f.

§ 18. Units of Magnetic Force.

If we choose a unit of magnetic force such that along any closed curve the product of the mean magnetic force, expressed in terms of this unit, into the length of the curve gives us the m.m.f. in ampere-turns around it, then in terms of this unit the magnetic force will be equal to the ampere-turns, divided by the length of curve. Thus the unit of magnetic force corresponding to the ampere-turn as a unit of m.m.f. is the *ampere-turn per unit of length.* Two such units of magnetic force have been used, corresponding respectively to the cm. and the inch as units of length. The relations connecting these units with the C. G. S. unit of magnetic force are as follows :—

1 ampere-turn per cm. length = 1·257 C. G. S. units;

1 ampere-turn per inch length $= \frac{1}{2\cdot54}$ or ·3937 of an ampere-turn per cm. length = ·4947 C. G. S. unit;

and 1 C. G. S. unit of magnetic force = ·7955 of an ampere-turn per cm. length = 2·021 ampere-turns per inch.

If, therefore, we denote the magnetic force when expressed in C. G. S. units by H, then the factors converting H into ampere-turns/cm. and ampere-turns/inch are ·7955 and 2·021 respectively.

The C. G. S. unit of magnetic force has been termed a *gauss.**

§ 19. Variation of H over Cross-section of Ring.

The expression obtained in § 17 for the value of H inside our ring-shaped coil shows that H is not uniform over the cross-section of the ring, but decreases in the inverse ratio of the distance from the axis of the ring as we proceed from its inner to its outer boundary. This variation of H is graphically represented in Fig. 3 for a ring the ratio of whose external and internal radii

* In honour of the great German mathematician *Gauss,* who was one of the earliest workers in the theory of magnetic units and measurements.

is 10 : 6. It will now be understood why in using ring-shaped specimens it is advisable to make the radial thickness of the ring small in comparison with its mean radius. For in that case the value of H will vary but little over the cross-section of the ring, and we may without serious error assume it to be nearly uniform and equal to the value which it has along the mean circle. We therefore assume the value of H to be given by

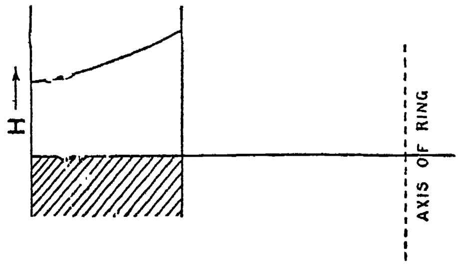


FIG. 3.—Variation of H over cross-section of ring.

$$H = \frac{2\, S_1\, i'}{10 r_m},$$

where r_m is the mean radius of the ring, in cms., i' the current in amperes, and S_1 the total number of turns in the exciting coil.

§ 20. Determination of B.

Let the cross-section of the ring be a sq. cms., and let us suppose that the induction obtained in the ring with a certain current flowing round the exciting coil is B. Then the total number of magnetic lines, or the *total magnetic flux*, linked with each turn of either coil is Ba, and if the number of turns in the secondary or exploring coil (§ 16) be S_2, the total flux linked with this coil = flux per turn × number of turns = BaS_2. Let this coil be connected to a ballistic galvanometer (i.e., one having a long period or time of swing), and, if necessary (in order to reduce the swing or throw to a readable amount), in series with an additional resistance, the connections being arranged as in Fig. 4

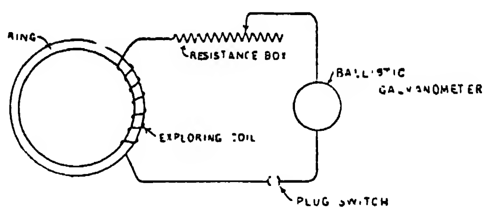


FIG. 4.—Connections of secondary circuit in ballistic ring tests.

(in which the exciting or primary coil is omitted, for the sake of simplicity). Let the *total* resistance of this secondary circuit, including secondary coil, galvanometer and additional resistance, be r' ohms, or $10^9 r'$ C. G. S. units. Let now the exciting current be reversed, so that the flux linked with the secondary coil changes from $+ \mathrm{B}aS_2$ to $- \mathrm{B}aS_2$, corresponding to a total change of flux $= 2\,\mathrm{B}aS_2$. In accordance with the principle explained in § 6, this change of flux will cause a discharge of $\frac{2\,\mathrm{B}aS_2}{10^9 r'}$ C. G. S. units of electric quantity through the galvanometer, producing a momentary deflection or "throw" of a certain number of scale divisions. If the galvanometer has been standardised as explained below, the quantity q (in C. G. S. units) corresponding to this number of scale divisions is known, and hence we have

$$q = \frac{2\,\mathrm{B}aS_2}{10^9 r'},$$

or

$$\mathrm{B} = \frac{10^9 r'}{2aS_2} q,$$

which enables us to calculate B.

§ 21. Standard Solenoid.

The most satisfactory method of standardising a ballistic galvanometer is by means of a standard solenoid. Such a solenoid may be constructed by winding about four or six layers of No. 16 S.W.G. double cotton-covered copper wire on a tube of vulcanised fibre or other insulating material, about 3 feet long and 3 inches in external diameter, care being taken to make the winding perfectly even and uniform. The value of H inside such a solenoid is practically uniform except in the vicinity of the ends, and may be easily calculated by an application of the great theorem regarding m.m.f. (§ 3). For if we take a closed rectangle A B C D, as shown in Fig. 5 (where,

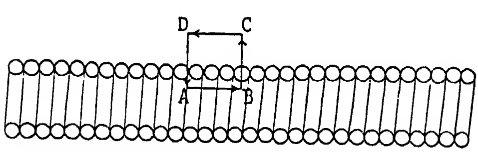


FIG. 5.—To illustrate determination of H inside solenoid.

for the sake of simplicity, the solenoid is shown as consisting of a single layer of wire), with the sides A B and C D parallel to the axis of the solenoid, then since (so long as we are not too near the ends) there are no components of magnetic force along the radial directions B C and A D, the m.m.f. around the rectangle is

$$H \times AB + H' \times CD,$$

where H is the magnetic force inside the solenoid, and H′ that along C D, outside the solenoid. If we make A B = C D = 1 cm., then the m.m.f. becomes H + H′. But since H′ in the case of a long solenoid is negligible in comparison with H, we may take the m.m.f. around the rectangle considered to be equal to H. Applying now the principle of § 3, we have

$$\text{m.m.f.} = H = \frac{4\pi}{10} S i',$$

where S is the number of turns linked with the rectangle, and i' the current in the solenoid, in amperes. Now since A B = 1 cm., $S i'$ represents the ampere-turns per cm. length of solenoid. Thus, at any point in the interior of the solenoid not too near its ends, we have

$H = 1{\cdot}257 \times$ ampere-turns per cm. length of solenoid . . . (1).

A secondary coil is next constructed, by winding a suitable number of turns (of, say, No. 30 S.W.G. double silk-covered copper wire) in a single layer on a carefully turned bobbin of marble* having a diameter of 2 or 2½ inches. The exact diameter of the bobbin is carefully measured before winding it and also over the winding, and the mean of the measurements is taken to be the mean diameter of a turn of the secondary coil. From this the mean area of a turn is obtained. The number of turns in the coil may be from 100 to 500.

§ 22. Calibration of Ballistic Galvanometer.

The secondary coil is slipped into the standard solenoid so as to occupy a position of symmetry, and is connected in series with the galvanometer to be calibrated and an additional resistance of such amount as to make the total resistance of the secondary

Boxwood or other well-seasoned hard wood might be used instead of marble if price is a consideration, but marble is preferable as being less liable to change its form.

circuit equal to r' in § 20—the total resistance of the secondary circuit used in connection with the ring test. A known current is then sent through the solenoid, and (the galvanometer being allowed to come to rest) the current is suddenly reversed. Then the throw obtained corresponds to the discharge of a quantity q_0 given, as in § 20, by

$$q_0 = \frac{2 \cdot H a_0 S_2}{10^9 r'},$$

where H is given by (1) of § 21, a_0 = mean area of each secondary turn, S_2 = number of turns in secondary coil.

In many cases, the throws will be found to be nearly proportional to the quantities, but the law of proportionality should not be assumed, and the galvanometer should be calibrated throughout its entire scale by varying the current through the standard solenoid, and plotting a curve connecting throw with quantity.

The most suitable form of galvanometer for use in connection with the above measurements is a *moving-coil* galvanometer.

§ 23. Precautions to be observed in Ballistic Ring Test.

In order to secure consistent results, certain precautions must be observed. The ring should first be magnetised very strongly, and the current reversed a number of times; the current reversals are then continued while at the same time, by gradually introducing resistance into the exciting circuit, the current is slowly weakened and finally reduced to zero. After this preliminary magnetic treatment of the ring, the connections for obtaining the B-H curve may be made and the readings commenced, the current being in each case reversed several times before the galvanometer throw is taken.

§ 24. Numerical Data for Materials used in Dynamo Construction.

The most important materials used in dynamo construction are cast iron, dynamo cast steel, and the "Swedish" sheet iron or mild sheet steel employed in the construction of armature

cores (§ 64). The relations connecting B with H for these three materials are given in the following table:—

H	B			H	B		
	Cast Iron.	Dynamo Steel Castings.	Armature Stampings.		Cast Iron.	Dynamo Steel Castings.	Armature Stampings.
2	1,400	1,800	2,500	32	6,590	16,390	15,600
4	2,500	6,800	6,000	34	6,680	16,500	15,700
6	3,400	9,300	7,500	36	6.760	16,600	15,790
8	3,900	11,000	9,000	38	6,830	16,690	15,870
10	4,400	12,200	10,500	40	6,890	16,780	15,940
12	4,800	13,100	11,500	45	7,040	16,950	16,070
14	5,200	13,750	12,400	50	7,190	17,100	16,190
16	5,500	14,250	13,200	55	7,340	17,250	16,300
18	5,700	14,700	13,900	60	7,490	17,400	16,400
20	5,850	15,000	14,500	70	7,790	17,700	16,600
22	6,000	15,300	14,800	80	8,050	18,000	16,800
24	6,150	15,600	15,000	90	8,290	18,200	17,000
26	6,280	15,850	15,180	100	8,500	18,400	17,180
28	6,400	16,060	15,340	130	9,100	19.000	17,600
30	6,500	16,240	15,480	200	10,150	20,000	18,300

The ratio $\mu = \frac{B}{H}$ is defined to be the permeability of the material for the given value of B or H.

A material which has within recent years been sometimes used in the construction of armature cores is the special alloy of iron and silicon (containing about $3\frac{1}{2}$ per cent. of silicon) known as "stalloy." For this alloy, the following table gives the relation connecting H and B:—

H	1	2	3	4	5	6
B	5,000	7,900	9,750	10,600	11,100	11,560

H	8	10	12	15	20	30
B	12,150	12,600	13,000	13,400	13,580	14,580

H	40	50	70	100	150	200
B	15,000	15,340	15,920	16,660	17,540	18,000

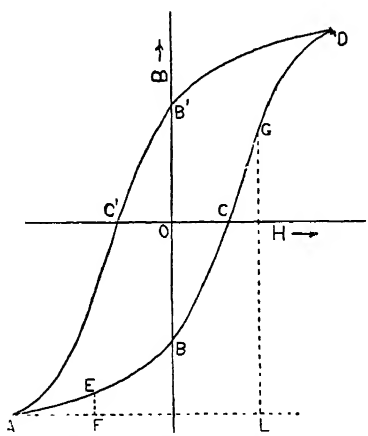


FIG. 6.—Hysteresis loop.

§ 25. Hysteresis Loop.

When a reversal of current takes place in the magnetising coil surrounding the ring, the law according to which the induction changes during this reversal is found to be represented by a curve such as the curve A B C D in Fig. 6, the curve not passing through the origin. If the current be again reversed, so as to reduce the specimen to its original magnetic state (represented by the point A in Fig. 6), it follows by symmetry that the curve along which the induction passes from D to A must be similar to the curve A B C D, and must therefore be represented by D B′ C′ A, the latter curve being obtained by a rotation of A B C D about O as centre through 180°. Thus during a double reversal, which restores the specimen to its original state, and which may therefore be spoken of as a complete *magnetic cycle* of changes, the point connecting the induction with H traces out a closed loop. This loop is spoken of as a *hysteresis loop*. It will now be seen that in determining the B-H curve by the method of reversals explained above we are really finding the positions of the peaks (A or D) of a succession of hysteresis loops.

§ 26. Determination of Hysteresis Loop.

In order to trace out a complete hysteresis loop, it will obviously be sufficient to determine one branch (such as A B C D in Fig. 6) of it, the other branch being simply obtained by a rotation through 180° of the first branch about O as centre, as already stated. The principle of the method employed is precisely similar to that underlying the determination of the B-H curve, the only difference being that instead of producing a complete reversal of current, we start from the point A, and by suddenly introducing resistance into the circuit diminish the current, thereby passing from A to some point such as E along the portion A B of the loop. The throw of the galvanometer now measures F E, which corresponds to the sudden change of induction. Having obtained the throw, we complete the reversal by passing from E to D, and then produce another full reversal, thus getting back to A. In this way a number of points along the portion A B of the loop may be obtained. In order to deter-

mine points along B C D, we have to arrange matters so that when the reversing switch is thrown over, a resistance is introduced which only allows a smaller reverse current to flow through the coil. We thus reach points such as G, the galvanometer throw measuring the change of induction G L. As before, the reversal is completed by short-circuiting the extra resistance, so as to reach the point D, when another complete reversal brings us back to A. By this means, starting each time from A, we determine a number of points along the portion B C D of the loop.

The diagram of the primary or exciting coil connections used in tracing a hysteresis loop is shown in Fig. 7. The connections of the secondary or exploring coil are the same as those shown in Fig. 4. The method of experimenting is as follows. The variable resistances R_2 and R_3 being short-circuited, and the reversing switch being in the position shown by the full lines, R_1 is adjusted until the current has the value corresponding to the peak of the hysteresis loop (A in Fig. 6). The resistance R_2 is then unshort-circuited, and adjusted to the value required to reduce the current to the desired amount. The reversing switch is next thrown over into the position shown by the dotted lines in the diagram, the resistance R_2 is short circuited, and the reversing switch thrown backwards and forwards a number of times, being finally left in the forward position corresponding to the full lines. The short-circuiting plug of the resistance R_2 is now suddenly pulled out and the galvanometer throw observed. In this way, a number of points along the portion A B of the hysteresis loop are determined. In order to find points along B C D, the resistance R_2 is short-circuited, R_3 unshort-circuited, and, with the switch in the dotted position, the current is adjusted to the desired positive value. R_3 is next short-

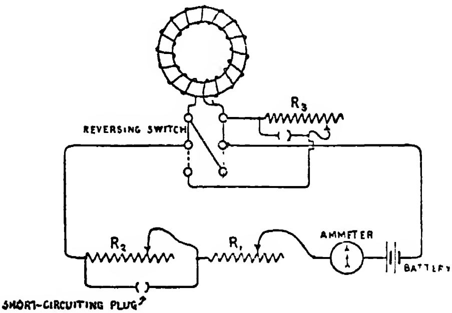


FIG. 7.—Connections of primary circuit for determining hysteresis loop.

circuited, and, after a number of reversals, the switch is left in the full-line position. R_8 is now unshort-circuited, and the switch thrown over into the dotted-line position, the galvanometer throw being observed.

§ 27. Calculation of Energy Dissipated during a Magnetic Cycle.

We shall next show that during each magnetic cycle a certain amount of energy is dissipated in every cubic cm. of the material. Let us suppose that, starting from the peak A of the hysteresis loop (Fig. 8), we have decreased H to the value P Q, the induction having the value O Q. Let now a further reduction of H from P Q to R S take place, the induction decreasing from O Q to O S, i.e., by an amount Q S. Let us suppose that this decrease is effected during t seconds. Then, in accordance with what has been said in § 5, there will be an e.m.f. induced in each turn of the exciting winding of amount $\frac{a \cdot Q\,S}{t}$, and since the total number of turns is S_1, the total e.m.f. induced in the winding is (in C. G. S. units) $S_1 \frac{a \cdot SQ}{t}$.

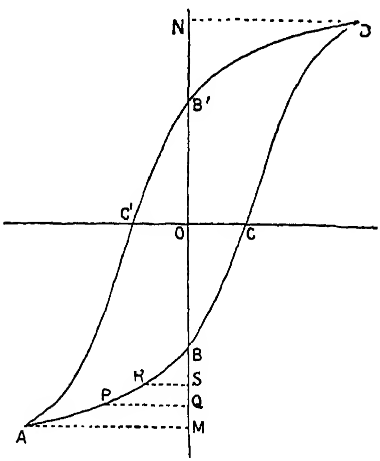


FIG. 8.—To illustrate dissipation of energy by hysteresis.

Again, this e.m.f. has, by Lenz's law (§ 5), the same direction as the current, since the current is decreasing. Hence it will help to maintain the current, and will supply energy to the circuit. The rate of supply of energy, or power, corresponding to this induced e.m.f., is given by the product (§ 1) of e.m.f. into current. Now (§ 17) the current i and the magnetic force H are connected by the relation

$$i = \frac{H\,l}{4\pi\, S_1},$$

a being, as in § 20, the cross-sectional area of the ring, and hence $a \cdot Q\,S$ = change of magnetic flux through each turn.

where $l = 2\pi r_m$ is the mean circumference of the ring. During the small change considered, the current changes by a small amount, and we may use its mean value during the change in calculating the power. But the mean value of H (Fig. 8) is $\frac{1}{2}$ (P Q + R S). Hence the mean value of the current is, by the above formula,

$$\tfrac{1}{2}(\mathrm{P\,Q} + \mathrm{R\,S})\,\frac{l}{4\pi\,\mathrm{S}_1},$$

and the mean power due to the induced e.m.f. is

$$\mathrm{S}_1\,\frac{a\,.\,\mathrm{Q\,S}}{t}\,.\,\tfrac{1}{2}(\mathrm{P\,Q} + \mathrm{R\,S})\,\frac{l}{4\pi\,\mathrm{S}_1} = \frac{la}{4\pi}\,.\,\frac{\mathrm{Q\,S}}{t}\,.\,\tfrac{1}{2}(\mathrm{P\,Q} + \mathrm{R\,S}).$$

Since this power is maintained during t seconds, the energy supplied to the circuit during that time is

$$\text{power} \times \text{time} = \frac{la}{4\pi}\,.\,\mathrm{Q\,S}\,.\,\tfrac{1}{2}(\mathrm{P\,Q} + \mathrm{R\,S}).$$

Now la represents the volume of the ring, in cubic cms. Hence the energy contributed by every cubic cm. of the ring during the small change considered is

$$\frac{1}{4\pi}\,.\,\mathrm{Q\,S}\,.\,\tfrac{1}{2}(\mathrm{P\,Q} + \mathrm{R\,S}) = \frac{1}{4\pi} \times \text{area of strip P Q S R},$$

this area being measured with due attention to the scales of H and B used in plotting the diagram.

Since similar reasoning applies to every consecutive small change in H and B, it follows that in passing from the point A to the point B in Fig. 8, we recover or gain from every cubic cm. of the material an amount of energy which, expressed in ergs, is equal to

$$\frac{1}{4\pi} \times \text{area A M B}.$$

If we now pass from B through C to D, it is evident that since the induced e.m.f. retains its original direction, while the current has changed sign, the e.m.f. will be an *opposing* or counter-e.m.f., and will act as a sink instead of as a source of energy, work having to be done *against* it by the battery. Applying, therefore, the reasoning previously used in connection with the portion A B of the loop to the portion B C D, we find that, in passing from B to D, we have to communicate to every cubic cm. of the ring

an amount of energy equal to $\frac{1}{4\pi} \times$ area B D N.

Similarly, considering the descending branch D B′ A of the loop, we find as before that in passing from D to B′ we gain or recover an amount of energy equal to $\frac{1}{4\pi} \times$ area B′ D N, and in passing from B′ to A we lose or store up an amount equal to $\frac{1}{4\pi} \times$ area B′ A M.

Hence during a complete magnetic cycle there is a balance of *lost* or unrecovered energy, representing energy dissipated in producing heat in the ring, of amount

$$\frac{1}{4\pi} \times \text{area A B D B}' \text{ A of the hysteresis loop}$$

per cubic cm. of the material. It is to be particularly noted that in measuring the area of the loop due account must be taken of the different scales used for B and H.

§ 28. Steinmetz's Law.

The great practical importance of the above result lies in the fact that in many practical applications of electro-magnetism we have to deal with large masses of iron which are rapidly undergoing periodic magnetic reversals. Such cases occur in connection with dynamos, motors, transformers, &c. Both the efficiency and the temperature rise of the apparatus will depend on the rate at which energy is being converted into heat by hysteresis. Now since, as we have seen, the energy dissipated per cycle is proportional to the area of the hysteresis loop, the question arises as to the connection which exists between this area and the maximum value of B corresponding to the peaks of the loop. This problem was first investigated by C. P. Steinmetz, who found that, for values of B up to about 9,000, the following relation holds good—

$$\text{energy, in ergs, dissipated per c.c. per cycle} = \eta\, B^{1\cdot6},$$

where η is a constant depending on the material, and is known as Steinmetz's *hysteretic co-efficient*. The above relation is known

as *Steinmetz's Law*. In the case of the sheet iron or sheet steel used in the construction of transformer and dynamo armature cores, η ranges from ·001 to ·0012.

28a. Kennelly's Law.

A simple approximate relation connecting the reciprocal of the permeability (or *reluctivity*, § 31) with the magnetic intensity when the intensity exceeds the value corresponding to maximum permeability was discovered by A. E. Kennelly. This relation, which is frequently referred to as Kennelly's law, is given by the equation

$$\frac{1}{\mu} = a + b\,\mathrm{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where a and b are constants for a given material.

Instead of this simple form of equation, the equation

$$\frac{1}{\mu - 1} = a + b\,\mathrm{H} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

may be used, as it gives better agreement with experimental results.*

Kennelly's law is very useful for extrapolation purposes, as, having determined the values of a and b by using an experimental curve which does not carry us to very high values of B or H, we can extend the curve to such values by the aid of (2).

A careful study of experimental results has established the fact that in the case of most materials the relation connecting $\frac{1}{\mu - 1}$ and H, instead of being represented by a single straight line in accordance with (2), is in reality represented by a broken line consisting of two straight lines making a small angle with each other.

Examples.

1. A ring of wood, whose external and internal diameters are respectively 10 inches and 8 inches, and whose length in an axial direction is 4 inches, is uniformly wound over with 800 turns of insulated wire. Find the values of H at the external and internal surfaces of the ring, and the

* According to (1), as H is indefinitely increased, μ approaches a zero value, which is incorrect; according to (2), however, it approaches a value of unity, as should be the case.

total magnetic flux through the cross-section of the ring, when a current of 5 amperes is sent round the coil.

2. The difference of magnetic potential between the polar surfaces of a permanent magnet is 300 ergs per unit pole. If the length of the magnet be 8 inches, what is the mean value of the demagnetising force?

3. A standard solenoid contains 20 turns of wire per cm. of its length. A secondary coil the mean circumference of each turn of which is 4·7 inches, and which consists of 600 turns, is placed inside the standard solenoid with the axes of the two coils parallel, and is connected in series with a galvanometer whose resistance is 15 ohms, and an additional resistance of 2,967 ohms. The resistance of the secondary coil is 18 ohms. If a current of 2 amperes circulating round the solenoid be suddenly interrupted, what is the quantity, in micro-coulombs, discharged through the galvanometer?

4. A coil of 500 turns, connected in series with a suitable resistance and a ballistic galvanometer, is placed at the middle of a bar magnet of cross-section 1 inch $\times$ $\frac{1}{4}$ inch, the coil being wound on a light rectangular frame which fits easily over the magnet. The magnet is suddenly withdrawn, and a throw of 180 scale divisions is observed. If the galvanometer constant is ·1 micro-coulomb per scale division, and if the total resistance of the galvanometer circuit be 1,000 ohms, what is the induction at the middle of the magnet?

5. The value of Steinmetz's hysteretic coefficient is ·0015 for a certain kind of iron, the specific gravity of which is 7·7. Find the power, in watts, dissipated by hysteresis in 220 lbs. of this iron when subjected to magnetic reversals between the limits of B $\pm$ 3,000, the number of cycles per second amounting to 25.

CHAPTER III.

§ 29. Hysteresis—§ 30. Effects of hysteresis in soft iron instruments—§ 31. The magnetic circuit—§ 32. Differences between electric and magnetic circuits—§ 33. Determination of ampere-turns to produce given flux—§ 34. Magnetic leakage—§ 35. Excitation with constant current and at constant p.d.—§ 36. Stress between magnetised surfaces—§ 36a. Permanent magnets—§ 36b. Steels for permanent magnets. Examples.

§ 29. Hysteresis.

In considering the behaviour of a magnetic material when subjected to a series of recurrent cycles of magnetisation between given limits of magnetic force (or induction), we found that the path followed by the curve connecting B and H consisted of two distinct portions enclosing a loop. This case is only one example of the characteristic behaviour of magnetic materials when subjected to the action of a varying magnetic force. We find that in all such cases the material responds much more readily to a change in the magnetic force which is in the same direction as the immediately preceding changes than to a change taking place in the opposite direction. This general tendency of a material to respond less readily to a change of magnetic force when the change takes place with reversal of sign is spoken of as *hysteresis*. If, for example, we suppose that the specimen is originally demagnetised, and that we apply to it a *steadily increasing* magnetic force, then the curve connecting B and H is the normal B-H curve. But if at any stage of the process, such as the point P in Fig. 9, we *decrease* H, then the series of magnetic changes immediately preceding is *not retraced:* the curve connecting B and H leaves the normal B-H curve, the material exhibiting a more or less strong tendency to oppose the change from increment to decrement of magnetic force. If the magnetic force is decreased to zero, the induction retains a considerable value—represented by O R in Fig. 9 and spoken

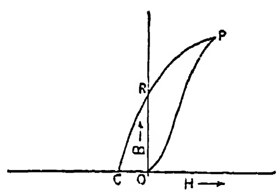


FIG. 9.—Residual induction and coercive force.

of as the *residual* induction—and in order to reduce the induction to zero a definite negative magnetic force must be applied to the specimen. This magnetic force, known as the *coercive force*, is represented by O C in Fig. 9. It must be understood that both the residual induction and the coercive force depend on the maximum induction previously reached by the specimen, and that they increase with increase of this maximum induction.

If the applied magnetic force varies irregularly, increasing, decreasing and again increasing, the material behaves in the manner represented by the curve in Fig. 10. It will be seen that, for a given value of H, the value of B is indeterminate, depending on the previous magnetic history of the specimen. We might define hysteresis as that property in virtue of which a magnetic material is capable, under the action of a given magnetic force, of assuming an infinite number of values of B lying between certain limits, the particular value of B assumed by the specimen depending on the manner in which it has been reached.

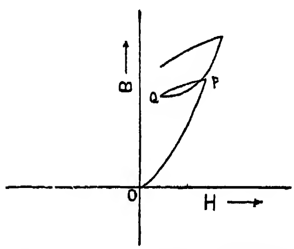


FIG. 10.—To illustrate hysteresis effects.

From what has just been said it will be seen that in general there is no perfectly definite relation connecting B and H. Out of the infinite number of values of B which are possible with a given value of H, we select, in carrying out a magnetic test, that particular value of B which is reached by starting with the specimen thoroughly demagnetised, and by steadily increasing H (without allowing any decrease to take place) until the given value is reached. The particular curve so obtained is the ordinary or normal B-H curve or magnetisation curve, and it approximately represents the locus of the peaks of consecutive hysteresis loops.

When the maximum induction reached by the specimen exceeds a certain limit, any further increase of maximum induction does not appreciably increase either the residual induction or the coercive force, which acquire practically constant values. It is these maxima values that are meant when the terms *residual induction* and *coercive force* are used without any qualification—they denote the highest values reached by magnetising the material as strongly as possible. These highest values of the residual induction and coercive force are also termed the *remanence* and *coercivity* respectively of the material. Numerical values of remanence and coercivity for the more important magnet steels are given in § 36b.

Although over a very wide range of values of H the effects of hysteresis are strongly marked, there are two special conditions under which the specimen may, within certain narrow limits of H, exhibit no appreciable hysteresis—i.e., yield a definite value of B for a given value of H (the loop P Q in Fig. 10 in this case collapsing into a single line). This happens when H has either a *very small* (less than 1 C. G. S. unit) or a *very large* (>100) value.

§ 30. Effects of Hysteresis in Soft Iron Instruments.

The effects of hysteresis become particularly troublesome in connection with measuring instruments (ammeters, voltmeters, supply meters) in which soft iron is used. Owing to hysteresis, an instrument of this type cannot be made to read quite correctly, and always gives a higher reading with decreasing than with increasing current. The error arising from hysteresis may, however, be reduced to a permissible value for a commercial instrument by one or other of the following expedients: (1) the use of a very weak field; (2) the use of a very strong field; (3) some arrangement for compensating the effect of hysteresis.

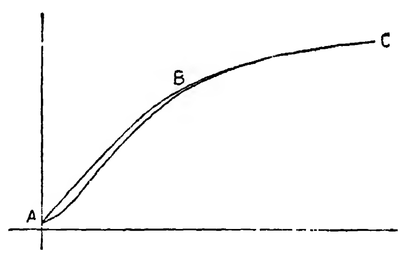


FIG. 11.—Disappearance of hysteresis in strong fields.

When a very strong field is used, the lower part of the scale of the instrument is not reliable, as it corresponds to the region within which hysteresis effects are very strongly marked. This region is represented by A B in Fig. 11; beyond B, the two branches of the hysteresis loop nearly coalesce, so that for any points in the region B C the value of B, and hence the reading of the instrument, will be nearly the same with an ascending as with a descending current. When strong fields are used, it is an obvious advantage to shape the iron part of the instrument so as to reduce the demagnetising effect due to its ends as much as possible—i.e., it is an advantage to make the length of the iron part large in comparison with its cross-section; since a smaller

number of ampere-turns will then be sufficient to produce the necessary field.

A serious trouble with instruments in which weak fields are employed is that if accidentally a very much stronger current than that corresponding to the highest reading be sent through the instrument, the strong residual magnetism imparted to the iron subsequently causes the instrument to give readings which are much too high. In many instruments, this difficulty is met by making the length of the iron part short in comparison with its cross-section, so as to make the demagnetising effect due to the ends (§ 13) very large, and thus prevent the iron from retaining an excessive amount of magnetism.

In some cases, however, the last-mentioned arrangement cannot be conveniently used. Recourse may then be had to some sort of compensating device. One such device, used in the old type of Ferranti meter, is illustrated in Fig. 12. In this instrument, the magnetic field in the gap is required to be as nearly as possible proportional to the exciting current, and a weak residual field is also required. In order to secure the former condition—proportionality between current and field—the induction is normally kept at a low value; and in order to provide a sufficiently strong residual field, the central portion of the cylindrical core is made of hard steel. The accidental passage of an abnormally strong current would, however, make the residual field so strong as to cause the readings to be much too high. This is prevented by fitting the central steel core with an outer sleeve of soft iron. Even if the central core does get strongly magnetised, most of the lines of induction will, when the current is arrested, pass from the steel core into the soft iron sleeve, whose magnetisation becomes reversed. The sleeve acts, in fact, as a keeper towards the steel core, and prevents the excessive crowding of the magnetic lines of the core into the gap, the lines being diverted into the sleeve.

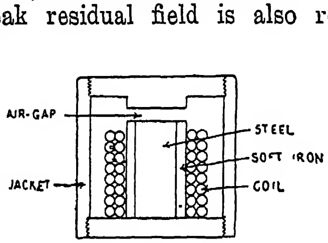


FIG. 12.—Method of compensating hysteresis.

See, in this connection, *The Electrician*, vol. xxxii., pp. 11, 93, and 186 (1893).

31. The Magnetic Circuit.

Returning to the consideration of a ring (§ 16) uniformly wound with S turns of insulated wire, we have, if l = mean circumference of ring, i = exciting current, in amperes, Φ = magnetic flux through cross-section of ring, and a = cross-sectional area,

$$H\, l = 1{\cdot}257\, S\, i \quad . \quad . \quad . \quad (1)$$

and

$$\Phi = B\, a = \mu\, H\, a,$$

μ being the value of the permeability (§ 24) at induction B.

We thus have

$$\Phi = \frac{\mu a}{l} . \; H l$$

$$= \frac{1{\cdot}257\, S\, i}{\frac{1}{\mu} \cdot \frac{l}{a}} \quad . \quad . \quad . \quad (2),$$

by (1).

Consider next an electric circuit consisting of a conductor of length l, cross-section a, and resistivity ρ. If the e.m.f. around this circuit be denoted by E, then the current i is given by

$$i = \frac{E}{r} = \frac{E}{\rho \frac{l}{a}} \quad . \quad . \quad . \quad . \quad (3),$$

r being the resistance of the circuit.

There is a great external similarity between equations (2) and (3), which has led to the adoption of a suggestive analogy between the magnetic case represented by (2) and the electric case represented by (3). Every magnetic line forms a closed loop: every electric current flows round a closed circuit. The assemblage of closed magnetic lines is usually spoken of as the *magnetic circuit*. The flow of a current is caused by the application of an e.m.f. to an electric circuit; a magnetic flux is produced by the application of an m.m.f. to a magnetic circuit. The ratio $\frac{\text{e.m.f.}}{\text{current}}$ is defined to be the electric resistance of the circuit; the

ratio $\frac{\text{m.m.f.}}{\text{magnetic flux}}$ is defined to be the *reluctance* of the magnetic circuit. Corresponding to the relation: resistance = resistivity $\times \frac{\text{length}}{\text{cross-section}}$, we have: reluctance $= \frac{1}{\mu} \times \frac{\text{length}}{\text{cross-section}}$, and $\frac{1}{\mu}$ is by analogy termed the *reluctivity* of the material. The reluctivity is thus the reciprocal of the permeability.

§ 32. Differences between Electric and Magnetic Circuits.

In spite of this apparent close similarity between an electric and a magnetic circuit, there are two important points in which the analogy entirely breaks down. In the first place, Ohm's law—the constancy of the resistivity of a conductor maintained at a fixed temperature—does not hold in the case of magnetic substances, whose reluctivity varies with the induction. And in the second place, there is no such thing as a magnetic insulator: we are, in general, unable to confine the magnetic flux to a definite channel.

The two important differences just mentioned render problems relating to the magnetic circuit much more complicated than those relating to the electric circuit. The magnetic problem arising most frequently in practice is that of determining the ampere-turns necessary to produce a given magnetic flux through a given area. In general, the magnetic circuit is, unlike the simple case of the closed ring, a *heterogeneous* one—i.e., it is made up of several sections having different magnetic permeabilities. The method of solution adopted is as follows:—

§ 33. Determination of Ampere-turns to Produce given Flux.

A closed line is drawn, to scale, on a sheet of paper to represent, as nearly as possible, the average path of the magnetic lines. Let, in Fig. 13, A B C D represent this line. The cross-sections of the various portions A B, B C, C D and D A

Thus, in the case of a simple closed ring such as we have repeatedly considered, this line would be the mean circumference of the ring.

being known, as well as the magnetic fluxes, we can calculate the value of B $\left(=\frac{\text{flux}}{\text{cross-section}}\right)$ for each portion. The magnetisation curves for the various substances must also be known. By reference to these, we find the values of H or, more conveniently, of the ampere-turns per unit of length, corresponding to the various portions. By multiplying the ampere-turns per unit of length by the total length of each portion, we find the drop of magnetic potential, in ampere-turns, over that portion; the sum of all such drops gives us the total ampere-turns required to produce the given flux.

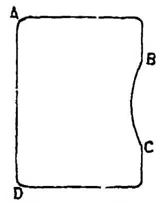


FIG. 13.—Diagram of magnetic circuit.

§ 34. Magnetic Leakage.

The problem, therefore, would admit of a very accurate solution if the flux through each portion of the magnetic circuit were known with accuracy. Such, unfortunately, is but seldom the case, owing to the non-existence of a magnetic insulator. The case of a closed ring which we have already studied in detail is very exceptional in that the magnetic flux is confined to a definite channel—none of the lines straying outside the ring. When such straying does take place, we speak of it as *magnetic leakage*, and a magnetic circuit in which leakage of lines takes place is described as a *leaky* circuit. Most circuits occurring in practice exhibit strong leakage. As an example of a leaky magnetic circuit, we may consider the simple case shown in Fig. 14, where an iron ring is shown provided with an exciting coil *concentrated* over a small portion of the ring, the ring being cut open on the side diametrically opposite to the coil, an air-gap being thereby formed. Let the object of the arrangement be to produce a definite flux through an area *a*

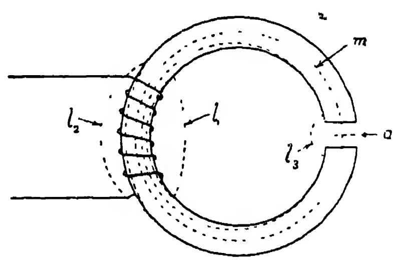


FIG. 14.—To illustrate magnetic leakage.

equal to the cross-section of the ring and placed symmetrically between the poles. The mean path of the flux is indicated by the dotted circle marked m, and it falls into (1) the portion across the air-gap and (2) that inside the iron. Now in addition to the magnetic lines passing through a, where they are wanted, there are others, such as those marked l_1, l_2 and l_3, which do not reach a, and which, so far as the object of the arrangement (viz., the production of a definite flux through a) is concerned, are useless. These useless lines form the *leakage flux*. It will be seen that leakage takes place continuously along the entire surface of the ring as we proceed from the middle of the exciting coil towards either pole, and that the maximum flux occurs through the cross-section at the middle of the coil. The ratio of this maximum flux to the useful flux through a is termed the *leakage coefficient*. Owing to the continuity of the leakage, it is obvious that the value of B varies from point to point of the mean path m. As it would be extremely troublesome to take into account this variation, we generally content ourselves with assuming—in order to be on the safe side—that the value of B remains equal to its maximum value (at the middle of the coil) as we proceed along the portion of m which lies in iron. If ν = leakage coefficient, and if Φ = total useful flux through the area a, then we assume the flux through the iron part of the circuit to be equal to $\nu\Phi$, and (which is not correct) *to remain constant at all the cross-sections*. Assuming the leakage coefficient to be known, we can calculate the ampere-turns by the method explained in § 33.

§ 35. Excitation with Constant Current and at Constant P.D.

The ampere-turns required to produce a given flux in a magnetic circuit have to be provided under one or other of two conditions, viz., with a given current through the exciting coil, or with a given p.d. across its terminals. When the current is known, the number of turns is immediately given by the quotient $\frac{\text{ampere-turns}}{\text{amperes}}$, and the size of wire is chosen with reference to the permissible temperature rise (§ 124). When however, the p.d. across the coil is given, the ampere-turns are,

as we shall see presently, determined by the *size* of wire. Let V = p.d., and r = resistance of the mean turn. Then amperes

$$= \frac{V}{\text{total resistance}} = \frac{V}{r \times \text{No. of turns}}. \quad \text{Hence}$$

$$r = \frac{V}{\text{amperes} \times \text{turns}} = \frac{V}{\text{ampere-turns}}.$$

We thus arrive at the important result that, in order to obtain a given number of ampere-turns at a given p.d., the size of wire must be so chosen that *the mean resistance of a turn equals the quotient of the p.d. by the ampere-turns.*

§ 36. Stress between Magnetised Surfaces.

The stress which exists between two magnetised surfaces of opposite polarity when these are either in contact with each other or are separated by a short air-gap is practically applied in the construction of lifting electro-magnets for handling iron and steel castings, plates, &c. In designing such a magnet, we have to determine the stress intensity, or the stress per unit area, corresponding to a given value of B. If we imagine the surfaces separated by a short gap, then the force which would be exerted on a unit pole placed in the gap is (by the definition of induction, § 10) equal to B. This force may be regarded as due to the joint action of the two magnetised surfaces, each surface contributing half the total amount, i.e., $\frac{1}{2}$ B. If now we imagine our unit pole placed *on* one of the surfaces, this surface will clearly exert no force on the unit pole in a normal direction; hence the force is only that due to the opposing surface—i.e., $\frac{1}{2}$ B. Thus the force per unit pole placed on either surface is $\frac{1}{2}$ B. But since the number of magnetic lines proceeding from a unit pole is 4π, and since there are B lines going across from every sq. cm. of one surface to every sq. cm. of the other, it follows that the pole-strength of every sq. cm. of either surface is $\frac{B}{4\pi}$. Now since the force corresponding to every unit pole-strength is, as we have

With a given p.d., the m.m.f. is *independent of the number of turns* (so long as the mean length of a turn remains the same). Increasing the turns reduces the exciting current, and hence the power required to maintain the excitation, but makes the coil more expensive.

just seen, $\frac{1}{2}$ B, it follows that the total force exerted on every sq. cm. of either surface is $\frac{B}{4\pi} \times \frac{B}{2} = \frac{B^2}{8\pi}$. *The stress between two magnetised surfaces is thus seen to be* $\frac{B^2}{8\pi}$ *dynes per sq. cm.*

§ 36a. Permanent Magnets.

Permanent magnets now play so important a part in various branches of electrical engineering that a great deal of study has within recent years been devoted to their behaviour under various conditions, and to the production of the most suitable materials for their manufacture. Besides being used in telephone receivers, relays and many types of measuring instruments, they are employed in the construction of the numerous types of small generators, known as *magneto-generators* (or "magnetos" simply), which are used for various purposes; one of their most important applications being the ignition of the explosive charge in the cylinders of internal combustion engines.*

Permanent magnets are used for the purpose of producing a given magnetic flux over a given area. For the sake of simplicity, we shall consider the case of a simple *ring* magnet, such as that shown in Fig. 14A, having an air-gap in which a certain flux Φ_a is required.† The length of air-gap l_a and its area a_a (at right angles to the direction of the flux) are given. We shall suppose the magnet to be provided with polepieces of area a_a. The length l_m and cross-section a_m of the magnet itself are at our disposal. Now although actually magnetic leakage takes place continuously along the surface of the magnet, we shall, for the sake of simplicity, assume (as in § 34) that no leakage takes

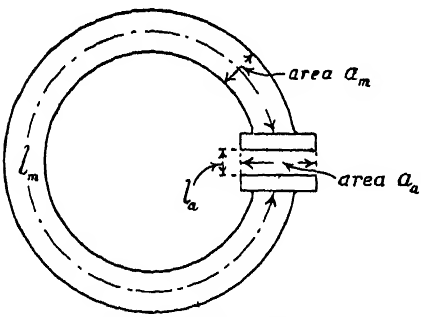


FIG. 14A.—Ring magnet.

* The rapid growth of the motor-car industry is mainly responsible for having created an enormous demand for the ignition type of "magneto."

† Such magnets are commonly used in many types of moving-coil galvanometers.

place until we reach the edges of the polepieces, and that then the entire leakage lines emerge suddenly. If $\nu =$ leakage coefficient, then the flux Φ_m through the magnet is equal to $\nu\,\Phi_a$.

The induction in the gap is given by $B_a = \Phi_a/a_a$, and the magnetic p.d. across the gap by $B_a\,.\,l_a = \Phi_a\,l_a/a_a$. Now if we imagine a tunnel of infinitely small cross-section formed in the magnet following the dotted circular arc l_m in Fig. 14A, and if this arc be continued across the polepieces and air-gap so as to form a complete circle, then since this circle is not linked with any current, the m.m.f. around it must vanish (§ 3). From this it follows that the magnetic p.d. across the short arc which forms the air-gap is equal to the p.d. across the remainder of the circle. Hence, if we denote the mean demagnetising force (§ 13) along the magnet by H, we have

$$H = \frac{\text{magnetic p.d. between ends of magnet}}{\text{length of magnet}}$$

$$= \frac{B_a\,l_a{}^*}{l_m} = \frac{\Phi_a\,l_a}{a_a\,l_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

Again, if B denote the induction in the magnet, $\Phi_m = B\,a_m$, and since $\Phi_a = B_a\,a_a$, and $\Phi_m = \nu\,\Phi_a$, we have

$$B\,a_m = \nu\,B_a\,a_a,$$

so that

$$a_a = \frac{B\,a_m}{\nu\,B_a},$$

Substituting this value for a_a in (1), we find

$$H = \frac{\nu\,\Phi_a\,B_a\,l_a}{B\,l_m\,a_m},$$

or

$$B\,H\,l_m\,a_m = \nu\,\Phi_a\,B_a\,l_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

Now Φ_a, B_a and l_a are given, and since ν may be regarded as approximately constant, the right-hand side of (2) is constant. Again, $l_m\,a_m$ represents the volume of the magnet. We may therefore re-write (2) in the form

$$B\,H \times \text{volume of steel in magnet} = \text{constant} \quad . \quad . \quad . \qquad (3)$$

* Strictly speaking, $B_a\,l_a$ represents the magnetic p.d. between the polar surfaces, and not between the ends of the *magnet* (the two becoming identical if no polepieces are used); but the approximation is valid because in general the length of the polepieces is small compared with that of the magnet itself.

In the above, it must be carefully noted, B and H represent the induction and the *negative* or *demagnetising* magnetic force respectively inside the magnet. We are at liberty to use any value of B numerically less than the highest possible residual induction, for we may so choose the length of the magnet that after it has been magnetised as strongly as possible, the demagnetising force which appears will be such as to reduce the induction to the desired value; the relation between B and H being given by that portion of the hysteresis loop (corresponding to maximum magnetisation) which lies in the second quadrant, as shown by the line marked "B" in Fig. 14B.* As B and H are varied, the product B H also varies, and hence, as follows from (3), the volume or amount of steel required. It is also obvious that the least volume of steel corresponds to the largest value of B H. Hence, *for any given magnet steel, the most economical magnet is that for which the product* B H *has a maximum value.* The values of B and H which yield a maximum value of B H are easily ascertained by taking various points on the demagnetisation curve, obtaining the products B H and plotting these against H (or against B). This has been done in Fig. 14B, the result being shown by the curve marked "B H." The point P determines the most economical values of B and H. The value of B determines the cross-section a_m of the magnet $\left(a_m = \frac{v\,\Phi_a}{B}\right)$; and its length l_m must, by means of equation (1), be so chosen as to give us the desired value of H.

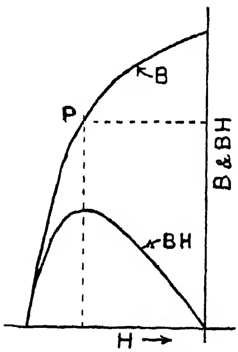


FIG. 14B.—Variation of B H with H.

We have hitherto supposed that the material of which the magnet is to be made is given, and we have seen how to use the material in the most economical manner. Recent metallurgical research has, however, placed quite a number of different kinds of magnet steel at our disposal, and so the further question arises as to which material gives maximum economy. To investigate this, let us determine, for each material, the volume v which gives maximum

* This portion of the loop is generally spoken of as the *demagnetisation curve*, and is the only portion which is of practical interest in connection with permanent magnets.

B H, and let c_1 stand for the cost of unit volume. Then if c be the total cost of material,

$$c = v\,c_1.$$

But by (3), $v = \dfrac{\text{constant}}{\text{B H}}$, hence

$$c = \text{constant} \times \frac{c_1}{(\text{B H})_{\text{max.}}},$$

where $(\text{B H})_{\text{max.}}$ denotes the maximum B H for each material. From this we see that c will be least for that material for which $\dfrac{c_1}{(\text{B H})_{\text{max.}}}$ is least, or for which $\dfrac{(\text{B H})_{\text{max.}}}{c_1}$ is greatest. In other words, *the most economical material to use is that for which* $\dfrac{(\text{B H})_{\text{max.}}}{c_1}$ *has the greatest value.*

If size and weight alone are important considerations, and cost a secondary matter, then the best material is that for which $(\text{B H})_{\text{max.}}$ has the greatest value.

§ 36b. Steels for Permanent Magnets.

The earliest permanent magnets were made of tool steel, containing about 1·2 per cent. of carbon. Later, it was found that the admixture of various other constituents greatly improved the magnetic quality of the steel. Plain tool steel has now been discarded as a material for permanent magnets, the newer magnet steels having taken its place. Of these, the one most generally used in England is tungsten magnet steel, which contains about 0·7 per cent. of carbon and about 6 per cent. of tungsten. It is capable of giving a remanence of approximately 10,000 and a coercivity of 70. Tungsten steel is generally supplied by the makers in the form of rolled bars which have to be cut and bent to the shape required, and in order to allow of the bending being done the bar has to be heated to about 800° C. Recently, successful attempts have been made to produce *cast* magnets of tungsten steel.*

Another important magnet steel, containing chromium and known as *chrome* steel, is very largely used in the United States and on the continent of Europe.† It contains about 1 per cent. of carbon and

* See S. Evershed, *Journal of the Institution of Electrical Engineers*, vol. 63, p. 779 (1925).

† E. A. Watson, *Journal of the Institution of Electrical Engineers*, vol. 63, p. 822 (1925).

2 per cent. of chromium, has a remanence of about 9,500 and a coercivity of 60. For a given weight, it is much cheaper than tungsten steel, although inferior to it in its magnetic properties.

For special purposes, magnet steels containing *cobalt* are sometimes used. Cobalt steels have an enormously high coercivity, of the order of 210, but a somewhat lower remanence (about 8,500) than tungsten steel. There is a whole series of these steels, containing varying percentages of cobalt, from 6 up to 35 per cent.; they also contain carbon (0·8 per cent.), chromium and tungsten. In spite of their remarkable magnetic qualities, cobalt steels are not very largely used, on account of their very high price; but where small size and weight are more important than cost of material, they find an application, and are used in wireless headgear receivers, in some types of ignition magnetos, and instead of electromagnets in small generators and motors where weight is the dominant consideration (as in the case of aeroplane apparatus).

The behaviour of a magnet steel is determined, not solely by its chemical composition, but also by the thermal treatment to which it has been subjected. An elaborate investigation of the effect of thermal treatment on tungsten steel has within recent years been carried out by S. Evershed.

The prices of magnet steels per lb. are approximately as follows: tungsten steel, 1*s.*; chrome steel, 6*d.*; cobalt steels, from 2*s.* 6*d.* to 6*s.*, according to the cobalt content.

Examples.

1. A certain magnetic circuit is made up of three portions, consisting of dynamo steel, cast iron, and air-gap. The lengths of the various portions, their cross-sections, and the magnetic fluxes through them are given by the following table.

—	Length.	Cross-section.	Total Flux.
Cast steel . .	20 cms.	120 sq. cms.	$2{\cdot}06 \times 10^6$
Cast iron . . .	30 cms.	240 sq. cms.	2×10^6
Air gap . . .	·6 cm.	200 sq. cms.	$1{\cdot}6 \times 10^6$

Journal of the Institution of Electrical Engineers, vol. 63, p. 725 (1925).

Using the data of the table in § 24, find the ampere-turns required to maintain the flux.

2. A ring of cast (dynamo) steel, 10 inches in external and 9 inches in internal diameter, and 1 inch wide in an axial direction, is slit in a radial direction, the slit forming a gap of uniform width equal to $\frac{1}{8}$ inch. Find the ampere-turns which must be provided by an exciting coil wound on the ring in order to produce a total flux of $44{\cdot}1 \times 10^3$ C. G. S. lines across the gap. The leakage coefficient is 1·06 (for the relation connecting B and H, see table in § 24).

3. A certain coil is required to produce a m.m.f. of 5,000 ampere-turns with a p.d. of 50 volts across its terminals. Find the mean resistance of a turn of the coil.

4. Two magnetised surfaces, each 4 sq. inches in area, are in contact with each other, and the total magnetic flux between them is 250,000 C. G. S. lines. Assuming this flux to be evenly distributed over the surfaces, find the total pull, in lbs. weight, exerted between them.

5. Four coils, each having a length of 10 inches and an external diameter of 2 inches, and wound with 500 turns of wire, are placed so that their axes lie along the four sides of a square and that their ends just touch each other. The coils are connected in series, and a current of 10 amperes is sent through them. Find approximately the mean value of the magnetic force in the space bounded by the coils.

CHAPTER IV.

§ 36c. Measurement of resistances by Wheatstone's bridge—§ 36d. Measurement of very high resistances. Evershed's "Megger"—§ 36e. Measurement of very low resistances. Kelvin's double bridge—§ 36f. Prices of instruments for the measurement of resistance.

§ 36c. Measurement of Resistance by Wheatstone's Bridge.

The range of resistances which the electrical engineer is called upon to measure is extremely wide, varying from a few ten-thousandths (or even less) of an ohm to many megohms. It is, therefore, not surprising that the particular method of measurement adopted is largely determined by the order of magnitude of the resistance. For resistances ranging from about 0·1 ohm to about 100,000 ohms, the most useful is the well-known Wheatstone's bridge method, with which the reader is supposed to be familiar, and which will therefore not be discussed in detail here. It may, however, be useful to recall the fact that the sensitiveness of the Wheatstone's bridge method is at its highest when the four arms of the bridge are as nearly equal as possible. It will thus be found that, with a given galvanometer and an arm ratio of, say, unity, the sensitiveness will depend on the *absolute values* of the ratio arms.

When dealing with very high resistances, the currents with the low voltages commonly employed with the standard types of Wheatstone's bridge become very small, and the sensitiveness is greatly reduced. Other methods of measurement then become necessary.

Difficulties also occur when an attempt is made to measure very low resistances with a Wheatstone's bridge. These are due to the uncertain resistance of the contacts and the difficulty of eliminating the resistances of the connecting leads. Here, also, it is necessary to have recourse to other methods.

§ 36d. Measurement of very High Resistances. Evershed's "Megger."

Several special methods have been devised for the measurement of resistances exceeding about 1 megohm. Here we shall only consider the method commonly employed in engineering practice, as opposed to laboratory methods. This method involves the use of a

special instrument known under the trade name of "Megger." The development of this instrument is due to Mr. S. Evershed.

The "Megger" consists essentially of a combination of two distinct elements: (1) a small high-voltage generator, and (2) an instrument constructed so that its deflection is determined by the value of the high resistance under test.

In the earlier forms of the instrument, the generator and the indicating instrument were kept entirely separate, each being enclosed in its own case. A great reduction in size and weight resulted by utilising the two bar magnets of the indicating instrument shown in Fig. 14c as the field-magnets (§ 63) of the generator, the polepieces of the instrument being at one end, and those of the generator at the other, of the pair of bar magnets.

The generator may be constructed for voltages up to 2,500.*

The construction of the indicating instrument will be understood by reference to Figs. 14c and 14d. Between the polepieces of the magnets is placed a soft iron hollow cylindrical core, with a portion removed to allow of the introduction of the moving part of the instrument. This moving part is of the *moving-coil* type, and consists of three distinct coils, mounted together on a common spindle which carries a pointer moving over a scale. One of the coils, known as the *pressure coil*, has one side attached to the spindle, while the other is capable of moving outside the cylindrical core. In the position of the coils shown in Fig. 14c, the pressure coil when carrying a current is not subject to any torque, and this position is one of *stable* equilibrium, since the connections are so arranged that a displacement in either direction gives rise to a torque tending to restore the coil to its original position. Further, since the radial component of the magnetic field steadily increases as the coil is displaced, the torque acting on the coil increases with the displacement. The second coil, known as the *current coil*, completely embraces the core, and is arranged with its plane at right angles to that of the pressure coil. The connections of the current coil are such that a current passing through it gives rise to a clockwise torque; further, since the sides of this coil are well under cover of

* The generator is provided with an *armature* (§ 63) which differs from the standard type commonly employed in large modern generators, and which closely resembles the "open-coil" armatures of the now obsolete arc-lighting generators used in the early days of electric lighting, and the "open-coil" armatures of certain types of modern motor meters (§ 59b). Essentially, the armature consists of a number of coils, the ends of each coil being connected to a two-part *commutator*, and the various coils being connected in series.

the polepieces, and hence move in a region where the field is uniform, a displacement of the coil will leave the torque acting on the coil unaltered.

Let us now suppose that there is a current in the pressure coil, but none in the current coil. Then, as we have seen, the system will assume the position shown in Fig. 14c. Next, let a current be allowed to flow in the current coil, the current in the pressure coil remaining unaltered. Owing to the clockwise torque now acting

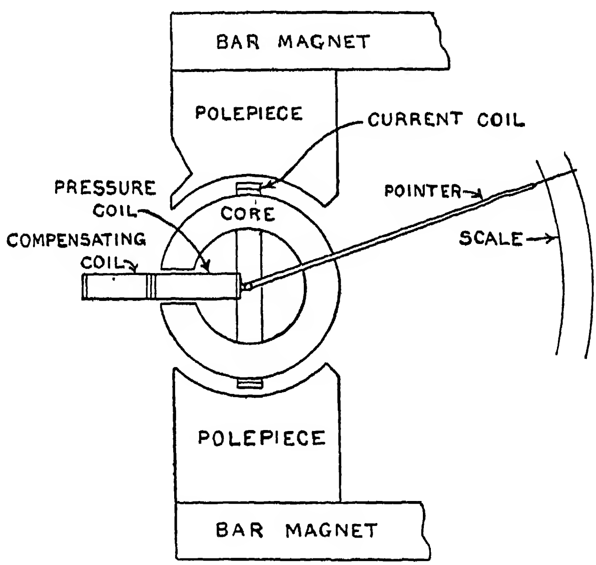


Fig. 14c.—Indicating instrument of Evershed's "Megger."

on the current coil, the system will be displaced in a clockwise direction. As the displacement takes place, the clockwise torque on the current coil will remain unaltered (unless the coil begins to pass into the magnetic "fringe" outside the polepiece, where the field begins to fall off rapidly and the torque on the current coil to decrease), while the counter-clockwise torque on the pressure coil steadily increases. A position will thus be reached where the two torques balance each other and where the system comes to rest. If we suppose that the current through the pressure coil is maintained constant while that through the current coil is varied, then for each value of the current through the current coil there will be a definite

position of equilibrium of the system. Again, if we suppose that, equilibrium having been reached, the current in each coil is increased in the *same ratio*, the torques will also be increased in the same ratio, and will thus still remain equal, and no change of deflection will result. We thus see that the deflection of the system depends on the *ratio* of the currents, and is independent of their *absolute values* so long as this ratio remains unaltered.

Since the sides of the current coil move in a very intense field, the torque acting on this coil will not be appreciably affected by any stray or external magnetic field of moderate amount. It is otherwise, however, with the pressure coil, since this coil moves in a comparatively weak field. In order to render negligible the effect of stray fields on the torque of the pressure coil, use is made of a

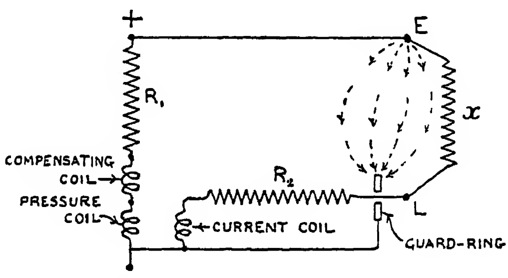


FIG. 14D.—Circuits of Evershed's "Megger."

third coil, known as the *compensating coil*. This coil is attached to the pressure coil, is connected in series and is co-planar with it. The area and number of turns in the compensating coil are such, and the connections of the two coils are so arranged, that the torques acting on the coils when placed in a uniform field are equal and opposite, the coils forming an *astatic* combination. Hence an external field will not affect the torque acting on the pressure coil, so long as this field is approximately uniform over the space occupied by the coils.

The diagram of connections is shown in Fig. 14D, and from it the action of the instrument will be readily understood. The generator terminals are connected to the points marked + and —. The resistance R_1 is intended to limit the current through the pressure and compensating coils; similarly, the safety resistance R_2 is intended to limit the current through the current coil in case of

short-circuit between the terminals L ("line") and E ("earth"), across which the unknown resistance x is connected.

As already explained, the position taken up by the moving system depends on the ratio of the currents through the pressure and current coils, and this ratio is obviously determined by the value of x. By using various *known* high resistances in place of x, and noting the corresponding positions of the pointer on the scale, the scale may be calibrated.

Since the resistance x is generally of a very high order, it is evident that the insulation between the instrument terminals E and L must be as perfect as possible, otherwise any leakage current passing between E and L will be added to the current through x, and the current coil will be traversed by the *sum* of these two currents. It is, however, extremely difficult to secure sufficiently good insulation between E and L under all conditions, and the error arising from leakage currents between E and L may be eliminated by the use of a very ingenious device, originally introduced by Price, and known as Price's guard-ring. The device consists of a metal ring which completely surrounds the terminal L, the ring being connected to the further terminal of the current coil. Any leakage currents passing from E towards L are intercepted by the ring and thence pass through the low-resistance path offered by the connecting wire to the further terminal of the current coil, without passing through the coil itself, which is traversed solely by the current flowing in x.

The small generator whose terminals are connected to the points marked + and − in the diagram (Fig. 14D) is driven by hand, through spur gearing, by means of a folding handle which, when the instrument is not in use, may be folded so as to fit into a groove, without projecting outside the instrument case. So long as the resistance x under test has no capacity, the instrument reading will be steady even if the speed of the generator is variable, for, as we have seen, the equilibrium position of the moving system depends on the ratio of the currents through the pressure and current coils, and this ratio is independent of the testing voltage. But if x is represented by the insulation resistance of a long length of cable, which forms a condenser of considerable capacity, then in addition to the variation in the current through x, which will take place when the voltage is altered, there will be a momentary charging or discharging current which passes into or out of the cable regarded as a condenser. These momentary capacity currents will render the

reading of the instrument unsteady if the speed is variable. In such cases a constant-speed or constant-pressure type of "Megger" should be used. In this, a special friction clutch is interposed between the driving gear and the generator; the clutch consists of a copper driving drum and carbon blocks which, so long as the speed does not exceed a certain limit, are pressed against the drum by the action of springs on the weighted levers which carry the blocks. When the speed exceeds the above limit, the centrifugal action of the weights on the levers reduces the pressure of the blocks on the drum to such an extent that *slipping* begins to take place, and no further increase of generator speed can occur. In order to maintain the generator speed (and hence its voltage) constant, it is therefore sufficient to turn the handle at a speed exceeding that at which slipping begins to take place.

§ 36e. Measurement of Very Low Resistances. Kelvin's Double Bridge.

Examples of the very low resistances which have to be measured at times in electrical engineering practice are afforded by the armature and series field windings of generators and motors, and short samples (in the form of wires or bars) of copper or other conductors, including sample lengths of the steel conductor rails employed in connection with the *third-rail* system of electric traction. Before considering the measurement of such low resistances, it may be well to draw the reader's attention to certain general points which have to receive careful consideration when dealing with very low resistances, but which cease to be of any importance when the resistance to be measured is high.

When we speak of the resistance of a relatively short length of conductor, such as a cylindrical rod of copper, or a few feet of steel conductor rail, we tacitly assume that the current is *uniformly distributed* over the cross-section of the conductor along the entire length considered. Unless this condition is fulfilled, the value of the *resistance* ceases to be definite, as the ratio p.d./current will depend on the particular way in which the current is distributed over the conductor cross-section. This point will easily become clear by considering a special case. Suppose we have a short copper cylinder of large cross-section, and suppose that we attempt to measure its resistance (from end to end) by sending a current through it and

measuring the p.d. across the ends. Imagine that for the purpose of leading the current into and out of the cylindrical conductor we drill and tap holes axially in its ends, and screw copper plugs of much smaller cross-section into the ends, as shown in Fig. 14E. Then a current entering and leaving the large cylinder will, near its ends, be distributed roughly as shown by the dotted lines. As we recede from the ends, the current distribution becomes more and more nearly uniform, and if instead of dealing with a short length of conductor we had a length great in comparison with its diameter, then over the greater part of the length the current might be regarded as uniformly distributed. But this uniformity is seriously disturbed in the *end regions.*

It is easy to see that, owing to the greater current density which exists along the axis of the conductor in its end regions, there will be

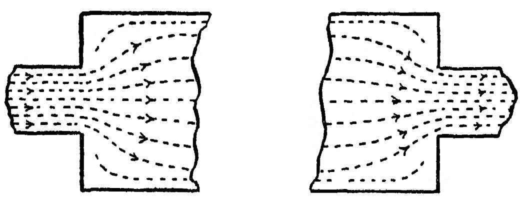


FIG. 14E.—Stream-line distribution.

a greater potential drop per unit length in those regions than in the middle portion of the conductor. Hence the total p.d. across the ends, and the value of the resistance (p.d./current) deduced from it, will be *higher* than the true resistance corresponding to entire uniformity of current distribution from one end to the other.

In order to overcome this difficulty when dealing with conductors of large cross-section, we must (1) obtain as great lengths of conductor as practicable, so as to make sure that there will be a middle region over which the condition of uniform current distribution is fulfilled; (2) use a method of leading the current into and out of the conductor which will yield the smallest possible disturbance near the ends; and (3) determine the resistance by using the middle portion only, where the current distribution is practically uniform.

For the purpose of ascertaining whether the current distribution is sufficiently uniform over the "middle" region, a pair of "knife-edge" contacts maintained at a fixed short distance apart and connected to a suitable galvanometer may be applied to the conductor

under test, the position of the knife-edges being varied and the galvanometer deflection noted. Over the region of uniform current distribution the deflection will remain constant.

From a theoretical point of view, the simplest method of measuring a resistance consists in determining the p.d. across the resistance, and the current flowing through the resistance. The quotient of the former by the latter immediately gives the value of the resistance. This simple method is, as a matter of fact, sometimes used in practice for the measurement of low resistances, but its accuracy is obviously limited by that of the measuring instruments employed, and for measurements of the highest degree of accuracy it is unsuitable.

The most accurate and satisfactory method of measuring low resistances is one originally devised by Lord Kelvin, and known as

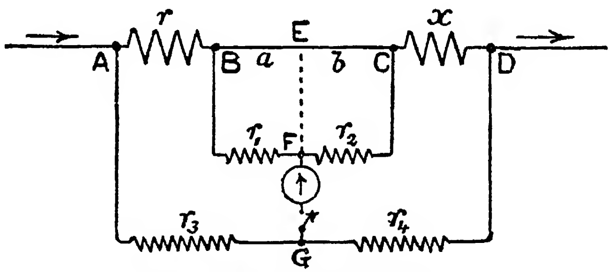


FIG. 14F.—Diagram of Kelvin double bridge.

the Kelvin *Double Bridge*. The principle of this method will be understood by a study of the diagram shown in Fig. 14F, where x is the unknown low resistance, and r a known low resistance, these two being connected in series with each other by means of a short length of cable or other conductor B C, and a (large) current being sent through them, as shown by the arrows. The " bridge " is completed by the four adjustable known resistances, r_1, r_2, r_3 and r_4, which it is convenient to make much higher than r and x. A sensitive galvanometer is connected between the junction F of r_1 and r_2 and the junction G of r_3 and r_4, and the values of r_1, r_2, r_3 and r_4 are varied until the galvanometer deflection is reduced to zero. That this is possible is evident from the fact that since F is a point intermediate in potential between B and C, and since the drop along $r_3 + r_4$ is the same as that along A D, $r_3 + r_4$ must include points whose potentials have all possible values lying between the potentials of B and C, and must therefore include a point which is at the

same potential as F. It is also obvious that the connecting conductor B C must contain some point E which is at the same potential as F. Let E divide B C into two parts, B E and E C, whose resistances are a and b respectively. Then the four conductors r_1, r_2, a and b may be regarded as forming a balanced Wheatstone bridge, and by the well-known condition for balance

$$\frac{a}{r_1} = \frac{b}{r_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

Again, since E and F are at the same potential, we may suppose them to be connected by a conductor E F of negligible resistance without altering any of the existing conditions in the circuit; E and F may now be supposed to have coalesced into a single point, and

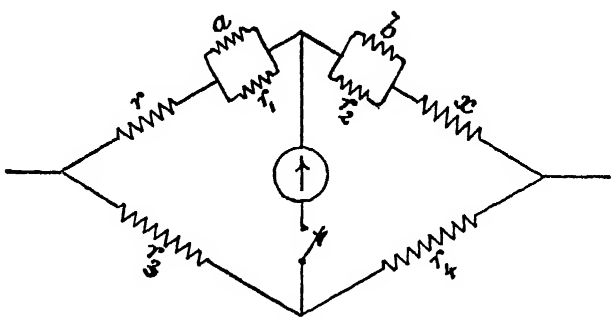


FIG. 14G.—To illustrate theory of double bridge.

the system of conductors now forms a *second* balanced Wheatstone bridge, shown, for the sake of clearness, separately in Fig. 14G. Applying the condition of balance to this second bridge, we have

$$\left(r + \frac{a\,r_1}{a + r_1}\right) \Big/ \left(x + \frac{b\,r_2}{b + r_2}\right) = \frac{r_3}{r_4} \quad . \quad . \quad . \quad (2).$$

Now from (1) we deduce, by adding unity to each side,

$$\frac{a + r_1}{r_1} = \frac{b + r_2}{r_2}$$

Let us put

$$\frac{r_1}{a + r_1} = \frac{r_2}{b + r_2} = c$$

(2) now becomes

$$\frac{r + a\,c}{x + b\,c} = \frac{r_3}{r_4}.$$

Solving for x, we find

$$x = \frac{r_4}{r_3} r + c\left(\frac{r_4}{r_3} a - b\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

But since by (1) $b = \frac{r_2}{r_1} a$, (3) may be written

$$x = \frac{r_4}{r_3} r + \left(\frac{r_4}{r_3} - \frac{r_2}{r_1}\right) a\, c \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Now if, while altering r_1, r_2, r_3 and r_4 in order to obtain balance, we alter them in such a manner as to maintain $\frac{r_4}{r_3} = \frac{r_2}{r_1}$, then the second term on the right-hand side of (4) disappears, and we get the very simple expression

$$x = \frac{r_4}{r_3} r = \frac{r_2}{r_1} r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5),$$

which enables us to find x if r and the ratio $\frac{r_4}{r_3} = \frac{r_2}{r_1}$ are known.

If ordinary resistance boxes were used for the resistances r_1, r_2, r_3

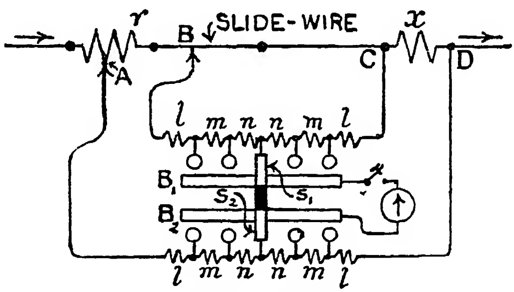


FIG. 14H.—Practical arrangement of double bridge.

and r_4, care would have to be taken always to satisfy the relation $r_1/r_2 = r_3/r_4$, and if r were fixed, the ratio r_1/r_2 would in general be represented by an awkward fraction. Hence in the practical forms of the Kelvin Double Bridge, arrangements are provided for varying r, and for *automatically* maintaining equality of the ratios r_1/r_2 and r_3/r_4. Further, in order to eliminate unnecessary arithemetical complications, these ratios are arranged to be integral powers of 10.

Fig. 14H shows diagrammatically the actual arrangement adopted in practice. The standard variable resistance r consists of a carefully adjusted resistance subdivided into a number of sections and

connected in series with a calibrated slide-wire by means of which fine adjustment may be obtained. The two mechanically coupled switches, S_1 and S_2, maintain equality of r_1 and r_3, and also of r_2 and r_4; S_1 bridges across the bar B_1 and the series of contact studs connected to the upper set of ratio resistances l, m, n, while S_2 similarly bridges across B_2 and the contact studs in connection with the lower set of resistances l, m, n. The double switch is shown in the position corresponding to a ratio r_1/r_2 or r_3/r_4 of unity. We shall now investigate what the relative values of l, m and n must be in order that the ratios corresponding to the remaining four positions of the switch may be 0·01, 0·1, 10 and 100. Taking the first two switch positions from left to right, we must have

$$\frac{l}{2m + 2n + l} = 0{\cdot}01$$

and

$$\frac{l + m}{2n + m + l} = 0{\cdot}1.$$

It is obvious that if these two equations are satisfied, then we shall also have, for the last two positions on the right,

$$\frac{l + m + 2n}{l + m} = 10$$

and

$$\frac{l + 2n + 2m}{l} = 100.$$

Let $l + m + n = p$. Then the above equations may be written

$$\frac{l}{2p - l} = 0{\cdot}01 \text{ and } \frac{l + m}{2p - (l + m)} = 0{\cdot}1.$$

The first equation gives $l = \frac{2}{101}p$, and the second, $l + m = \frac{2}{11}p$, hence $m = \frac{180}{1111}p$. Lastly, $n = p - (l + m) = \frac{9}{11}p$. Reducing the above expressions to the common denominator 1111, we have

$$l = \frac{22}{1111}p;\ m = \frac{180}{1111}p;\ n = \frac{909}{1111}p.$$

In order, therefore, that the required values of the ratios may be obtained, l, m and n must be proportional to 22, 180 and 909 respectively. As regards the absolute values, it is convenient to make the smallest resistance l of the order of 1 ohm.

§ 36f. Prices of Instruments for the Measurement of Resistance.

Wheatstone bridges vary widely in price, according to the number of coils and their accuracy of adjustment. The cheaper sets cost about £10, while those adjusted with the highest precision may be priced at £100. Galvanometers suitable for use with Wheatstone bridges range from about £5 to about £20.

"Meggers" cost from about £30 to about £60, according to the range and type of instrument.

Kelvin double bridges cost from about £20 to about £50.

CHAPTER V.

§ 37. Classification of measuring instruments—§ 38. Multicellular electrostatic voltmeter—§ 39. Kelvin electric balance—§ 40. Moving-coil instruments—§ 41. Moving-coil voltmeters—§ 42. Moving-coil ammeters—§ 43. Technical data of moving-coil instruments—§ 44. Soft iron instruments—§ 45. General conditions to be fulfilled by ammeters and voltmeters. Accuracy obtainable—§ 46. Illuminated dial and edgewise instruments—§ 47. Recording instruments—§ 48. Relative merits and prices of different types of instruments—§ 49. Limited scale voltmeters.

§ 37. Classification of Measuring Instruments.

There are various ways of classifying electrical measuring instruments. Thus, we may consider them from the point of view of the accuracy attainable, and divide them into *standard instruments*, whose design and construction are such as to render them thoroughly reliable in the most delicate measurements; and into *commercial instruments*, whose accuracy in many cases need not be of a very high order, and whose construction may therefore be simpler and cheaper. Again, we may base our classification on the nature of the particular quantity which the instrument is intended to measure: we thus have voltmeters for measuring p.d., ammeters for measuring current, quantity and energy meters, &c. Another system of classification may be based on the particular effect of the p.d. or current which is utilised in the construction of the instrument: we thus have electrostatic, electro-dynamic, electro-magnetic and thermal (or "hot-wire") instruments. We may further divide instruments into *deflectional* or *indicating* and *zero* instruments. Those of the first type give a reading without requiring any preliminary adjustment, while in zero instruments such an adjustment, consisting in reducing the movable part of the instrument to its initial or zero position, must be made before a reading can be taken. Lastly, we may adopt as our basis of classification the nature of the controlling couple which resists a deflection of the movable part of the instrument: we are thus led to consider instruments as having magnetic, spring, and gravity control.

Standard instruments are used either in connection

measurements where extreme accuracy is required (as in efficiency tests of various kinds), or for the purpose of checking, from time to time, the readings of the cheaper and less reliable commercial measuring instruments used on switchboards, &c. It is mainly for this latter purpose—the periodic calibration of switchboard instruments—that most generating stations are provided with one or more standard instruments. We shall commence our study of instruments by considering two well-known types of standard instrument—the Kelvin Multicellular Electro-static voltmeter and the Kelvin Electric Balance.

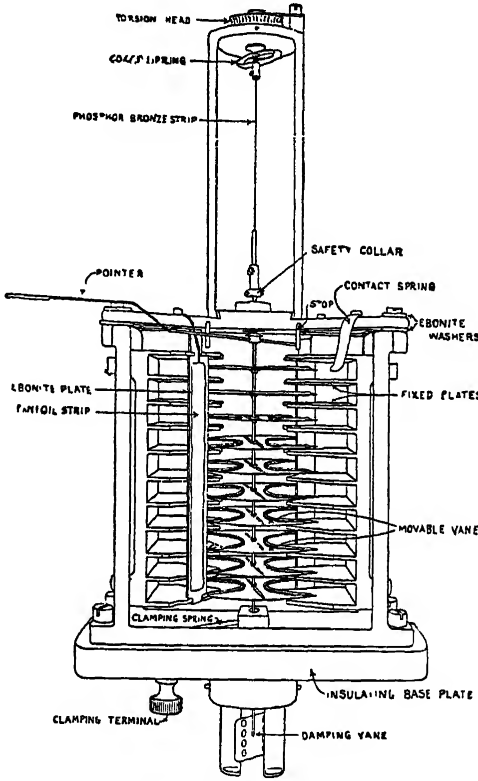


FIG. 15.—Multicellular electrostatic voltmeter.

§ 38. Multi-cellular Electrostatic Voltmeter.

Lord Kelvin's multicellular voltmeter is an *electrostatic* instrument—i.e., its action depends on the electrostatic stress between the fixed and movable parts of the instrument: it therefore takes no current; the controlling couple is furnished by the torsion of a phosphor-bronze strip. The construction of the instrument will be understood from Figs. 15 and 16. Fig. 15 shows the working parts of the instrument in perspective. The system of fixed plates, which is in connection with one terminal of the

instrument, consists of two similar aluminium castings, each having the form of a stout vertical plate from which project a series of equidistant horizontal shelves, the spaces between which form a number of "cells"—hence the name of the instrument. These plates are shown in plan in Fig. 16. They are screwed to a strong metal frame which at the top receives the framework supporting the movable system. This framework is insulated, as shown in Fig. 15, by means of ebonite washers, and is in metallic connection with the other terminal of the instrument and with the outer case, contact with which is maintained by means of a light metal spring shown in the figure and marked "contact spring." Arising from the framework are two pillars which end in a horizontal circular plate supporting a "torsion head," which consists of a worm-wheel geared with a worm (not shown), by means of which the movable system may be adjusted to its zero position. Interposed between the torsion head and the phosphor-bronze suspending strip is a coach-spring which provides an elastic support, and which is intended to prevent breakage of the suspension if owing to a sudden jerk the suspending strip should be stretched with a force exceeding its normal load. The safety collar shown in the figure in that case comes against the perforated circular stop placed immediately below it, and prevents any further stretching of the filament. Attached to the safety collar is the long vertical spindle which carries the series of movable horizontal vanes which are free to swing round into the "cells" formed by the fixed plates. The construction of these vanes will be understood by reference to Fig. 16, which shows a vane in plan and section, and the relative position of the vanes and fixed plates when the pointer is at zero. It will be noticed that the vanes are stamped with ridges down their middle and along each side—an arrangement intended to secure stiffness.

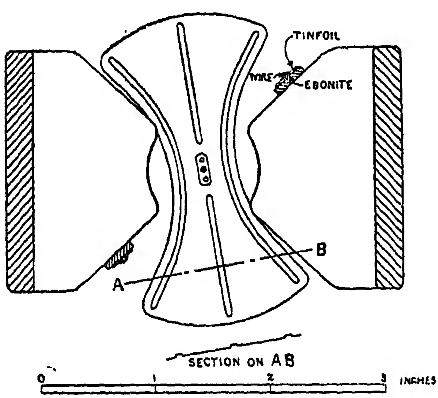


FIG. 16.—Plan of vanes and fixed plates.

Connected with the movable system are two vertical strips of tinfoil supported by plates of ebonite, which fit into dove-tailed recesses in the highest and lowest fixed plate of each set. A thin copper wire runs from a small screw in the framework down the strip of ebonite, and the tinfoil is pasted over the wire, as shown clearly in Fig. 16.

At its lower end, the vertical spindle carries a perforated metal plate which is completely immersed in a small oil-vessel, and which forms the damping device for rapidly reducing the movable system to its position of equilibrium, the damping vane encountering considerable resistance as it moves through the oil,

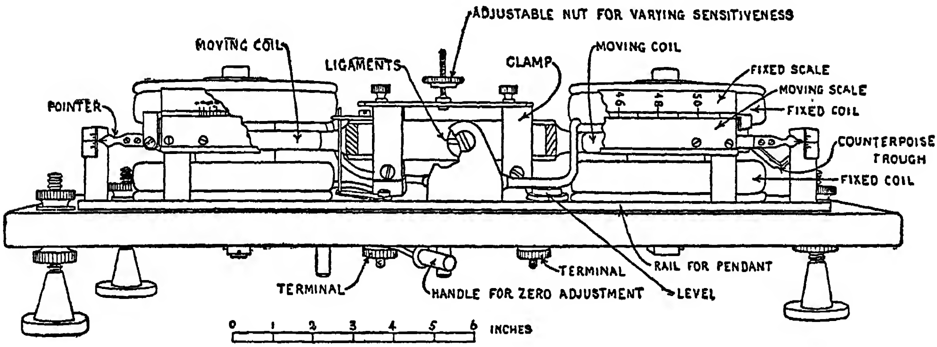


FIG. 17.—Kelvin electric balance.

and so checking the oscillations which would otherwise persist for a long time.

The range of motion of the movable vanes is limited by two wire stops against which the pointer strikes when it passes outside the limits of the scale.

When the instrument is not in use, the movable system may be clamped by means of the clamping spring shown in the figure. This spring is a flat strip of metal with a forked end embracing the spindle. On tightening the clamping terminal, the forked end of the spring is made to press against a collar mounted on the spindle, thereby bodily raising it, and forcing the upper conical part of the collar supporting the pointer against a conical recess in the lower surface of the supporting frame-work. The

movable system is thus securely clamped, and the suspending strip is freed from tension.

The action of the instrument is as follows: On connecting the instrument to a source of e.m.f., the fixed and movable systems acquire charges of opposite sign (with the exception of the two vertical strips of tin-foil, which, being in metallic connection with the movable system, acquire charges of the same sign as this latter), and owing partly to the unsymmetrical position of the movable relatively to the fixed plates in the zero position, partly to the presence of the repelling vertical tin-foil strips, a couple is produced on the needle. One effect due to the strips of tin-foil

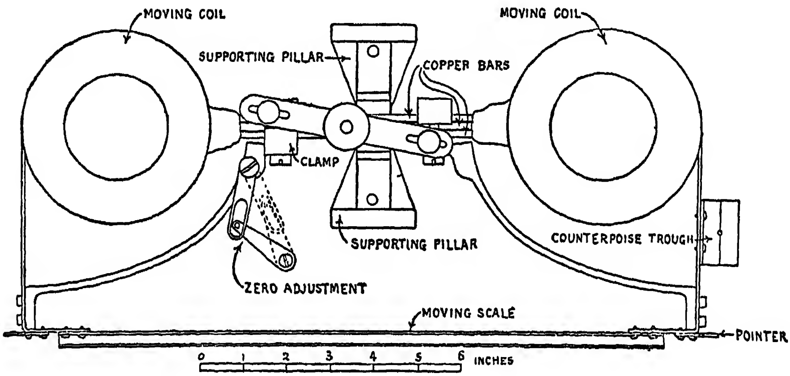


FIG. 18.—Movable system of Kelvin balance.

is an improvement in the scale, the lower portion of which is thereby considerably opened out.

§ 39. Kelvin Electric Balance.

The Kelvin Electric Balance is primarily intended to be used as an ammeter; it is an electro-dynamic instrument, i.e., its action depends on the mutual forces or stresses exerted between wires (or coils of wire) conveying currents (§ 4). It is further an instrument of the zero type, i.e., the movable system is brought back to its original or zero position before a reading is taken. The controlling couple is provided by gravity, a sliding weight being employed to restore balance.

The general arrangement of the instrument will be understood

from Fig. 17,* which is a perspective view, with parts of the instrument shown cut away so as to exhibit the construction more clearly. Fig. 18 shows the movable system in plan. There are in all six coils in the instrument, four of which are fixed and two movable. The movable coils are supported at the ends of a heavy compound bar (see Figs. 18 and 19) consisting of three bars of copper insulated from each other, and held together by means of two heavy metal clamps lined with ebonite so as to insulate them from the bars. Across the clamps is placed a thick strip of brass slotted at its ends so as to make it adjustable, and from its centre there arises a screwed vertical shank fitted with an adjustable nut for varying the sensitiveness. The centre bar of copper connects the two coils, the remaining coil ends being in connection with the outer bars. Fixed to the middle of each outer bar is a horizontal cylinder of brass, the greater part of which is cut away so as to leave rather less than a semi-cylinder (see Figs. 17 and 19). Soldered to each semi-cylinder are numerous very fine copper wires or *ligaments*, by means of which the movable system is flexibly suspended, and which form one of the most important features of the instrument, allowing of very large currents being sent into the movable coils through a suspension of extreme flexibility. The upper ends of the ligaments are attached to semi-cylinders supported by two heavy brass pillars mounted on the slate base of the instrument. These pillars are provided with bolts and nuts for receiving the ends of the conductors leading to the fixed coils. The current enters the movable system by one set of ligaments, and leaves by the other. The connections are such that the current circulates *in opposite directions* around the two movable coils—an arrangement which renders them *astatic*, or not liable to be disturbed by any *uniform* external field. Each movable coil is placed

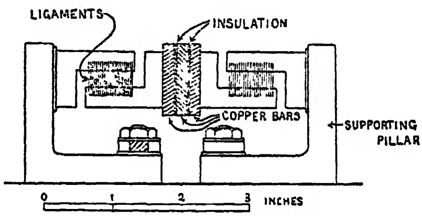


FIG. 19.—Method of suspension used in Kelvin balance.

* The instrument to which the description and illustrations refer is a *deka-ampere* balance. The details of construction vary slightly in instruments of different ranges.

between two fixed coils, one of which repels while the other attracts it, there being a resultant downward force on the left-hand moving coil and a resultant upward force on the right-hand one If the movable system is horizontal (as indicated by the pointers attached to the moving scale—Figs. 17 and 18) with no current passing through the instrument, then on sending a current through it the coils will be deflected. Balance is then restored by sliding a weight along the movable scale, the weight being supported by a carriage (Fig. 20); the current is determined from the displacement of the weight. The carriage and weight are slid along the scale by means of the self-releasing pendant, shown in Fig. 21, which fits over a guide-rail (seen in Fig. 17) and is provided with silk cords passing through the glass case of the instrument; by means of the cords, the pendant may be pulled one way or the other. In order to secure stability, the movable coils are normally displaced from their midway positions between the fixed coils (in which positions the forces exerted on them are at a minimum), each movable

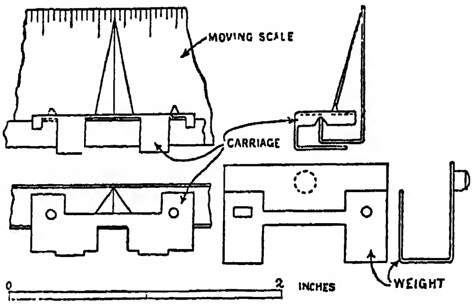


FIG. 20.—Sliding carriage and weight of Kelvin balance.

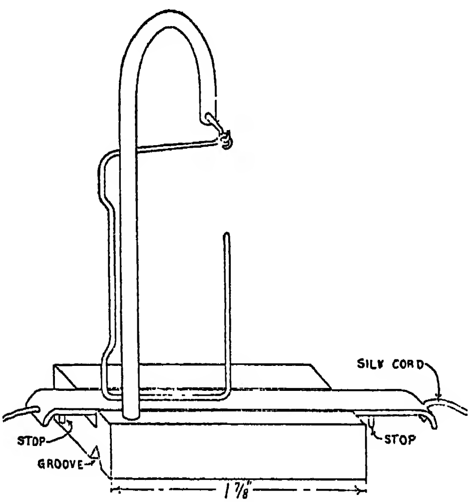


FIG. 21.—Self-releasing pendant.

coil being somewhat nearer the repelling fixed coil than the attracting one.

Provision is made for accurately adjusting the zero position by means of a light slotted metal flag projecting from one of the clamps which hold together the copper bars (see Figs. 17 and 18). By means of a lever arranged underneath the base of the instrument, a vertical pin which passes freely through the slot in the flag may be moved one way or the other, and so the flag itself displaced, the centre of mass of the suspended system being correspondingly displaced one way or the other. Care must be taken, after displacing the flag, to move the pin quite clear of it.

One advantage of a zero electro-dynamic instrument such as the Kelvin Balance is that it obeys a definite law. The deflecting couple acting on the movable system is proportional to the force on either movable coil. This force is proportional to the product of the current in the coil into the field in which the coil is placed. Since, however, the field itself is produced by the fixed coils, which carry the same current as the movable ones, it follows that the deflecting couple varies as the *square* of the current. The balancing or controlling couple brought into play by the displacement of the weight varies directly as the displacement. Hence for every position of balance, we must have

$$\text{square of current} \propto \text{displacement of weight}$$
$$\text{or current} \propto \sqrt{\text{displacement}},$$
$$\text{i.e., current} = \text{constant} \times \sqrt{\text{displacement}}.$$

The displacement of the weight is read off on the finely divided movable scale, and on multiplying the square root of this by a definite constant (which is, for the sake of convenience, arranged to have some simple value—such as 1, $\frac{1}{2}$ or $\frac{1}{4}$), the corresponding current is obtained. For the sake of greater convenience, a fixed or inspectional scale is provided, arranged immediately behind the movable scale, and divided so that the numbers corresponding to the consecutive divisions are proportional to the square roots of the numbers along the equally divided movable scale. Rough values of the square roots and currents may thus be obtained by inspection, without reference to the tables which are provided with each instrument.

In order to extend the range of accurate measurement, each instrument is provided with several sliding weights and a corresponding set of counterpoise weights, the heavier weights being

used when measuring larger currents. The counterpoise weights are cylindrical, and are fitted with cross-pins which pass through the bottom of the triangular trough (see Figs. 17 and 18) intended to receive them, a perfectly definite position of the counterpoise in the trough being thereby secured—the arrangement forming a "geometrical clamp." A similar "geometrical clamp" is employed between the sliding carriage and the weight supported by it (see Fig. 20), the "clamp" consisting of two conical pins in the carriage, and a hole and slot in the weight.

§ 40. Moving Coil Instruments.

The favourite switchboard type of instrument for continuous currents is the "moving coil" type. These instruments may be

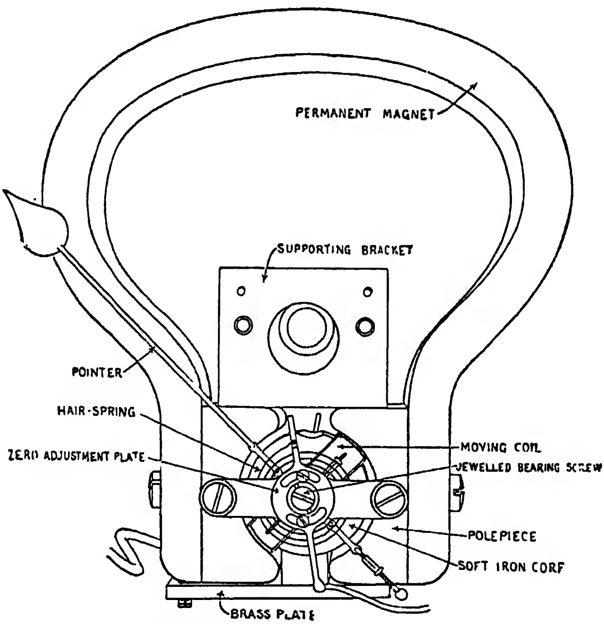


FIG. 22.—Weston moving coil instrument.

constructed either as ammeters or as voltmeters. They form simple modifications of the moving coil galvanometer, originally

devised by Lord Kelvin (who used a moving coil in his "syphon recorder"—an instrument for recording the signals transmitted through a submarine cable), and adapted for laboratory use by Deprez and D'Arsonval.

The construction of the instrument will be readily understood from Figs. 22 and 23, Fig. 22 being a perspective view of the instrument as it appears when taken out of its case. A permanent steel magnet, whose length is great in comparison with its cross-section in order to secure constancy (§ 13), is fitted with soft iron pole-pieces; these are bored out to receive a solid cylindrical core of soft iron, of diameter less than the bore of the pole-pieces, which is supported by a brass plate screwed across the pole-pieces. An annular gap is thus

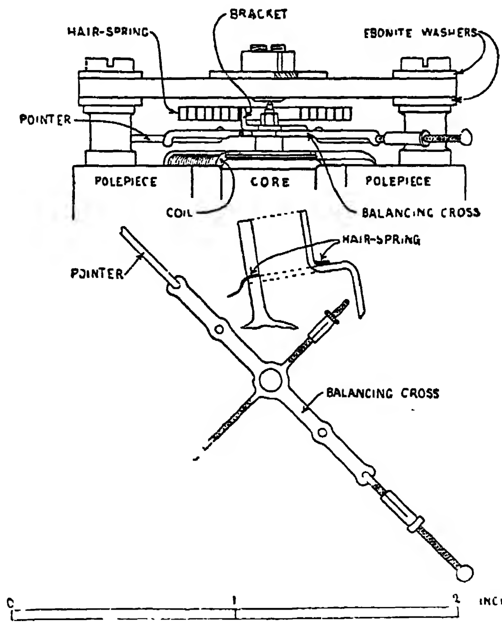


FIG. 23.—Details of Weston moving coil instrument.

formed around the core, and in this gap is pivoted a coil of fine silk-covered copper wire wound on a light aluminium frame, the frame causing the instrument to be "dead-beat" by powerfully damping the motion, owing to the strong currents induced in the frame as it moves across the field. Cemented to each end of the moving coil are aluminium plates which carry the pivots and the attachments for the hair-springs. These latter serve the double purpose of providing the necessary controlling couple and leading the current into and out of the coil. The details of the method of attaching the hair-spring

will be understood from Fig. 23. As it is essential to have the coil accurately balanced, in order that the control may be a pure spring control and unaffected by gravity, a special balancing cross is provided, one arm of which carries the pointer while its prolongation on the other side supports a small screwed shank having a counterpoise (a drop of solder) at its end, and fitted with an adjustable nut. The remaining two arms consist of screwed shanks, one of which carries a balancing nut. On the top of the balancing cross is placed a small bracket, to one end of which is soldered the inner end of the phosphor-bronze hair-spring, while to the other end is soldered one end of the coil. The balancing cross and bracket are secured to the spindle by means of a small nut, beyond which projects the pivot; the latter fits into a jewelled bearing at the end of a screw which passes through a brass plate supported by two brass pillars arising from the pole-pieces, but insulated from them by means of ebonite washers. The outer end of the hair-spring is soldered to an arm projecting from a circular slotted plate (marked "zero adjustment plate" in Fig. 22), whose position may be varied for the purpose of accurately adjusting the zero. The diametrically opposite arm of this plate is connected to one of the instrument terminals. Similar arrangements are provided at the other end of the coil, except that there is no balancing cross. In order to prevent creeping of the zero, the two springs are coiled in opposite directions, so that when the coil is deflected, one spring is coiled up and the other uncoiled.

§ 41. Moving Coil Voltmeters.

If the instrument is intended to be used as a *voltmeter*, it is, as shown in Fig. 24 (*a*), connected in series with a very high resistance consisting of some alloy having a low temperature coefficient (such as German silver, platinoid, "eureka," constantan, or manganin), so as to render the total resistance of the instrument nearly independent of temperature; otherwise, a change of temperature only, without any change of p.d. across the instrument terminals, would affect the reading. At first sight, it might for this reason appear advisable to wind the moving coil itself with some alloy having a low temperature coefficient. This, however, is found to be inconvenient, as low temperature coefficient is

always associated with high resistivity. Hence, in order to prevent excessive heating of the coil, a much larger size of wire would have to be used than when the coil is of copper. An increase in the thickness of the wire would, however, necessitate either an increase of gap length, with a corresponding decrease of field intensity, or a reduction in the number of turns in the coil without any change of gap length. Either of these changes would reduce the sensitiveness of the instrument. It is for this reason that the moving coil itself is wound with copper wire.

§ 42. Moving-Coil Ammeters.

In a moving-coil *ammeter*, the moving coil is joined in series with a resistance, and is then connected across a suitable low resistance which conveys the main current, as shown in Fig. 24 (*b*). The error due to temperature changes, which may be made quite negligible in the case of voltmeters, presents a serious difficulty in ammeters. If the ammeter shunt were constructed of copper, and the moving coil connected directly across it, without the use of any series resistance, the reading would remain correct at all temperatures provided the moving coil and the shunt could always be maintained at the same temperature. Since, however, it is in most cases impossible to secure this condition, the temperature error will not be eliminated. Hence it is safer to make the shunt of an alloy with a small temperature coefficient, so as to render the p.d. across the shunt practically constant at all temperatures for a given current, and to connect in series with the moving coil as large a resistance of negligible temperature coefficient as possible, in order to reduce the joint temperature coefficient of the coil and its series resistance to the lowest possible value. On the other hand, the resistance of the shunt should be kept as low as possible in order (1) that the power lost in it may not be excessive, and (2) that

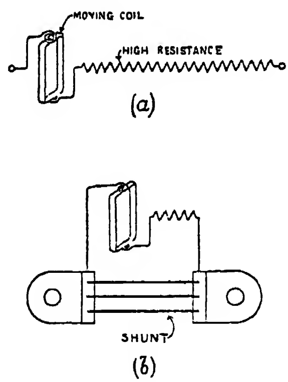


FIG. 24. — Connections of moving-coil instrument as (*a*) voltmeter and (*b*) ammeter.

the shunt may not be of inconveniently large dimensions and expensive. In practice, the series resistance is from 1 to 4 times that of the moving coil itself (which ranges from 1 to 3 ohms), while the current taken by the moving coil is from ·02 to ·1 ampere.

An extremely ingenious compensating device for ammeters has been invented by Mr. A. Campbell. This is represented diagrammatically in Fig. 25. The ammeter shunt has a negligible temperature coefficient, while the Wheatstone's bridge connected across it consists partly of copper, and partly of a material having a negligible temperature coefficient. The moving coil occupies the galvanometer branch of the bridge. The bridge is, of course, unbalanced, and the moving coil receives the out-of-balance current. When a rise of temperature takes place, the resistance of the moving coil increases; but at the same time, owing to the increase of resistance in the opposite copper arms of the bridge, which are of higher resistance than the high-resistivity arms, the out-of-balance p.d. across the moving coil rises, and by properly proportioning the resistances of the various branches it is possible to arrange matters so that the rise of p.d. across the moving coil is proportional to its increase of resistance, and hence that the current flowing through it remains unaffected by temperature changes.

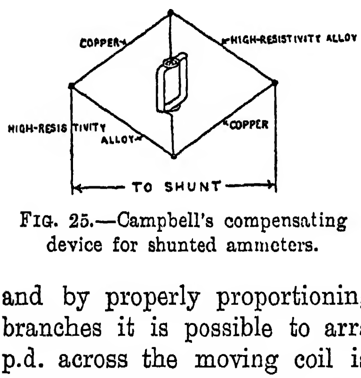


FIG. 25.—Campbell's compensating device for shunted ammeters.

The shunts used in connection with ammeters generally consist, as shown in Fig. 24 (*b*), of a number of thin plates of a high-resistivity alloy (manganin in most cases), held between two massive blocks of copper or gunmetal, which are grooved to receive them, and to which they are soldered. The shunts should be fixed with the plates vertical, so as to maintain good ventilation. When the current to be measured does not exceed about 100 amperes, the shunts are frequently fixed at the back of the instrument case itself, rendering the instrument self-contained.

Instead of a special shunt, a length (about 5 feet) of copper

Journal of the Institution of Electrical Engineers, vol. xxxv., p. 197 (1905).

conductor forming part of the main circuit is sometimes made use of—an arrangement found convenient in the case of switchboard instruments. The temperature error in this case may, however, be considerable.

The reading of a shunted ammeter may in certain cases be seriously affected by thermo-electric effects. If owing to a defective joint between the main cable and one end of the ammeter shunt this end becomes hotter than the other, a thermo-electric e.m.f. may result in the local circuit formed by the ammeter and its shunt, and this may be of sufficient magnitude to alter the reading of the ammeter.* One method of dealing with this difficulty—suggested by Edgcumbe †—is to use connecting wires between the instrument and its shunt of the same material as the shunt itself. The ends of the wires connected to the instrument terminals will be practically at the same temperature, and there will be no resultant thermo-e.m.f. in the local circuit of the instrument. A more satisfactory solution is to avoid, in the construction of shunts, the use of alloys (such as constantan) giving a high thermo-e.m.f. with copper, and to use only such as have a neglible thermo-e.m.f. (*e.g.*, manganin).‡

§ 43. Technical Data of Moving-Coil Instruments.

The air-gap field in moving-coil instruments generally varies from about 600 to 1,000, although in some exceptional cases it may reach a value of 2,000. In order to obtain so high a value of the gap induction, the cross-section of the magnets must be made large in comparison with the polar area; special pole-pieces are in this case dispensed with, the tapered ends of the magnets themselves forming the polar surfaces. Intense gap fields are mostly used in instruments provided with very long pointers, and in recording instruments (§ 47) in which the pointer carries a pen, and in which, in order to reduce the frictional error caused by the pen, it is desirable to use a powerful deflecting torque.

* *Electrical Review*, vol. 64, p. 1061 (1909).
† *Electrical Review*, vol. 65, p. 129 (1909).
‡ Constantan gives a thermo-e.m.f. of about 35 microvolts per degree C. with copper, while manganin gives only about $\frac{1}{25}$th of this amount.

The induction in the cores of the permanent magnets varies from about 1,000 to about 4,000. The length of the magnets is from 20 to 35 cm., and their cross-section from 3 to 4 sq. cm. The area of a pole-piece varies from 9 to 14 sq. cm. The length of (single) air-gap is from 1 to 2 mm.

The weight of the moving element of a moving-coil instrument is about 3 grammes. The ampere-turns of the coil are of the order of unity, and the full-load torque is of the order of 1 gramme-cm. The torque is generally higher in voltmeters than in ammeters.

The resistance of a moving coil voltmeter is at the rate of about 70 to 120 ohms per volt of maximum scale reading.

The current taken by a moving-coil voltmeter is of the order of ·015 ampere, while the drop of potential over the shunt of an ammeter varies from ·05 to ·1 volt.

§ 44. Soft Iron Instruments.

In cases where a high degree of accuracy is not of importance, "soft iron" electro-magnetic instruments may be used. Of these there are numerous types in existence. The general principle on which they are based is that a mass of soft iron if placed in a magnetic field which is not uniform tends to move from weak to strong regions of the field, or tends to assume a position corresponding to a maximum flux through it. The actual arrangement may consist either of a coil which sucks in a movable core, or of a coil with two cores, one fixed and the other movable, the function of the fixed core being to modify the field due to the coil in such a manner as to produce the necessary gradient of field intensity in the direction of motion of the movable core.

As an example of a soft iron instrument, we may consider the O-K type manufactured by Messrs. Nalder Bros. and Thompson, Ltd. The essential parts of this instrument are shown in Fig. 26. A wire of soft iron is fixed against the inner surface of the bobbin on which the coil is wound, while the movable core consists of a tiny bar of soft iron supported by a spindle moving in jewelled bearings. The fixed wire produces a local weakening of the field, and the movable core is displaced towards the stronger

regions of the field* against the action of a small counterweight (shown in Fig. 27) which furnishes the gravity control. The

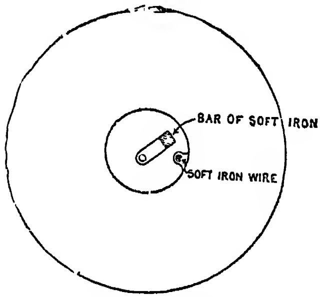


FIG. 26.—Nalder Bros. and Thompson's soft iron instrument.

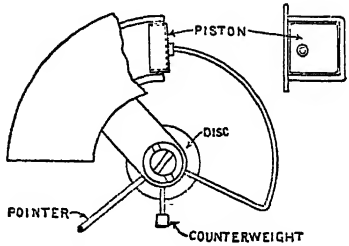


FIG. 27.—Details of soft iron instrument, showing air damping device.

fixed and movable cores extend the entire length of the coil. Outside the coil, the spindle supporting the movable core is fitted at its front end, as shown in Fig. 27, with a metal disc which carries (1) the pointer, (2) the counterweight, and (3) the piston of the air dash-pot. The piston is in the form of a light square box open inwards, and moving with very little clearance within an annular channel of square cross-section. This air damping device is very effective, and has been adopted very generally by instrument makers.

§ 45. General Conditions to be Fulfilled by Ammeters and Voltmeters. Accuracy Obtainable.

In order that a measuring instrument may not affect the existing conditions of a circuit, and that it may not absorb an excessive amount of power, its resistance must have a suitable value. The condition to be fulfilled by a voltmeter is in this respect of an opposite nature to that required of an ammeter. If a voltmeter is connected across any two points of a circuit, and if the mere fact of connecting the voltmeter is not to affect the value of the p.d. which previously existed between the two points, the voltmeter must have a resistance so *high* that the small current taken by it does not materially alter the distribution of the p.d.'s in the various parts of the circuit. Again, since the

* An alternative method of regarding the matter is as follows :—The coil magnetises the two cores in the same direction, and hence, owing to the "repulsion" between the like magnetic poles at each end of the coil, the movable core is driven away from the fixed one.

power taken by a voltmeter at its maximum reading is $\frac{V^2}{r}$, where V is the maximum reading and r the resistance of the instrument, it follows that r must be as *high* as possible in order that the power taken by the instrument may be small. From both points of view, therefore—maintenance of the originally existing conditions in the circuit, and smallness of power absorbed—it is desirable to make the resistance of a voltmeter circuit as *high* as possible. In an ammeter, on the other hand, the conditions are reversed. For if the introduction of an ammeter into a circuit is to leave the originally existing current unaltered, it is obvious that the ammeter resistance must be negligible in comparison with that of the rest of the circuit, and hence must be made as *low* as possible. Similarly, since the power absorbed at the maximum reading i of an ammeter of resistance r is ri^2, r must be made *small* in order to keep down the power taken by the instrument. Hence ammeters should have as *low* a resistance as practicable.

As regards accuracy, standard instruments are capable of giving results correct to within $\frac{1}{10}$th to $\frac{1}{4}$ per cent. at maximum scale reading. Good commercial instruments may, when giving their maximum reading, be relied upon to within 1 or 2 per cent., and cheaper ones to within 2 to 4 per cent. The accuracy of ammeters, owing to the use of shunts, is always inferior to that of voltmeters of the same type.

§ 46. Illuminated Dial and Edgewise Instruments.

Instruments intended for switchboard use should have a large, boldly marked scale and conspicuous pointer, so as to enable them to be easily read from any part of the engine-room. An arrangement frequently used for this purpose is that of an *illuminated dial* or scale. This is shown in Fig. 28. The scale is of opal glass, and is strongly illuminated from behind by means of one or more incandescent lamps, mirrors being sometimes used to increase the illumination and render it more uniform. The black pointer of the instrument stands out strongly against the bright background of the scale, and the instrument is easily read from a considerable distance.

Where economy of space is of great importance, and the switchboard dimensions have to be kept as small as possible, the *edgewise* type of instrument is employed. In instruments of ordinary

construction, the scale lies in a plane parallel to the plane of motion of the pointer. In the edgewise type, the end of the

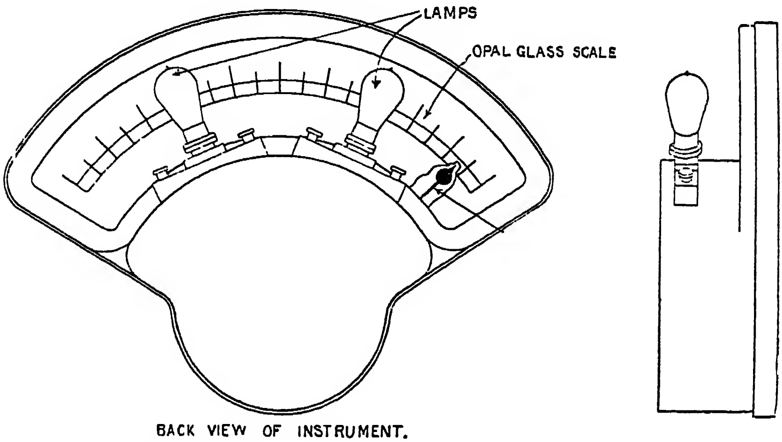


FIG. 28.—Showing arrangement of illuminated scale.

pointer is bent at right angles, so that as the pointer moves its bent end sweeps out a cylindrical surface. The scale of the instrument is accordingly made cylindrical, the axis of the cylinder coinciding with the axis of rotation of the movable part of the instrument. The general arrangement of an instrument of this type will be understood from Fig. 29, and since the flat instrument case stands out at right angles to the plane of the switchboard instead of lying flat against it, it will readily be seen that a large number of such instruments may be crowded into a comparatively small space.

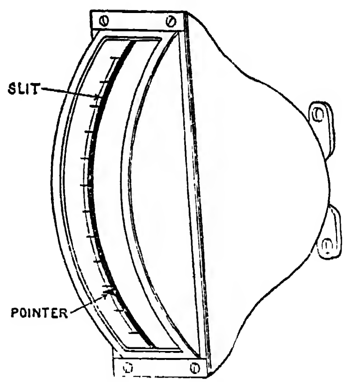


FIG. 29.—Edgewise type of instrument.

§ 47. Recording Instruments.

In some cases, a *continuous record* of the fluctuations in the p.d. or current is required, and for this purpose *recording* instruments

are employed. The pointer of the instrument carries a small reservoir pen whose tubular point presses lightly against a sheet of paper driven at a uniform rate by clockwork in a direction at right angles to the direction of motion of the pen. The general arrangement of the instrument and details of one form of pen will be understood from Fig. 30, which shows an instrument by Messrs. Elliott Brothers. The main trouble in connection with such instruments is the friction between the pen and paper, and although this may be made small, it is always liable to interfere with the accuracy of the instrument. For this reason, it is advisable to use powerful deflecting and controlling torques, so as to render the frictional torque due to the pen insignificant in comparison with them.

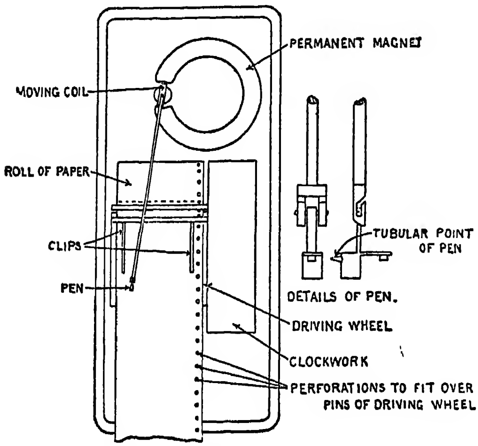


FIG. 30.—Recording instrument.

§ 48. Relative Merits and Prices of different types of Instruments.

The electrostatic type of instrument forms the ideal voltmeter. It takes no current and hence wastes no power; it is practically quite free from temperature errors; and an additional advantage is that it may be used on alternating as well as on continuous current circuits. The only disadvantage of the instrument as compared with other types is its high price. A multicellular electrostatic voltmeter costs £22.

Moving-coil instruments, which cost from £3 10*s*. to £15, depending on the range, size of scale, and quality of workmanship, are at present more widely employed on switchboards

than any other type. They have a uniformly divided scale, are dead-beat in their indications, and are free from the hysteresis error which affects soft iron instruments.

Soft iron instruments cost from £2 upwards, and are the cheapest class of instrument obtainable. They are extensively used in all cases where low first cost is an important consideration, and where a high degree of accuracy is not desired—as in the case of small motors owned or hired by private consumers, on private lighting or power switchboards, &c.

Electro-dynamic instruments are expensive, and are, generally speaking, of more importance in connection with alternating than with continuous currents. In connection with the latter, they are chiefly used as standard instruments. The best known and most important representative of this class of instrument is the Kelvin Balance, which costs from £70 to £100, according to the range.

§ 49. Limited Scale Voltmeters.

Switchboard voltmeters are generally required to give readings with a high degree of accuracy over a limited range only, since the p.d. at a generating station fluctuates only between narrow limits. In order to allow of greater ease and accuracy in taking the readings, the scale of such instruments is sometimes confined to the limits required, and the divisions can consequently be made large and bold. This end may be attained by using comparatively weak springs in the instruments, and by having them "set up" or wound up to start with, the pointer being pressed against a stop, so that the deflecting couple is unable to overcome the control of the spring until the p.d. reaches a certain minimum value, beyond which the pointer will begin to move across the scale; owing to the weakness of the springs, the displacement of the pointer corresponding to a given percentage change of p.d. will be much larger than in instruments of ordinary construction. Although such instruments of limited range are easy to read, their accuracy is not greatly superior to that of instruments which start reading at zero, owing to the fact that the errors of the instrument are magnified in about the same ratio as its scale divisions.

CHAPTER VI.

50. Calibration of ammeters and voltmeters—§ 51. Use of copper voltameter for calibrating ammeters—§ 52. Principle of potentiometer method—§ 53. Crompton potentiometer—§ 54. Calibration of voltmeters and ammeters by potentiometer method—§ 55. Electric supply meters—§ 56. Bastian electrolytic meter—§ 57. Reason Manufacturing Co.'s meter—§ 58. Chamberlain and Hookham meter—§ 59. Ferranti meter—§59a. Theory of mercury motor meters—§ 59b. Commutator motor meters—§ 60. Aron clock meter—§ 61. Limits of accuracy and prices of electric supply meters.

§ 50. Calibration of Ammeters and Voltmeters.

One of the simplest and most convenient methods of "calibrating" an ammeter or voltmeter, i.e., of ascertaining the relation connecting its scale readings with the true values of the current or p.d., consists in comparing its readings with those of some standard instrument which may be taken as correct. As suitable standards, we may mention Lord Kelvin's multicellular voltmeters and current balances (§§ 38 and 39), or a high-class moving-coil instrument (§ 40). In the case of ammeters, the instrument under test is connected *in series* with the standard, so that the same current flows through both, and a double set of readings (instrument and standard) is taken at regular intervals along the scale of the instrument. In the case of voltmeters, the instrument under test and the standard are joined *in parallel*, so as to obtain the same p.d. across each, and as before a double set of readings is obtained.

From the two sets of readings the differences between the scale readings of the instrument and the true readings are easily obtained. This difference will in general vary at different parts of the scale. If s be the scale reading, and t the true reading or correct value (as given by standard instrument) corresponding to it, and if c represent the difference between the two, so that $t = s + c$, then c may be termed the *correction* at the part of the scale considered, and may obviously be positive or negative.

Care must, of course, be taken to arrange the instruments so that they do not affect each other. If, for example, both instruments are of the moving-coil type, they should be placed several feet apart.

Having found the values of c from our double set of readings, we may next plot a *curve of corrections* (sometimes also called *error curve*) connecting values of s (plotted horizontally) with values of c (plotted vertically), positive values of c being plotted upwards, and negative downwards. Such a curve of corrections immediately enables us to ascertain the true value corresponding to any given scale reading. For the sake of convenience, it is advisable to make the vertical or c-scale much larger than the horizontal or s-scale.

§ 51. Use of Copper Voltameter for Calibrating Ammeters.

A method of calibration applicable to ammeters which does not involve the use of any standard instrument is that of the copper voltameter. This voltameter has formed the subject of numerous careful researches, and the conditions which must be complied with in order to secure reliable results with it are now well established.

The solution is prepared by dissolving pure re-crystallised copper sulphate in ordinary tap-water, the amount of water being such as to give a density lying between 1·15 and 1·18. To this solution is added 1 per cent. (by volume) of strong sulphuric acid. Each cathode plate is arranged between two anode plates, the distance apart of the cathode and anode on either side of it being about half an inch. In calculating the current from the amount of copper deposited, the following values of the apparent electro-chemical equivalent of copper* for various current densities at the cathode may be used :—

Square inches of cathode area per ampere	10	20	25	30	35	40	45
Apparent electro-chemical equivalent	$328{\cdot}6 \times 10^{-6}$	$328\ 2 \times 10^{-6}$	328×10^{-6}	$327{\cdot}8 \times 10^{-6}$	$327\ 6 \times 10^{-6}$	$327\ 4 \times 10^{-6}$	$327{\cdot}2 \times 10^{-6}$

* The true electro-chemical equivalent of copper is an *absolute constant* of nature; but the amount of copper actually obtained on the cathode plate of a copper voltameter due to the passage of one coulomb is slightly variable, owing to the fact that some of the freshly precipitated copper is attacked by the electrolyte, and goes into solution.

If the current density exceeds ·1 amp. / sq. in., the deposit is unsatisfactory, and does not adhere firmly to the plate.

The calibration of Lord Kelvin's current balances is carried out by this method, which is particularly applicable to instruments obeying a definite law. Where there is no simple relation connecting the deflection with the current, the method is not so convenient, as a single experiment enables us to obtain the correction at one part of the scale only.

§ 52. Principle of Potentiometer Method.

The principle of this important method is the fact that in any circuit conveying a steady current the drop of potential occurring in any part of the circuit is proportional to the resistance of that part. The voltage between two (variable) points in such a circuit may be adjusted as desired by selecting the positions of the points so as to include more or less resistance between them. Let us, for example, consider the case of a circuit in which there is a steady current of ·01 ampere. If we suppose that part of the circuit consists of a set of exactly equal resistances, each of 10 ohms, joined in series, then the drop of potential over each of these resistances will amount to $10 \times \cdot 01 = \cdot 1$ volt. We may thus obtain voltages of ·1, ·2, ·3, &c., of a volt, by taking the terminals of a single, or of two, three, &c., of our equal resistances in series. This gives us a sort of "volt scale," each division of which represents ·1 volt. For purposes of exact measurement, however, it is essential to have a more minute sub-division of our volt scale. This may be readily provided by including in our circuit a slide-wire of uniform diameter, so that the drop of potential over any part of it is proportional to the length of that part. Let a length of this wire be chosen such that its resistance is exactly equal to that of one of our set of coils of equal resistance, and let this length be provided with a scale divided into 100 equal parts. Let us further suppose that by the aid of a slider contact may be established with any point of the wire, and so any desired fraction of its length be included in the part of the circuit across which the desired voltage is to be obtained. It is now evident that we can obtain any voltage from the maximum corresponding to the entire set of coils and slide-wire in series, down to a voltage represented by one

hundredth part of the wire—i.e., by $\frac{1}{100} = \cdot 001$ volt—by steps of ·001 volt at a time; while by estimating tenths of a division of the wire, the voltage may be read to ·0001 volt.

The set of coils of equal resistance are termed the potentiometer coils, and the slide wire the potentiometer wire. Taken together, they constitute a *potentiometer.*

A potentiometer is thus simply a "volt scale," capable of furnishing us with accurately known voltages. We have next to consider how such a volt scale may be applied to the measurement of unknown voltages.

The measurement of an unknown voltage by the potentiometer method consists in finding a known voltage on the potentiometer which is exactly equal to the unknown voltage. The measurement therefore resolves itself into ascertaining whether two voltages —belonging to two distinct circuits—are equal or not. This is easily effected as follows:—

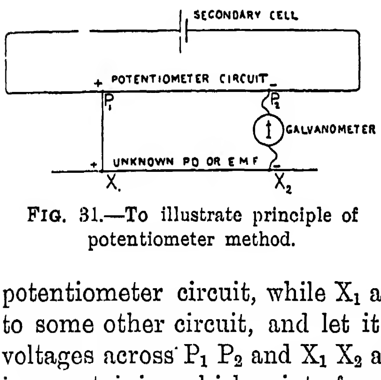


FIG. 31.—To illustrate principle of potentiometer method.

Let in Fig. 31 P_1 P_2 represent a given portion of the potentiometer circuit, while X_1 and X_2 are two points belonging to some other circuit, and let it be required to find whether the voltages across P_1 P_2 and X_1 X_2 are equal. The first step consists in ascertaining which point of each pair is at the higher potential. Let us suppose that P_1 is at a higher potential than P_2, and X_1 at a higher potential than X_2. P_1 is then connected to X_1, and so P_1 and X_1 are reduced to the same potential. If now the p.d. across X_1 X_2 is equal to that across P_1 P_2, then P_2 will be at the same potential as X_2. Hence P_2 may be connected to X_2 without causing any current to flow between them. In order to ascertain the equality in the potentials of P_2 and X_2, a galvanometer is connected across the two points as shown. If P_2 is at a higher potential than X_2, a current will flow from P_2 to X_2; but if P_2 is at a lower potential than X_2, the current will flow in the opposite direction. The measurement consists in adjusting the p.d. across P_1 P_2 until it is found to balance the unknown p.d. or e.m.f. between X_1 and X_2.

In order that the potential drop over each coil of the potentiometer may remain constant, the current must be maintained at a constant value. For this reason, a secondary cell is used as the source of current, as such a cell is capable of supplying a current which will remain practically constant over considerable periods.

It is evident that the first step in using the potentiometer is to adjust the current with a high degree of accuracy to the value required. Thus in the case considered above, in which each potentiometer coil has a resistance of 10 ohms, the current must be ·01 ampere. At first sight, the simplest plan would appear to be that of including a suitable ammeter in the potentiometer circuit. Even the best ammeter, however, could not be read with a degree of accuracy comparable with that which may be attained in testing the equality of the p.d.'s across P_1 P_2 and X_1 X_2 in Fig. 31. The accuracy of the potentiometer measurement would thus be limited by the accuracy of the ammeter included in its circuit. In order to avoid this disadvantage, the adjustment of the current is carried out by a different method, as follows. A standard cell—such as a Clark cell—whose e.m.f. is known with a high degree of accuracy, is connected between the points X_1 X_2, and the resistance between P_1 P_2 is arranged so that if the required current were flowing in the potentiometer circuit the drop of potential over P_1 P_2 would equal the e.m.f. of the standard cell. The current is next adjusted so that the drop over P_1 P_2 equals the e.m.f. of the standard cell. In the case of a Clark cell at 15° C., e.g., whose e.m.f. is 1·434 volts, and potentiometer coils of 10 ohms resistance each, the resistance between P_1 P_2 would be made 143·4 ohms —i.e., it would consist of 14 whole coils and 34 scale divisions of the slide-wire, as with this resistance between P_1 P_2 and a current of ·01 ampere the p.d. across P_1 P_2 would be 1·434 volts.

§ 53. Crompton Potentiometer.

One of the best known forms of potentiometer is the Crompton potentiometer, the connections of which are shown in Fig. 32. There are 14 potentiometer coils and a slide-wire (to

To 1 part in 1,000.

correspond to the e.m.f. of 1·434 volts of a Clark cell). In addition to these, a set of resistance coils for rough adjustment

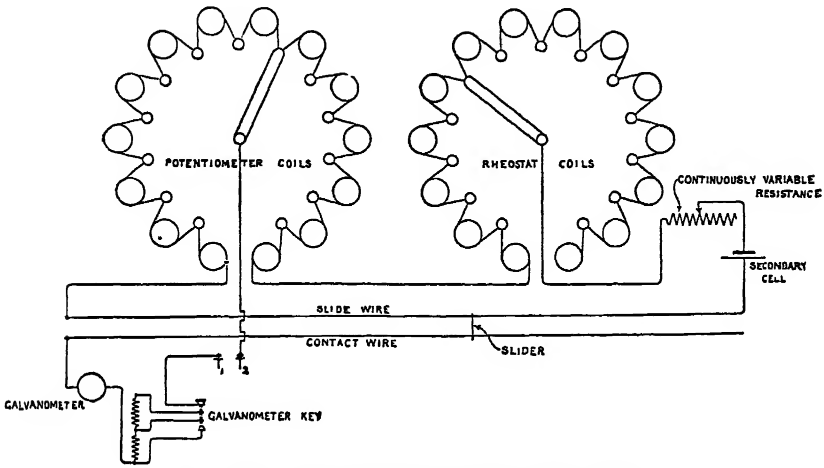


FIG. 32.—Connections of Crompton potentiometer.

of the current, and a continuously variable resistance (consisting of a bare wire wound in a spiral groove on the surface of

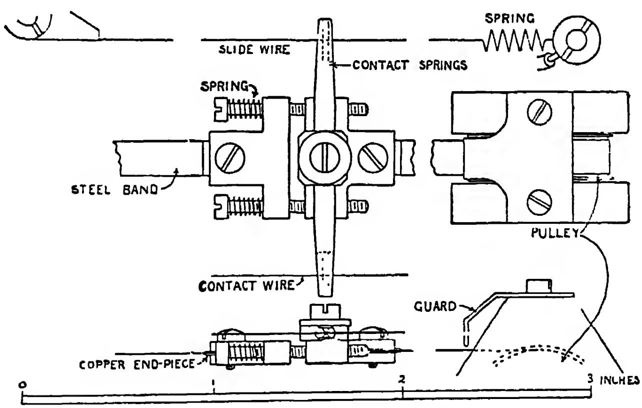


FIG. 33.—Details of Crompton potentiometer.

an ebonite cylinder) for fine adjustment are provided. Contact with the slide-wire is obtained by means of a slider which bridges across the potentiometer wire and a second wire, marked

"contact wire" in Fig. 32, stretched parallel to it. In order to minimise the wear of the slide-wire as much as possible, this wire is made of a nickel-steel alloy ("nickelin"), a material combining low temperature coefficient of resistance with great mechanical hardness. The other coils are made of the same material, are silk-covered, and wound on small boxwood bobbins arranged in a circle as shown. The method of fixing the slide-wire and details of the construction of the slider are shown in Fig. 33. The slider is mounted on a divided carriage formed by two cross-heads attached to the ends of a flat steel band passing over two pulleys, one at each end of the instrument base-board. By turning either pulley, the slider may be moved one way or the other. The slider itself consists of a couple of flat springs fitted with short lengths of silver wire along the end portions of their inner surfaces, the cylindrical surfaces of the silver wires bearing on the slide and contact wires.

The terminals T_1 T_2 in Fig. 32 correspond to the points X_1 X_2 in the diagram of Fig. 31. The galvanometer key, it will be noticed, is of special construction. If it is pressed only far enough to close the first contact, a sufficient amount of resistance is included to prevent an excessive current from passing through the galvanometer. If the deflection is very small, the key is pressed further, until the second contact is reached and the first section of the resistance short-circuited. Should the deflection be still inconveniently small, the key is pressed hard down, all the resistance being thereby short-circuited.

In order to save time when a number of measurements have to be made in succession, the points T_1 T_2 are connected through a multiple-contact switch to a number of pairs of terminals, so that the passage from one pair of terminals to the next is rapidly effected by simply turning the multiple-contact switch.

The first step in using the potentiometer consists in standardising it—i.e., in adjusting the current to the correct value—by means of a standard cell, in the manner explained in the latter part of § 52.

§ 54. Calibration of Voltmeters and Ammeters by Potentiometer Method.

By means of the potentiometer, we can directly measure voltages up to about 1·5 volts. But the range of measurement

may be readily extended to very much higher voltages by the simple expedient of subdividing the unknown voltage by means of a suitable resistance, and measuring a known fraction of it on the potentiometer. Suppose, for example, that the unknown voltage does not exceed 600. The voltage which we can directly measure does not exceed 1·5, which is $\frac{1 \cdot 5}{600}$ or $\frac{1}{400}$th of the unknown voltage. If, therefore, we apply our unknown voltage to a resistance of, say, 40,000 ohms, subdivided, as shown in Fig. 34, so that the resistance of one section, $X_1 X_2$, is only 100 ohms, or $\frac{1}{400}$th of the total, then it is evident that the p.d. across this section will also be only $\frac{1}{400}$th of the total unknown p.d. But this fraction of the total p.d. is within the range of direct measurement on the potentiometer. A subdividing resistance for use with the potentiometer is termed a *volt-box*, and for the sake of convenience several points of subdivision are provided, corresponding to different fractions of the total resistance. In calibrating a voltmeter, the volt-box is connected in parallel with the voltmeter.

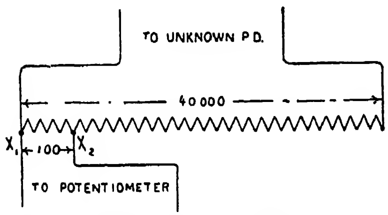


FIG. 34.—Measurement of high voltage by potentiometer.

The measurement of currents, and the calibration of ammeters, are readily effected by introducing a known standard resistance into the circuit, and measuring the p.d. across it by means of the potentiometer. If the current does not exceed 50 amperes, for example, a standard resistance of $\frac{1 \cdot 5}{50} = \cdot 03$ ohm would be suitable. Such resistances take various forms, according to the magnitude of the current to be dealt with. For small currents (up to about 10 amperes) spirals of manganin wire are used; while for larger currents, wide strips of the same metal suitably mounted are commonly employed, several strips being connected in parallel if the current is very large.

§ 55. Electric Supply Meters.

For the measurement of the total amount of energy supplied to a consumer, instruments generally known as supply meters, or

meters simply, are used. Let V_1, V_2, V_3, &c., and i_1, i_2, i_3, &c., respectively stand for the p.d.'s and currents during the successive time intervals t_1, t_2, t_3, &c., respectively. The total energy supplied to the consumer is

$$E = V_1 i_1 t_1 + V_2 i_2 t_2 + V_3 i_3 t_3 + \quad . \quad . \quad . \quad . \quad . \quad .$$

An instrument which measures the sum of all such expressions as $V_n i_n t_n$, and whose reading therefore depends on the voltage as well as on the current during any time interval, is termed an *energy meter*, *watt-hour meter*, or *integrating wattmeter*. Electric energy is generally supplied at an approximately constant p.d., so that if we neglect fluctuations in the p.d., and assume that it remains constant at a value V corresponding to its mean value the expression for the energy may be written

$$E = V (i_1 t_1 + i_2 t_2 + i_3 t_3 +). \quad . \quad . \quad . \quad . \quad .$$

An instrument whose reading depends simply on the expression within the brackets—which represents the total electric *quantity* supplied to the consumer—is termed a *quantity meter*, *coulomb-meter*, or *integrating ammeter*. Such a meter takes no account of fluctuations in the p.d., and from this point of view is a less perfect instrument than an energy meter. On the other hand, quantity meters are simpler and cheaper to construct than energy meters, and are for this reason much more extensively used, especially for small consumers.

Most electric supply meters may be referred to one or other of three main types, viz., (1) electrolytic meters, (2) motor meters, and (3) clock meters.

§ 56. Bastian Electrolytic Meter.

The electrolytic type of supply meter depends, as indicated by its name, on the chemical effect of the current. It is one of the earliest types introduced, various arrangements having been used by Edison at different times. These earlier forms were shunted copper or zinc voltameters, and most of them were not direct-reading, the quantity having to be determined periodically by weighing the plates. An electrolytic meter is, by its very nature, a *quantity* meter.

One of the simplest modern representatives of the electrolytic type of meter is the Bastian meter. This is in principle a water voltameter. The electric quantity which has passed through the

meter is measured, not by collecting the oxygen and hydrogen evolved, but by determining the amount of water which has disappeared by being electrolysed. The electrolyte employed is a solution of caustic soda in water, and the electrodes are made of nickel.* The essential parts of the meter are shown in Fig. 35. The containing vessel, Fig. 35 (*a*), is of cylindrical shape, with a bulb at the lower end. In order to prevent loss of water by evaporation, the top of the electrolyte is covered with ½ inch of a non-volatile oil, the junction of the electrolyte and oil forming the index, which is viewed through the central slot in the plane sheet-zinc scale attached to the front of the containing vessel. At the back of this vessel is a curved sheet of zinc silvered on the inside to form a mirror, and this mirror is held in position by four springs which in front are attached to the scale

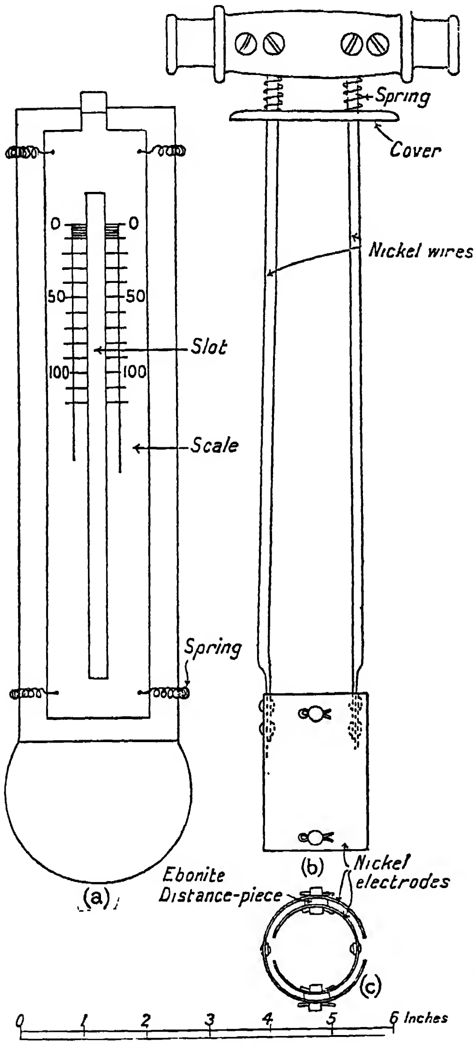


FIG. 35.—Bastian electrolytic meter.

* In the earlier forms of the Bastian meter, platinum foil electrodes in a solution of sulphuric acid were used. Owing to trouble with the electrodes, this arrangement was abandoned in favour of that now in use.

as shown. The scale is provided with a tag at the top which is bent over the top of the vessel and rests on the cover, thus affording additional security against any displacement of the scale. Figs. 35 (*b*) and (*c*) show details of the cylindrical sheet nickel electrodes, which are supported by two stout nickel wires passing through the cover and clamped to the terminal holder at the top. The cover is pressed down against the top of the vessel by means of two springs as shown. The meter is contained in a stout cast-iron case provided with a glass front. When the electrolyte has sunk to the lower portion of the scale, ordinary tap water is used for refilling the tube.

The great advantages of the Bastian meter are its simplicity, reliability and cheapness. Its main disadvantage is the relatively high voltage drop, which even with the smallest loads is of the order of two volts, and rises to as high a value as 2·7 volts at full load.

§ 57. Reason Manufacturing Co.'s Electrolytic Supply Meter.

Another example of the electrolytic type is the meter manufactured by the Reason Manufacturing Co., Ltd., of Brighton. Originally, this meter was the invention of A. Wright, but it has been considerably improved by Hatfield. The essential parts of this meter, and a diagram of its connections, are shown in Fig. 36. Cemented to a glass reservoir of the shape shown is a long glass tube, flattened sideways and provided with a scale (the scale is not shown in the diagram). The bulk of the reservoir and tube, which are hermetically sealed, is filled with the electrolyte—a double iodide of mercury and potassium. The current passes along a platinum wire sealed through the wall of the reservoir, and reaches the mercury anode, which is contained in an annular trough around the lower part of the reservoir. It then flows through the electrolyte to the thin ring-shaped iridium* foil cathode, and passes out by the wire connected to it. Mercury goes into solution at the anode, and is deposited on the cathode, where it collects in small drops which finally fall down the glass tube and collect in it. The

* Platinum foil cathodes were found to be attacked.

total electric quantity is proportional to the amount of mercury which has collected in the glass tube, and the corresponding energy is read off directly on the scale.

Means must obviously be provided for maintaining the supply of mercury in the annular trough. This is done automatically by providing the reservoir with a side vessel communicating with it and shaped somewhat like the inverted bulb of a retort. This is at the start completely filled with mercury, and the level of the mercury in the annular trough is sufficiently high to completely close up the orifice of the side vessel. As the mercury goes into solution, its level is gradually lowered, until finally the upper part of the orifice or channel connecting the two vessels becomes sufficiently exposed to enable a drop of the electrolyte to force its way along the channel and bubble up through the mercury to the top of the side vessel, an equal volume of mercury being thereby displaced which flows into the annular trough and again seals up the channel connecting the two vessels. In this way, the mercury in the anode trough is automatically maintained at an approximately constant level.

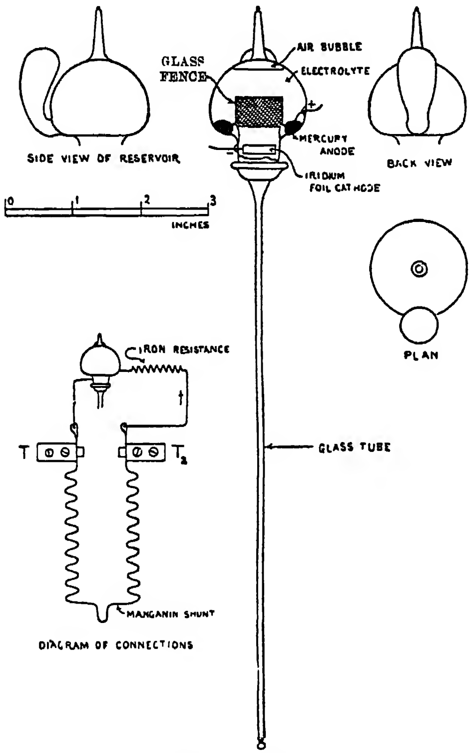


FIG. 36.—Wright electrolytic meter.

In order to prevent the mercury in the anode trough from being mechanically jerked over into the tube, a cylindrical glass fence is provided, having its bottom edge fused to the projecting glass lip

which forms the inner boundary of the trough. A further safeguard against the effects of vibration or blows is provided by a spring suspension of the working parts. The instrument is re-set very simply after a quantity of mercury has collected in the tube and the reading has been taken, by tilting the entire tube (which is suitably pivoted) so as to allow the mercury to flow back into the side vessel of the reservoir.

It would be impracticable to allow the entire current to flow through the voltameter, hence a manganin shunt is provided which takes the bulk of the current (see diagram of connections, Fig. 36). This shunt will have practically the same resistance at all temperatures. Since, however, the resistance of the voltameter changes considerably with temperature, decreasing as the temperature rises (by about 2½ per cent. per degree C.), it is obvious that a large temperature error would be introduced if the voltameter were connected directly across the shunt. This difficulty is overcome in a very simple and ingenious way by connecting in series with the voltameter, which has a negative temperature coefficient, a coil of iron wire, whose temperature coefficient is positive, the resistance of the wire being such that the total temperature coefficient of the voltameter and iron coil in series vanishes. The loose coil of silk-covered iron wire which forms this compensating resistance is immersed in oil contained in a small test-tube arranged at the back of the instrument case.

§ 58. Chamberlain and Hookham Meter.

The most numerous class of electric supply meters is that belonging to the *motor type*. The feature common to all such meters is the use of some form of electric motor. In some cases (as in the Elihu Thomson meter) the motor is of somewhat elaborate construction, resembling in general arrangement the ordinary forms of electric motors (without, however, any iron cores). But in most representatives of this type the motor is of a very simple and rudimentary form. The rotating part generally takes the shape of a cylinder or a flat disc.

As one example of a motor meter, we shall select for description one of the types manufactured by Messrs. Chamberlain and Hookham, the general arrangement of which is shown in Fig. 37.

A permanent steel magnet is fitted with soft-iron pole-pieces of the shape shown, terminating in cylindrical projections, between the opposing plane end surfaces of which an intense magnetic field is produced. The motoring and braking elements of the meter are both represented by a single disc mounted on a spindle which at its upper end carries a pinion gearing with the counting

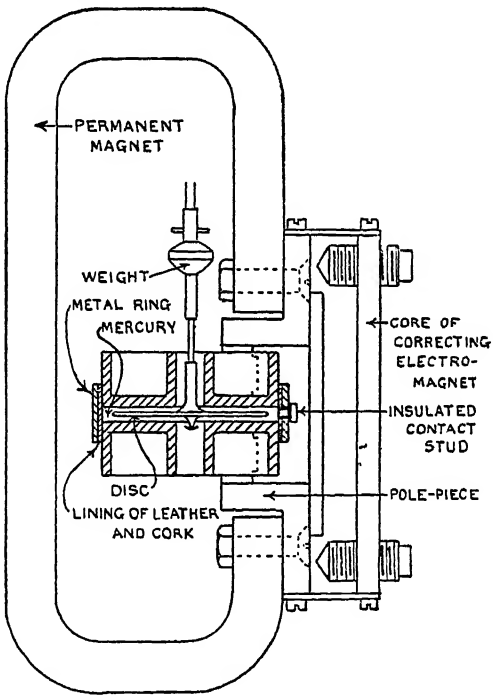


FIG. 37.—Chamberlain and Hookham meter.

mechanism. The disc is completely immersed in mercury contained in a chamber formed by two castings and a metal ring which may be slipped over them and tightened. The cylindrical pole-pieces of the permanent magnet are embedded in the castings. The surfaces of the castings are insulated. The metal ring which forms the side walls of the mercury chamber is lined with leather and cork, and at one portion of its surface is provided with a raised boss which carries an insulated contact stud. The line joining the centre of the

disc and the contact stud bisects the cylindrical pole-pieces. The edge of the disc and the area in the immediate neighbourhood of the centre are well amalgamated. In order to prevent the disc from floating on the mercury, the upper part of the spindle carries a weight. The object of having the disc immersed in mercury is to provide a convenient method of leading the current into the central area of the disc and leading it out of the edge of the disc to the contact stud. The current enters the meter by the screw (not shown in figure) which carries the jewelled cup supporting the lower end of the spindle carrying the disc, passes through the mercury into the central amalgamated area of the disc, then spreads over the disc and converges towards those portions of the edge which are opposite the contact stud. The main flow of current is roughly radial. A certain small portion of the current will, of course, flow through the mercury without entering the disc. Now the roughly radial horizontal current sheet which exists in the disc is under the influence of the vertical field between the cylindrical pole-shoes, and, in accordance with the principle of § 2, a tangential force will be exerted on the disc, producing rotation. As soon as rotation begins to take place, however, e.m.f.'s will be induced in the disc or rotor which produce a system of currents opposing the motion (in accordance with Lenz's law, § 5). Let us for a moment suppose that the magnetic field in which the disc is placed remains constant, and that the only resisting torque is that due to the induced eddy-currents. Then the driving torque is proportional to the current, and the resisting torque to the speed. Hence, the driving torque being equal to the resisting torque when a constant speed has been reached, it follows that the current will be proportional to the speed, and therefore the total quantity which has passed through the meter during a given time to the total number of revolutions made by the rotor during that time. The meter will therefore act as a *quantity meter*.

In the above, we have supposed that the *only* resisting torque is that due to eddy-currents. But in addition to this, we also have the frictional resisting torque due to friction at the rotor pivots and friction in the counting train, and further the resistance due to mercury friction, the rotor being immersed in mercury. The friction of the counting train is very small. The rotor pivot friction may be made practically negligible; not so, however, the

mercury friction, the retarding torque due to which increases rapidly with the speed. In order to compensate for the appreciable mercury friction at the higher speeds, a correcting electro-magnet is provided. This forms a variable magnetic shunt across the poles of the permanent magnet, and with increasing load diverts more and more of the magnetic flux which would otherwise pass through the disc into itself. Now if the gap field is weakened in a certain ratio, the driving torque will be reduced in the same ratio, but the eddy-current torque will be reduced in the *square* of this ratio,* and so an increase of speed will take place, the retarding effect of mercury friction being thereby compensated. The correcting electro-magnet is provided with movable poles which may be screwed into or out of the core, the gaps between the permanent magnet pole-pieces and those of the correcting magnet being thereby reduced or increased. By increasing the gaps, the magnetic shunting effect is reduced, and with it the speed; the opposite effect taking place when the gaps are reduced. This furnishes a convenient method of adjusting the speed to the desired value, and so making the meter register correctly.

§ 59. Ferranti Meter.

Another example of a motor meter is the Ferranti-Hamilton meter. This is shown in Fig. 38. The rotor is in the form of a copper disc. The rotor spindle carries a worm at the top, which gears with a worm-wheel; the latter being in direct connection with the counting train. The rotor disc is platinum-plated and enamelled, except at its centre and

* The driving current is obviously independent of the magnetic field; a change in the latter in a given ratio will therefore alter the driving torque in the same ratio. But the eddy currents are proportional to the field, and the product "eddy current $\times$ field" which determines the resisting torque, is thus proportional to $(\text{field})^2$.

along its edge, where it is amalgamated so as to make good contact with the mercury. The rotor disc is placed in a mercury bath formed by two nickel-plated brass plates separated by an intervening ring of vulcanised fibre. The inner surfaces of the brass plates are prevented from coming into direct contact with the

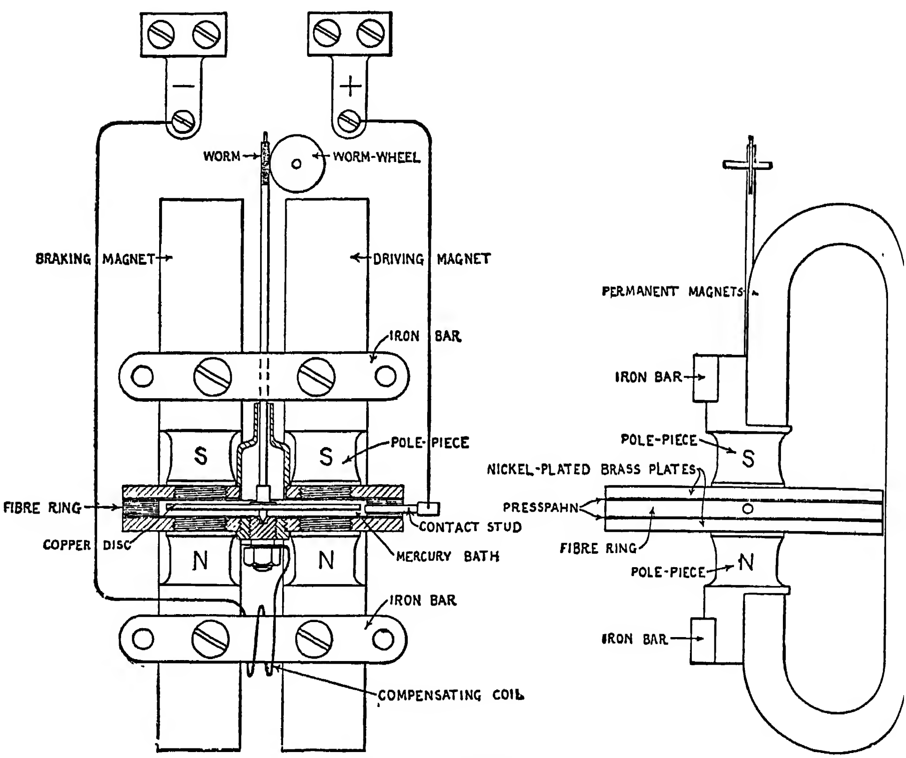


FIG. 38.—Ferranti-Hamilton meter.

mercury by two presspahn washers. The current enters the mercury bath by a horizontal contact-stud passing through the fibre ring, and the greater part of it flows in a radial direction through the rotor disc towards its central portion, where it enters the mercury and flows down it to the vertical contact

stud. From this it passes round the compensating coil and finally reaches the negative terminal.

Two powerful permanent magnets placed side by side, and of the shape shown in Fig. 38, are fitted with pole-pieces screwed into the brass plates forming the mercury bath. The two upper as well as the two lower poles are of the same polarity, and each pair is connected by a bar of soft iron, the bar connecting the lower pair being surrounded by the compensating coil. The right-hand magnet is the driving magnet, and provides a vertical field through that portion of the rotor disc which carries the greater part of the main current. The current flowing radially inwards, and the field being vertically upwards, the disc will (§ 2) be driven in a counter-clockwise direction when viewed from above. The other magnet provides a vertical field which induces eddy-currents in the disc and so supplies the necessary braking torque. As in the Chamberlain and Hookham meter, the driving torque is proportional to the current, and the braking torque to the speed. Hence the current varies as the speed, and the quantity as the number of revolutions, the meter being again a *quantity* meter.

Mercury friction is compensated for by an automatic strengthening of the driving field and an equal weakening of the braking field with increase of load. This is brought about by the compensating coil, which, as an inspection of Fig. 38 will show, gives rise to a local magnetic flux around the magnetic circuit formed by the two air-gaps, the two pairs of poles, and their soft iron connecting bars. The superposition of this local magnetic flux on the main flux produces a redistribution of the resultant flux of the nature mentioned, viz., a crowding of the lines into the driving gap, and a reduction in the lines of the braking gap. It is to be noted that this produces no magnetising or demagnetising effect on the permanent magnets themselves.

The magnet which we have called the "driving magnet" also induces eddy-currents in the disc, and so contributes towards the resisting torque. Let H stand for the field intensity in either air-gap when there is no current flowing through the compensating coil. The driving torque will then be proportional to H, and the eddy-current torque to H^2. If now a current be allowed to flow through the compensating coil, increasing the driving air-gap field by an amount h, and

§ 59a. Theory of Mercury Motor Meters.

In the ideal ampere-hour motor meter the speed is strictly proportional to the current, and hence the revolutions made during a given time to the ampere-hours which have passed through the meter during that time. No actual motor meter is capable of complying with this requirement, exact proportionality between current and speed being unattainable, owing to various disturbing influences.

We shall first consider the plain or uncompensated mercury motor meter. In this the driving torque is proportional to the current, so that we may write

$$\text{driving torque} = k_1 i,$$

where i is the current and k_1 a constant. The resisting torque is made up of the following components: (1) the eddy-current component, which is proportional to the speed; (2) the mercury friction component, which may be taken to be approximately proportional to the square of the speed; and (3) the solid friction component (bearings and counting train), which is independent of the speed, We may therefore write

$$\text{retarding torque} = k_2 s + k_3 s^2 + k_4,$$

where s denotes the speed and k_2, k_3 and k_4 are constants. Now, when the meter is running steadily, the driving torque must equal the resisting torque. We thus have

$$k_1 i = k_2 s + k_3 s^2 + k_4,$$

so that

$$\frac{i}{s} = \frac{k_2}{k_1} + \frac{k_3}{k_1} s + \frac{k_4}{k_1 s},$$

or, writing, for the sake of brevity, $\frac{k_2}{k_1} = a$, $\frac{k_3}{k_1} = b$, and $\frac{k_4}{k_1} = c$,

$$\frac{i}{s} = a + bs + \frac{c}{s},$$

hence

$$\frac{s}{i} = \frac{1}{a + bs + c/s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

decreasing the braking air-gap field by the same amount, then the driving torque will be increased in the ratio $\frac{H + h}{H} = 1 + \frac{h}{H}$, while the eddy-current torque will be increased in the ratio $\frac{(H + h)^2 + (H - h)^2}{2\,H^2} = 1 + \left(\frac{h}{H}\right)^2$. Thus, since $\frac{h}{H} < 1$, the driving torque will be increased in a greater ratio than the eddy-current torque, and the balance will be available for compensating mercury friction.

Now in the ideal meter, $\frac{s}{i}$ is a constant; but equation (1) shows that such is not the case with any actual meter, and that s/i cannot be made constant.

The general law according to which s/i varies with the speed may be inferred from a consideration of the right-hand side of equation (1). Of the two disturbing terms, bs and c/s, the former is insignificant in comparison with the latter at very low speeds. As the speed gradually rises from very low values, bs increases, while c/s decreases; but since, so long as c/s is greater than bs, a given small change of speed will produce a larger numerical change in c/s than in bs, the sum $bs + c/s$ will *decrease*, and the ratio s/i will, in accordance with (1), *increase*. This increase in s/i with increasing speed will go on until

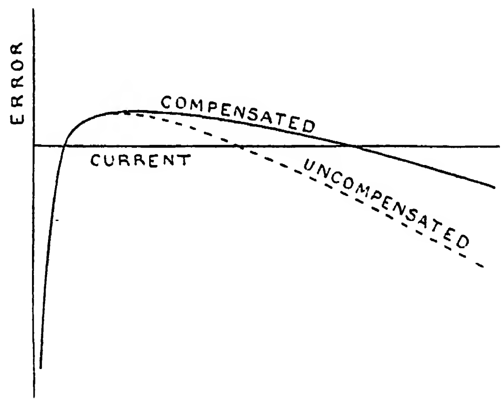


FIG. 38A.—Error curves of mercury motor meter.

bs becomes numerically equal to c/s, at which stage a small change of speed will result in approximately equal numerical changes in bs and c/s, so that their sum will remain unaltered. When bs becomes greater than c/s, a small change of speed will produce a larger numerical change in bs than in c/s, the sum $bs + c/s$ will increase, and the ratio s/i will *decrease*. The graph of s/i expressed as a function of s or i will therefore have the general shape shown by the curves in Fig. 38A, the curve rising to a maximum and then bending downwards.

By suitably choosing the wheels in the counting mechanism, the meter can be made to read correctly at one particular load or speed, corresponding to a definite value of s/i. If in the diagram of Fig. 38A we draw a horizontal line corresponding to this value of s/i, then

the amount by which the graph of s/i at other loads or speeds falls below or rises above this horizontal line will be proportional to the *error* of the meter, and hence, with a suitable change of scale, the graph will represent the percentage error of the meter, and we may then speak of it as the *error curve* of the meter.

Since with a curve shaped like that in Fig. 38A a horizontal line will in general intersect the curve *twice*, it will be seen that, in general, there will be *two* loads at which the meter reads correctly. Between these two loads it has a *positive error*, *i.e.*, it runs too fast; outside this range of load, its error is *negative*, *i.e.*, its speed is too low. In practice, the position of the horizontal axis corresponding to zero error is so chosen as to render the *average* error over the most important range of load as small as possible.

The use of the compensating devices already described results in a reduction in the droop of the error curve, as shown by the dotted and full-line curves of Fig. 38A.

§ 59b. Commutator Motor Meters.

Besides the mercury meter, another type of motor meter is widely used, viz., the commutator motor meter. In this, coils of fine wire arranged either in the plane of a disc or on the surface of a cylinder are used, a suitable fraction of the load current being allowed to flow through the coils, which are acted on by the field of a permanent magnet or magnets, and are thereby maintained in rotation so long as there is a current flowing through them.

The system of coils—usually three in number—is known as the *armature* of the tiny motor, and continuity of rotation is maintained by the use of a device known as a *commutator*, against which press contact springs known as *brushes*.

The principle of action of this type of meter will be understood by reference to Fig. 38B. Imagine three wires, marked 1, 2 and 3 in the figure, bent to the form of sectors as shown, and connected to three contact-pieces or *segments*, A, B and C, in the manner shown. A, B and C, which form portions of a cylindrical surface, constitute the *commutator* of the motor. The two contact springs or *brushes* are marked + and −, the current entering by the brush marked +, and leaving by that marked −. Consider now the position of the armature shown at (*a*). The conductor 1 is here short-circuited by the + brush, and carries no current, while in conductors 2 and 3

the currents flow in the directions indicated by the arrows. Imagine now that the dotted area M corresponds to the polar surfaces of a C-shaped permanent magnet, the direction of the magnetic field being vertically downwards through the surface of the paper, as indicated by the circle with a cross inside it. Then it is evident that since the radial portions of 2 and 3 which are under cover of M are placed in a magnetic field which is at right angles to them, they will experience a dynamical force which is at right angles both to the conductors and the field, and the sense in which this force acts will, by an application of Fleming's rule (§ 2), be found to be such as to cause a clockwise rotation of the armature. The armature will accordingly (provided the frictional resisting torque is not too great) be set in rotation. As rotation takes place, however, the active portions of 2 and 3 will gradually leave the field, as shown at (*b*), where the active portion of 3 is nearly entirely outside the field, while the active portion of (2) is on the point of beginning to move out of the field. A little later, as shown at (*c*), there are no conductors under cover of M (also no current in 2, since this is now short-circuited by the — brush), and hence no torque. Between the positions (*a*) and (*c*), therefore (corresponding to a rotation through 60 degrees), the torque decreases from a maximum to a zero

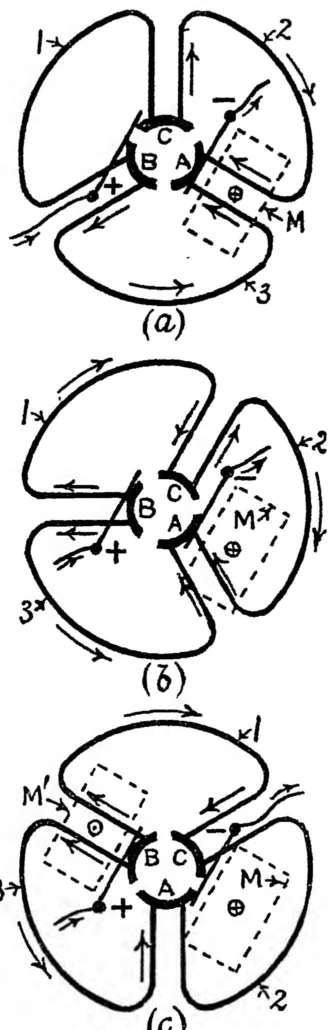


FIG. 38B.—To explain principle of commutator motor meter.

value. Position (c) thus constitutes a "dead point," and if the armature happened to be in this position when the current was switched on, it would not start. The difficulty is easily overcome by providing a second magnetic field, marked M′ in (c), diametrically opposite to M, and having an *upward* direction (circle with dot) through the plane of the paper. The torque due to M′ is at its maximum when that due to M is zero, and *vice versâ;* and in this way a continuous driving torque is obtained.

In practice, instead of the simple conductors of Fig. 38B, flat sector-shaped coils of fine wire are used. These are enclosed in a

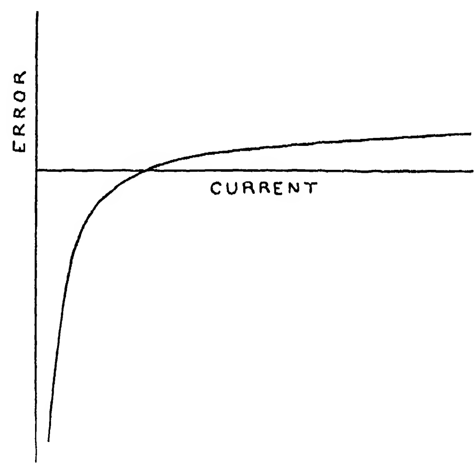


FIG. 38C.—Error curve of commutator motor meter.

shallow cylindrical box of aluminium, giving the armature the appearance of a solid aluminium disc. The aluminium casing forms the brake, the currents induced in it by the rotation of the armature furnishing the resisting torque.

Using the same notation as in § 59a, we have, in the commutator motor meter,

$$\text{driving torque} = k_1 i,$$

and

$$\text{resisting torque} = k_2 s + k_4,$$

the term $k_3 s^2$ which was present in the resisting torque of the mercury motor meter now dropping out, as there is no mercury friction; and the term k_4 now including brush friction as well as bearing and counting train friction.

When the motor is running steadily, we have

$$k_1 i = k_2 s + k_4,$$

and hence

$$\frac{i}{s} = a + \frac{c}{s},$$

or

$$\frac{s}{i} = \frac{1}{a + c/s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

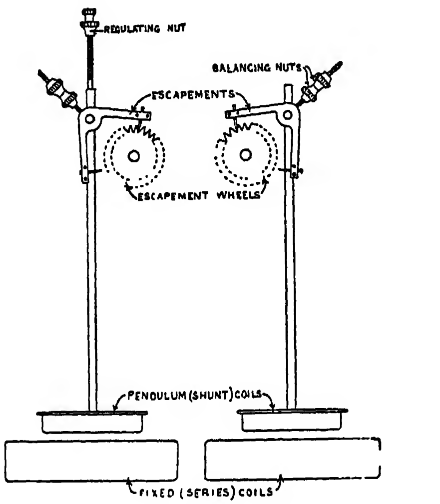
From this we see that with increasing load—*i.e.*, increasing s—the ratio s/i steadily increases, approaching the limit $1/a$ as s becomes very large. The graph of s/i will therefore be of the shape shown in Fig. 38c, and—unlike the graph of the mercury meter—exhibits no maximum point. By a suitable shift of the horizontal axis and change of vertical scale, the graph of s/i may be transformed into the *error curve* of the meter.

The most troublesome features of commutator motor meters are brush friction and uncertainty of brush contact. Owing to the necessity of reducing the dimensions of the commutator to the lowest practicable limit, commutator motor meters are all of the shunted type.

§ 60. Aron Clock Meter.

The principle of the clock type of meter is briefly as follows. Two pendulums, of similar construction and placed side by side, carry at their lower extremities fine-wire coils which are connected in series with each other and with a large non-inductive resistance, and are then bridged across the mains. Immediately underneath the pendulum coils are arranged two thick-wire or series coils which are inserted into one of the mains and convey the current which is being supplied to the consumer. The connections are made so that there is attraction between one of the pendulum coils and its series coil, resulting in a downward pull on the pendulum coil; and that there is repulsion between the remaining pendulum coil and its series coil, giving an upward thrust on that pendulum coil. Thus the first pendulum will be accelerated, and the second one retarded, when a current is flowing through all the coils. The accelerating and retarding forces will be proportional to the product of the currents in the pendulum and

fixed coils, but since the current through the pendulum coils is proportional to the p.d., the forces will vary as the product of the p.d. into the current—*i.e.*, as the power. Now it may be shown that so long as the accelerating and retarding forces do not exceed a certain limit, they are approximately proportional to the difference in the number of vibrations per second of the two pendulums, assuming that the periods of natural vibration of the pendulums—when the system is not acted on electro-magnetically—are approximately equal. From this it follows that the *energy* supplied to the consumer during a given period is approximately proportional to the difference in the number of vibrations made by the two pendulums during that time. Such a meter is, therefore, an *energy* and not a quantity meter like those hitherto considered.



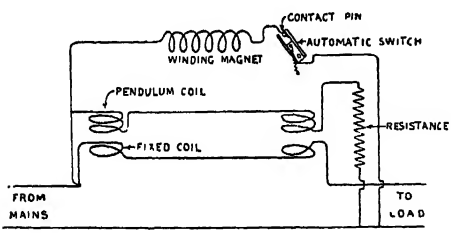


FIG. 39.—General arrangement and diagram of connections of Aron clock meter.

The general arrangement and electrical connections of the Aron clock meter will be understood from Figs. 39 (*a*) and (*b*). The natural periods of vibration may be adjusted to approximate equality (which is all that is necessary, exact isochronism not being required) by means of the regulating nut provided on one of the pendulums. Each pendulum is driven in the usual way by an escapement wheel. The spring driving

the escapement wheels is wound up by an electro-magnet about every half minute.

The winding magnet and its essential details are illustrated in Fig. 40. The mainspring consists of a flat strip of steel, one end of which is fixed to a pin mounted on a brass plate (not shown) screwed to the pole-pieces, while the other is attached to the Z-shaped armature of the magnet, which is mounted on an axle so as to be free to rotate between the pole-pieces. The core and

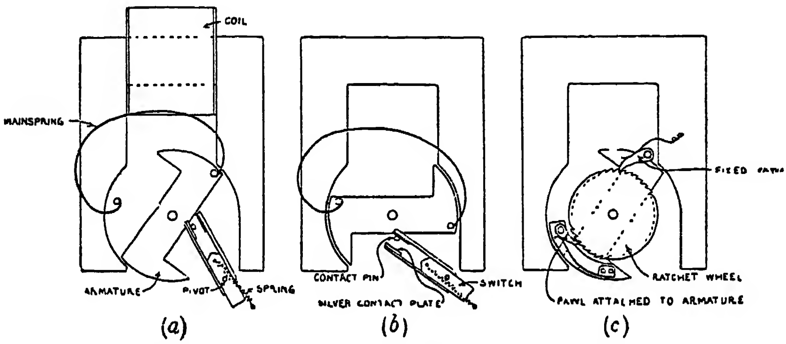


FIG. 40.—Winding electro-magnet of Aron clock meter.

armature of the magnet consist of thin iron stampings. Figs. 40 (*a*) and 40 (*b*) show the armature in its two extreme positions, (*a*) showing the mainspring unwound, and (*b*) fully wound. The torque due to the pull exerted by the mainspring on the armature is transmitted to the main driving axle by means of a pawl mounted on a brass segment screwed to one end of the armature, as shown in Fig. 40 (*c*), the pawl engaging a ratchet wheel mounted on the main driving axle. A second pawl (fixed to the front brass plate mentioned above) maintains the ratchet wheel in its position while the armature is winding up the mainspring and the pawl attached to it is slipping over the teeth of the ratchet wheel.*

The automatic switch which periodically (every 30 secs.) closes the circuit of the magnet coil and causes the mainspring to be

* Between that part of the driving axle which carries the ratchet wheel and the part carrying the driving differential gear is inserted an elastic coupling in the form of a spiral spring, the torque of which is sufficient to drive the pendulums during the very brief period that the mainspring is being wound up.

wound up is clearly shown in Fig. 40. The switch consists of a two-pronged fork, one of the prongs being a metal one and being fitted with a silver contact plate, while the other consists of two strips of vulcanised fibre, one shorter than the other, with a piece of clock-spring held between them. The switch is fitted with a spiral spring so arranged that if left to itself the switch will be pulled by the spring into one or other of its extreme positions, but will not remain in any intermediate position (on the dead centre, the equilibrium is unstable). In the position (*a*), the circuit of the coil is closed, a contact-pin attached to the armature (this pin projects from a brass plate, which has been omitted for the sake of clearness) bearing against the silver contact plate. As the armature moves round, the contact is maintained by the tension of the spiral spring, until—the armature having reached

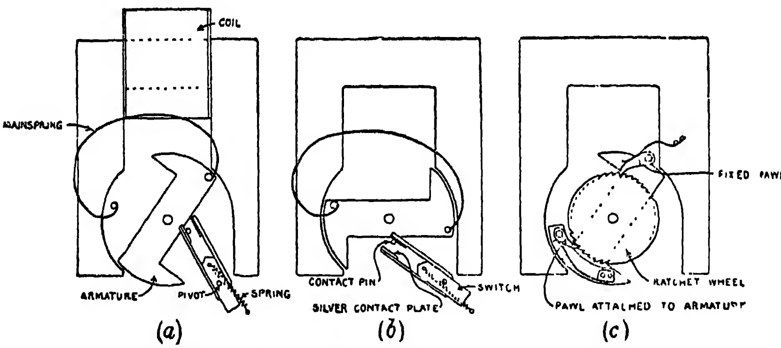


FIG. 40.—Winding electro-magnet of Aron clock meter.

its extreme position, as shown in (*b*)—the switch passes over its dead centre, and snaps away from the contact-pin, suddenly breaking the circuit of the coil.

The main driving axle carries the cross-arm of the "differential gear" shown in Fig. 41. At one end of this cross-arm is mounted a bevelled "planet" wheel which is free to revolve about the cross-arm as axis, while at the other end are arranged counterpoise nuts. The planet wheel engages two crown wheels, one

In the figure, for the sake of simplicity, the right-hand prong of the switch is shown as carrying a plate of insulating material. In the actual instrument, as already explained, the prong consists of two strips of vulcanised fibre, the inner one shorter than the outer, with a piece of clock-spring between them. The contact pin presses against the clock-spring, which takes the wear (the shorter inner strip of fibre allows the pin to clear it).

on each side, which are rigidly attached to toothed wheels. Each combined crown and toothed wheel rides loose on the main driving axle, and engages a pinion, as shown in Fig. 42, mounted on the escapement-wheel axle. It is evident that while the differential gear transmits a driving torque to each escapement wheel, it *leaves the escapement wheels free to move at different rates.* If each wheel has the same speed, there is no rotation of the planet wheel about the cross-arm; but if the speeds are different, then in addition to the rotation of the cross-arm we have a rotation of the planet wheel about its axis.

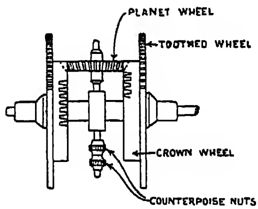


FIG. 41.—Differential gear of Aron meter.

In order to count the difference in the number of vibrations made by the two pendulums, a second differential gear, arranged as shown in Fig. 42, is employed. So long as the pendulums vibrate at the same rate, the side wheels of the driven differential gear will travel at the same speed in opposite directions, causing a rotation of the planet wheel without carrying round the cross-arm. But as soon as the rates are different, the cross-arm begins to move, driving the counting train.

If the natural periods of vibration differ appreciably—and any attempt to secure exact equality would greatly increase the cost of production of the meter—a record will be made even if there is no current through the series coils. In order to eliminate this source of error, a very ingenious method has been adopted, which consists in reversing the currents through the pendulum coils about about every ten minutes, and at the same time reversing the direction in which the counting train

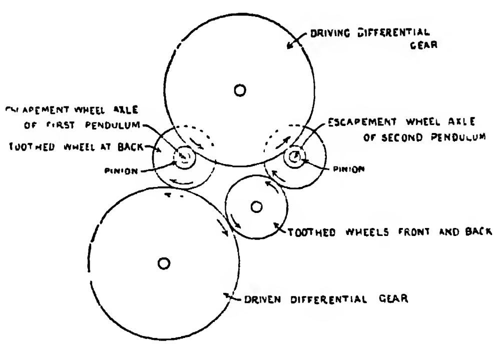


FIG. 42.—Arrangement of driving and driven differential gears.

is driven by the differential gear. Both reversals take place simultaneously, the electrical one by means of a two-part commutator which is suddenly rotated through 180° every ten minutes, and the mechanical one by a lever which is thrown one way or the other by a crank pin on the commutator axle, the lever in the first position introducing an extra toothed wheel into the set of intermediate wheels between the differential gear and counting train, and in the second position throwing this extra wheel out of gear. By means of this arrangement, the error arising from want of isochronism between the two pendulums is practically wiped out, each pendulum being alternately accelerated and retarded while the other is retarded or accelerated. The sudden rotation of the reversing gear axle is effected by means of a spring which is gradually wound up and suddenly released at the end of every ten minutes.

§ 61. Limits of Accuracy and Prices of Electric Supply Meters.

In the case of meters whose full load current does not exceed 3 amperes, the error at any load from $\frac{1}{10}$th of full load upwards should not exceed $\pm$ 3 per cent.

For meters whose full load current lies between 3 and 50 amperes, the error from $\frac{1}{10}$th of full load upwards should not exceed $\pm$ 2 per cent.

The error of meters whose full load current exceeds 50 amperes should not be greater than $+$ 2 per cent. between $\frac{1}{20}$th and $\frac{1}{10}$th of full load; and not greater than $\pm$ 2 per cent. from $\frac{1}{10}$th of full load upwards.

The price of an electric supply meter, like that of any other measuring instrument, varies a great deal, according to the size and type. The cheapest class is that of the electrolytic meters, which range from about £1 to £5. Motor meters range from about £3 to about £6 8*s*. A clock meter for the smaller ranges costs about £12. The above prices are for currents not exceeding about 100 amperes. For very large currents the prices will be much higher, a 1,000-ampere meter costing about £20, and a 10,000-ampere meter about £80. In the case of watt-hour meters the price also depends on the voltage, the cost of the shunt circuit of the meter rising with increasing voltage.

CHAPTER VII.

§ 62. Principle of a dynamo—§ 63. Component parts of dynamo—§ 64. Construction of armature core—§ 65. Law of eddy-current loss—§ 66. Nature of e.m.f. induced in armature conductor—§ 67. Problem of connecting armature conductors—§ 68. Lap winding.—§ 69. Balance of e.m.f.'s in lap winding—§ 70. Pitch of winding—§ 71. Wave winding—§ 72. Balance of e.m.f.'s in wave winding—§ 73. Comparison of lap and wave windings—§ 74. Commutator and brushes—§ 75. Path of current through lap winding—§ 76. Path of current through wave winding — § 77. Calculation of armature e.m.f. — § 78. Gramme winding—§ 79. Bar and coil windings—Examples.

§ 62. Principle of a Dynamo.

A *dynamo* may be defined as a machine for the conversion of mechanical into electrical energy. The action of a dynamo as a generator of continuous current depends on the principle of electromagnetic induction discovered in 1831 by Faraday. According to this principle (§ 5), a conductor moving with its length perpendicularly across a magnetic field becomes the seat of an e.m.f., the numerical value of which, in C. G. S. units, is equal to the number of C. G. S. magnetic lines cut by the conductor per second.

In order to apply this principle practically to the conversion of mechanical into electrical energy, we have to provide a magnetic field in which the motion can take place, a system of conductors in which the e.m.f.'s are induced by the motion, and suitable arrangements for maintaining contact between the system of moving conductors and the external circuit in which the current is utilised.

§ 63. Component Parts of Dynamo.

The magnetic field is provided by a system of powerful electromagnets known as the *field-magnets* or *field* simply. The general arrangement of these magnets is shown in Fig. 43. The magnet cores (cast steel) are of circular or rectangular cross-section, and each carries its own exciting coil (or *field coil*). These cores project radially inwards from a circular casting (of cast-iron or steel) known as the *yoke*. Each core terminates in an expansion known

as the *pole-shoe*. The function of the pole-shoe is to spread the magnetic flux over a larger area as it enters the air-gap, and so to reduce the reluctance (§ 31) of the gap. Within the cylindrical space bounded by the pole-shoes is placed a cylindrical iron core, known as the *armature core*, the primary function of which is to provide a path of low reluctance for the magnetic flux as it passes from one pole-shoe to the next. The importance of maintaining the reluctance of the magnetic circuit as low as possible will be evident when it is considered that the amount of copper on the field will depend directly on the reluctance of this circuit.

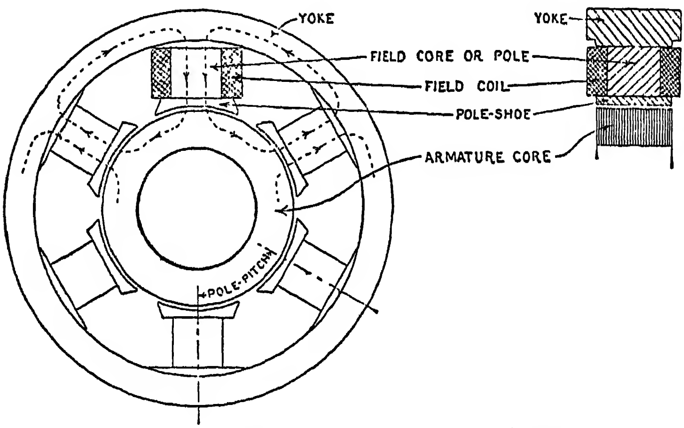


FIG. 43 —General arrangement of typical dynamo.

The connections of the field coils are such that the magnetic polarity of consecutive pole-shoes is alternately north and south. The mean path of the magnetic flux is indicated by the dotted lines in Fig. 43. It will be seen that the flux proceeding from a core through a north pole-shoe divides on entering the armature core, one half of it passing to each of the two neighbouring south pole-shoes. On passing through the corresponding field cores, the two fluxes enter the yoke and meet at the top of the core whence they started. It will be seen (1) that there are as many magnetic circuits as there are field cores; and (2) that the flux distributed over the yoke and armature cross-sections is (leakage neglected) only half that distributed over a field core. The field cores are frequently spoken of as the *poles* of the machine.

Between the pole-shoes and the surface of the armature core we thus have an annular air-space—the *air-gap* or *gap* simply—in which a system of conductors might be arranged to rotate, the armature core being stationary. Such an arrangement would, however, present considerable difficulty from the point of view of mechanical construction,* and would also necessitate the use of an abnormally large air-gap. Hence the conductors are mounted on the armature core itself, and the core is rotated bodily with the conductors, the whole forming the *armature* of the dynamo.

§ 64. Construction of Armature Core.

The mechanical construction of the armature may be greatly improved, and the air-gap of the machine reduced, by embedding the conductors in *slots* cut in the surface of the armature, as shown in Fig. 44, and this is the form of construction almost invariably used at the present time. Owing to the rotation of the armature core, the core itself is cutting magnetic lines, and as a result there will be e.m.f.'s induced in it in a direction parallel to the axis of rotation. These e.m.f.'s will, on account of the alternation in the polarity of the poles, alternate along the armature circumference, the e.m.f.'s in the portions of the armature core under cover of the north poles all having one direction, while those under cover of the south poles have an opposite direction. Since the armature core if solid would provide closed paths or circuits for these oppositely directed e.m.f.'s, enormous currents would circulate in it, causing very large waste of power and heating of the core. Such currents, circulating around closed paths in the mass of any solid conductor, are termed *eddy-currents.* The enormous loss which would be occasioned by solid armature cores renders the use of such cores practically impossible, and the eddy-currents are reduced to a sufficiently small amount by *laminating* the core, i.e., building it up of thin sheets or laminæ of iron or steel insu-

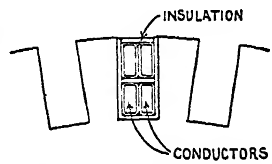


FIG. 44.—Arrangement of conductors in slots.

* This difficulty has, nevertheless, been overcome in a very ingenious manner in the Brooks Sayers dynamos, for a description of which the reader is referred to *Engineering*, vol. 119, p. 68 (1925).

lated from each other. These sheets are variously termed core *stampings*, core *discs* or *laminations*, and in dynamos are not more than ·025 inch thick. The direction of lamination being at right angles to the direction of the induced e.m.f.'s, the latter are prevented from giving rise to large current-densities in the core, as they can only act around circuits of very high resistance (due to small thickness of laminations). The insulation between the armature laminations consists either of very thin paper or of insulating varnish.

§ 65. Law of Eddy-current Loss.

It is easy to show that the eddy-current loss is proportional to the *square* of the thickness of the core-discs. In Fig. 45 (*a*) is represented a portion of a core disc of thickness t, the mean path

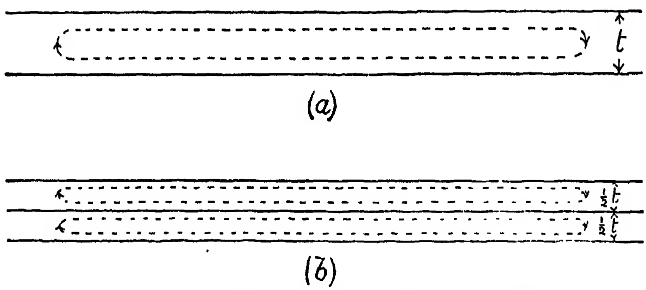


FIG. 45.—To illustrate law of eddy-current loss.

of the eddy-currents being indicated by the dotted line. In Fig. 48 (*b*) are represented two core-discs, each of thickness $\frac{1}{2}t$, with the mean eddy-current paths shown as before. It is evident from the figures that in each of the thin core-discs the total e.m.f. around this eddy-current path is only half as great, and the total resistance of this path twice as great, as they are in the single core-disc of thickness t. In the general case, by substituting n discs, each of thickness $\frac{t}{n}$, for a single disc of thickness t, we reduce the e.m.f. around each eddy-current path to $\frac{1}{n}$th of its original value, and increase the resistance of the path n times. Let e = e.m.f. around eddy-current path in single thick disc, and

r = resistance of this path. Then the eddy-current loss occurring in the path is $\frac{e^2}{r}$. The eddy-current loss in a corresponding path in each of the thinner discs is $\frac{(e/n)^2}{rn} = \frac{1}{n^3} \cdot \frac{e^2}{r}$. But since there are n thin core-discs corresponding to the single thick one, the total loss in the n discs is $\frac{1}{n^2} \cdot \frac{e^2}{r}$, or only $\frac{1}{n^2}$ times that in the single thick disc. Thus by reducing the thickness of the discs in the ratio $1 : n$, we reduce the eddy-current loss in the ratio $1 : n^2$.

Again, it is easy to see that the eddy-current loss varies as the square of the armature speed. For if the speed be increased m-fold, the eddy-current e.m.f. will also be increased m-fold, and hence (the resistance of the eddy-current paths remaining practically unaltered) the eddy-current loss will increase m^2-fold.

§ 66. Nature of E.M.F. Induced in Armature Conductor.

Let us now consider the e.m.f.'s induced in the armature conductors laid in the slots between the teeth of the core. As the armature rotates, the e.m.f. induced in each conductor alternates in sign as many times during each revolution as there are poles in the field. For every time that a conductor emerges from under a north pole and passes under cover of the neighbouring south pole, or *vice versâ*, the e.m.f. in it is reversed. Thus the e.m.f. induced in each armature conductor is an *alternating* one, and the problem now before us is to utilise these alternating e.m.f.'s in the individual conductors in such a manner as to obtain a uni-directional or continuous e.m.f. with respect to an external circuit.

§ 67. Problem of Connecting Armature Conductors.

Since the field intensity in the gap seldom reaches a value of 10,000 C. G. S. units, and the peripheral velocity of armatures does not generally exceed 5,000 feet per minute, or 2,540 cm./sec., we see that the e.m.f., in volts, per cm. length of

armature conductor does not ordinarily exceed ·254 (§ 5). This corresponds to about 7·74 volts per foot length of conductor. It is therefore evident that in order to obtain voltages of the order of 250 and 500, such as are commonly used nowadays, we must connect a number of armature conductors in series with each other in such a manner as to obtain addition of their e.m.f.'s. If at one end of the armature we connect a conductor to another which is separated from it by a distance approximately equal to the *pole-pitch*, i.e., the distance, measured along the armature circumference, between the centre lines of two neighbouring pole-pieces (see Fig. 43), the e.m.f.'s in the two conductors will at every instant be added (Fig. 46), since while one of them is

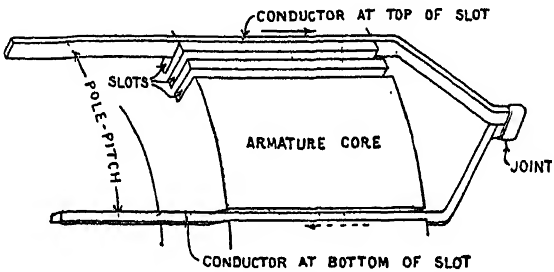


FIG. 46.—Method of connecting armature conductors.

moving under cover of a north pole-piece, the other is moving under cover of a south one, so that the two e.m.f.'s act in the same direction *around* the loop formed by the two conductors, as indicated by the arrows in Fig. 46. Such a loop forms a complete *turn* of the winding. For convenience in making the end connections between the conductors, one of each pair of conductors forming a loop or turn of the winding is placed at the top of a slot, and the other at the bottom, as shown in Fig. 46, the conductors in the slots being arranged in two layers, and the bent portions projecting outside the core lying in two different cylindrical surfaces.

By connecting the conductors at one end of the core in the above manner, we form a number of independent turns of the winding, and we have now to connect the turns to each other at the other end of the core. According to the way in which this is done, we arrive at one or other of the two types of winding

almost exclusively used at the present time, and known as the *lap* and the *wave* types of winding. In each case, the armature conductors are so connected as to form a *closed* coil or circuit ("re-entrant winding").

§ 68. Lap Winding.

For the sake of simplicity, we shall suppose each slot to contain only two conductors, arranged one above the other. Let the slots be numbered consecutively, and let us suppose that we start with the top conductor in slot 1 (Fig. 47), and connect this to a bottom conductor in a slot distant from slot 1 by an amount corresponding approximately to the pole-pitch. We thus, as already explained, form a single turn of the winding. Let the remaining end of the bottom conductor be bent back, as shown in Fig. 47, so as to meet the end of the top conductor in slot 2, which is bent forward, and let the conductors be joined as on the other side of the core. Now the top conductor in slot 2 forms the beginning of the next loop or turn, which is formed in exactly the same way. Proceeding in this manner, we obtain a succession of *overlapping* loops or turns—hence the name *lap winding*—and finally close our winding.

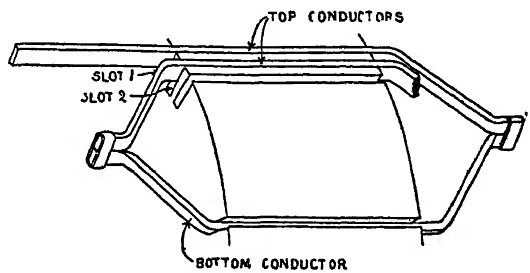


FIG. 47.—Lap winding.

§ 69. Balance of E.M.F.'s in Lap Winding.

Such a *closed-coil* or *re-entrant* winding must always fulfil the condition that the total resultant e.m.f. around the winding vanishes; otherwise, there would be local currents circulating in the armature winding even when there was no connection to an external circuit. As will be seen presently, this condition of a local balance of e.m.f.'s in the closed circuit of the armature is

fulfilled by our lap winding. Let in Fig. 48, N, S, N represent three consecutive field poles, and let A, B, C be points on the armature surface half-way between the pole-pieces. Let the direction of rotation be right-handed, as shown by the arrow in the figure. Then in any conductor under cover of a north pole the induced e.m.f. will act in a direction away from the observer (as shown by the circles with crosses in Fig. 48), and in any conductor under cover of a south pole it will act towards the observer (as shown by circle with dot).* It is therefore

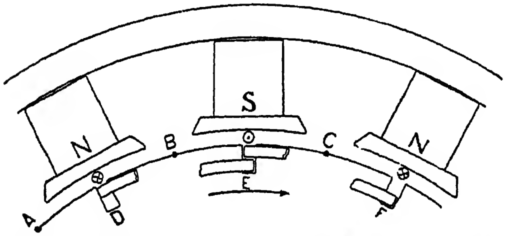


FIG. 48.—Showing direction of e.m.f. in conductors.

evident that, looking down at the loops on the armature surface from the side of the poles, the e.m.f. around any loop whose top conductor lies between A and B (such as the loop D E) acts in a clockwise direction, while the e.m.f. in any loop whose top conductor lies between B and C (such as the loop E F) acts in a counter-clockwise direction. We thus see that the loops whose upper conductors lie (at a given instant) within an arc A C corresponding to twice the pole-pitch—and hence embracing a pair of neighbouring poles with their interpolar spaces—fall into two equal groups, one corresponding to a north, the other to a south pole-piece, in which the e.m.f.'s are *equal but oppositely directed, and hence balance each other*. Diagrammatically, this result might be represented as in Fig. 49, where

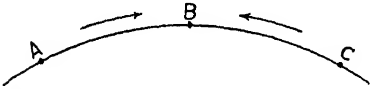


FIG. 49.—Balance of e.m.f.'s in winding.

A B represents that portion of the winding the upper halves of whose loops lie between A and B in Fig. 48, and B C the portion the upper halves of whose loops lie between B and C in Fig. 48 In the portion A B, the e.m.f. acts from A to B; in the portion B C, it acts from C to B. Since the rise of potential as proceed from A to B is equal to the fall of potential as we pas

* The dot represents the *point* of an arrow approaching the observer, the the *feathered end* of an arrow receding from the observer.

from B to C, it is evident that the points A and C will be at the same potential, or will be *equipotential.*

A lap winding such as the one described is therefore seen to fall into as many equal groups of conductors as there are poles in the machine, the e.m.f.'s being equal as regards magnitude in all the groups, but alternating in sign from group to group. The distribution of the e.m.f.'s in a six-pole armature with a lap winding might be diagrammatically represented as in Fig. 50, where A, B, C, D, E, F are points in the winding at which the e.m.f. changes its sign, and therefore correspond to conductors lying half-way between the pole-pieces. The points A, C and E are equipotential, and so are B, D and F; and the p.d. between any point in the first group and any point in the second is equal to the e.m.f. induced in any one of the six equal portions of the winding.

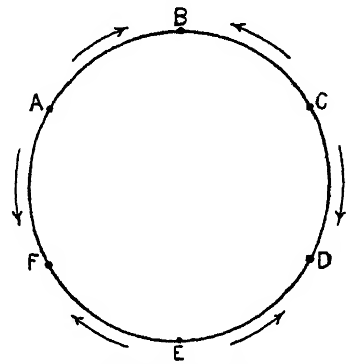


Fig. 50.—Distribution of e.m.f.'s in winding.

As the armature rotates, the points A, B, C, D, E, F change their positions relatively to the conductors, running round the winding in a direction opposite to, and with a speed equal to, that of the armature. It is to be noted, however, that the points A, B, C . . . are stationary *in space.*

The oppositely directed e.m.f.'s in the various portions of the winding balancing each other in pairs, it is clear that the resultant e.m.f. *around* the winding vanishes.

§ 70. Pitch of Winding.

In the case of a lap winding, there is no restriction on the number of conductors (except that this must necessarily always be *even*), as the winding can always be made symmetrical. It is convenient, in a tabular representation of the winding, to express the distance between any two conductors in terms of the number of conductors by which the one conductor is distant from the other. This number is known as the *pitch* of the winding.

An example will make this clear. Consider a 4-pole armature having 162 conductors arranged in two layers. Let the conductors be numbered consecutively, as in Fig. 51, the upper layer being denoted by odd and the lower one by even numbers. The number of conductors per pole-pitch is $\frac{162}{4} = 40\frac{1}{2}$. Let us start with conductor 1, and connect this to a conductor which is about a pole-pitch distant from it. The conductors $1 + 40 = 41$ and $1 + 41 = 42$ are those most nearly fulfilling the required condition. The first of these—41—could not, however, be used, as it is a *top* conductor; and, since 1 is also a top conductor, it must be connected to a bottom (or even) one. The second conductor—42—might be used; but it is preferable, in order to shorten the projecting ends of the conductors, to use a distance which, if anything, is less than the pole-pitch. We shall therefore connect conductor 1 to conductor 40 ($= 1 + 39$). The number 39 by which we proceed from conductor 1 to conductor 40 is the *forward pitch* of the winding. We next have to connect conductor 40 at the other end to the beginning of the next loop, which is represented by conductor 3. We thus take a backward step of $40 - 3 = 37$, and 37 represents the *backward pitch*. The difference between the forward and backward pitches is $39 - 37 = 2$. It will be noticed that both pitches are represented by *odd* numbers—a condition which must necessarily always be fulfilled, since any conductor in the top layer is always in connection with a conductor in the bottom layer. The connections of the conductors may now be represented in tabular form as follows* :—

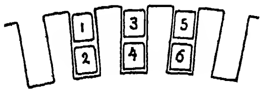


FIG. 51.—Method of numbering conductors.

1   3   5 . . . 123   125   127 . . . 159   161   1

40   42   44 . . . 160   162   2   4 . . . 36   38

One characteristic of the lap winding is the fact that it has two pitches, a forward pitch and a smaller backward pitch. In proceeding along the winding, we are alternately stepping forward and backward.

* The *top* line of the table refers to *top* conductors; the *bottom* line, to *bottom* conductors.

§ 71. Wave Winding.

A *wave winding* possesses only a *single pitch* which is nearly equal to the pole-pitch.* Fig. 52 shows the method of connecting the conductors, and it will be noticed that the winding proceeds in a zig-zag or *wavy* line around the armature circumference—hence the term *wave winding.* In order that the winding may be perfectly symmetrical, we must choose its pitch (which must, as in the lap winding, be necessarily an *odd* number) so that on completing each round or journey around the armature circumference we find ourselves either one conductor in advance of, or one conductor behind, that from which we started; so that the difference between the conductors representing the starting-points of two consecutive rounds must amount to 2 (this being the difference between two neighbouring conductors in the *same* layer). If, therefore, we suppose the pitch to be y, and the total number of conductors to be Z, then, 2 P standing for the number of poles, a round will be completed in $2\,P - 1$ steps,† and the beginning of the next round reached in 2 P steps. Let us start from conductor 1. After 2 P steps, we reach the conductor $1 + 2\,P\,y$. Since this represents the beginning of the second round, it must represent either conductor $Z - 1$, or else conductor $Z + 3$—i.e., conductor 3. We must therefore have either

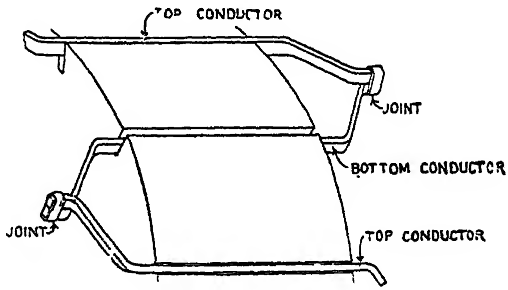


FIG. 52.—Wave winding.

$$1 + 2\,P\,y = Z - 1,$$

$$\text{or, } 1 + 2\,P\,y = Z + 3,$$

i.e., combining the two conditions,

$$y = \frac{Z \mp 2}{2\,P} \quad . \quad . \quad . \quad (1).$$

* Wave windings may have different pitches (both being *forward* pitches) at the front and back, but no advantage is gained by this arrangement.

† The first or starting conductor not being counted as a "step."

Now since y must necessarily be an integer, we see that the number of conductors Z cannot be chosen arbitrarily in the case of a wave winding. The number of conductors must be such that if we either add or subtract 2, the result is divisible by the number of poles. There is no such restriction in the case of a lap winding.

Taking the case already considered in connection with a lap winding, viz., a 4-pole armature having 162 conductors, we see that the condition represented by equation (1) is fulfilled, and that $y = 40$ or $y = 41$. But only the second value is admissible, y being odd. The winding therefore proceeds as follows :—

1 \ 42 / 83 \ 124 / 3 \ 44 / 85 \ 126 / . . . \ 162 / 41 \ 82 / 123 \ 2 / . . . \ 120 / 161 \ 40 / 81 \ 122 / 1

§ 72. Balance of E.M.F.'s in Wave Winding.

Let us now consider the distribution of the e.m.f.'s in a wave winding. Referring to Fig. 53, let us suppose that at a certain instant conductor 1 is in the position A. As we step round the armature circumference, following the zig-zag path of the wave winding, we find that we get addition of the e.m.f.'s during each journey round the armature until the conductor 41 is reached, which occupies the position B. Between A and B, or conductor 1 and conductor 41, we have half the total number of conductors. The remaining half is that lying between the conductors which are at B and C, i.e., between conductors 41 and 83—or, practically, between 41 and 1 (since 83 is, through conductor 42, connected to 1). It is easy to see that in this second half of the winding the e.m.f. has a direction opposed to that in the first half of the winding. For the magnetic polarity

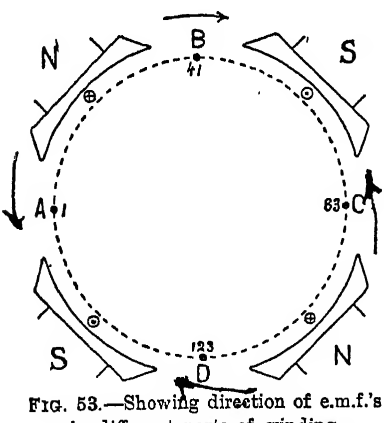


FIG. 53.—Showing direction of e.m.f.'s in different parts of winding.

Or, in the case of armatures having more than one turn per coil, the number of coil-sides.

between B and C is of opposite kind to that between A and B and the same holds for all the remaining corresponding regions of the armature. We thus arrive at the result that in a *wave* winding the armature conductors may at any instant be divided into *two* equal groups containing oppositely directed and equal e.m.f.'s. These e.m.f.'s will balance each other with respect to the closed circuit formed by the armature winding, and there will be no local current around the winding. The distribution of the e.m.f.'s is diagrammatically indicated in Fig. 54, where, as before, the closed armature winding is represented by a simple circle.

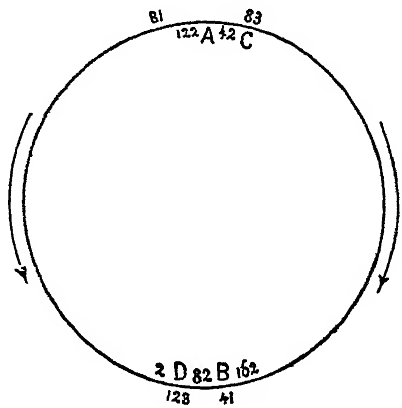


FIG. 54.—Balance of e.m.f.'s in wave winding.

§ 73. Comparison of Lap and Wave Windings.

Since in a wave winding each group of conductors consists of half the total number, the *e.m.f. per group* is equal to the e.m.f. induced in $\frac{1}{2}$ Z conductors, and is thus independent of the number of poles; whereas in a lap winding the e.m.f. per group is that induced in $\frac{Z}{2P}$ conductors.

In some points, the wave winding resembles the lap winding. Thus, a reference to Figs. 53 and 54 shows that the points A and C are practically equipotential, and also B and D. In general, any two conductors *in the same layer* which are distant from each other by twice the pole-pitch are equipotential, whereas between two conductors in the same layer which are distant from each other by a pole-pitch (such as conductors 1 and 41 in Figs. 53 and 54) there periodically occurs the maximum p.d. *The maximum p.d. also occurs periodically between two conductors of different layers in the same slot* (such as

conductors 1 and 2 in Fig. 54), whether the winding be of the lap or wave type. The necessity of very thorough insulation *between the two layers* will now be understood.

§ 74. Commutator and Brushes.

We have seen how, by using either the lap or the wave method of connecting the conductors, a closed winding is obtained in which there are balancing groups of conductors with oppositely directed e.m.f.'s. How may these e.m.f.'s be utilised in an external circuit? Considering first the case of a lap winding, and referring to Figs. 48 and 49, we see that a continuous current in an external circuit could be obtained if the circuit were always maintained in connection with the pair of points in the winding corresponding to A and B, which are separated from each other by a pole-pitch. But since the armature conductors are constantly changing their positions as the armature rotates, it is obvious that the contacts for the external circuit must be fixed in space, in the positions A and B of Fig. 48, and arranged to slip over the surfaces of the conductors as these pass under them. In order to prevent the conductors themselves from being worn away by the friction of the contact-pieces in connection with the external circuit, it is usual to provide a special structure which takes the wear. This structure is known as a *commutator*, because it effects the periodic changes in the connections between the rotating conductors and the external circuit which are necessary in order to obtain a uni-directional or continuous current from the alternating e.m.f.'s induced in the conductors (§ 66). In the simplest case, corresponding to each joint between two conductors at one end of the armature, there is provided a bar of copper which is left bare or uninsulated along its outer surface, and which, during its rotation, in certain

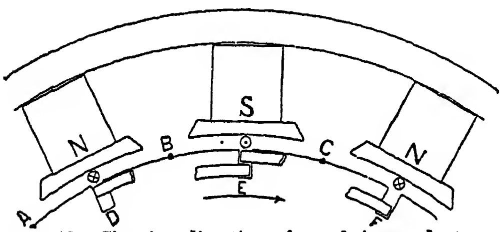


FIG. 48.—Showing direction of e.m.f. in conductors.

See § 89 for a further comparison of lap and wave windings.

positions establishes contact between the external circuit and the corresponding point in the winding. Fig. 55 illustrates the arrangement. The ends of the conductors are soldered and riveted to a thin connecting-strip of copper, whose other end is soldered into a narrow radial slit in the contact bar or *commutator segment.* These segments consist of tapering bars of hard-drawn copper insulated from each other with mica and mounted on a

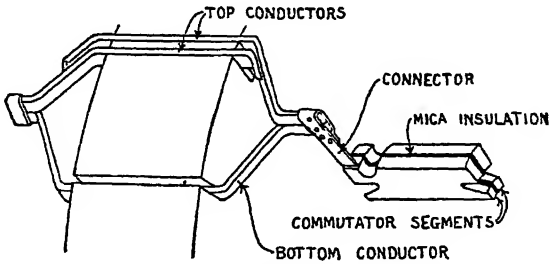


FIG. 55.—Connection of winding to commutator.

suitable support, from which they are also, of course, insulated. The commutator may be regarded as forming a dummy armature, each joint between two conductors being represented by a commutator segment. The fixed contacts (corresponding to A, B, C, &c., in Figs. 48 and 53) which are in connection with the external circuit and which press against the bare surfaces of the commutator segments are technically termed *brushes.*† In modern machines, these rubbing contacts consist of blocks of carbon, forming *carbon brushes.*

§ 75. Path of Current through Lap Winding.

By connecting the external circuit to two such "brushes" or groups of brushes, and applying these to the commutator segments which at any instant represent the points A and B of the winding in Figs. 50 and 54, it is evident that a current will be obtained in the external circuit. This current will start from

Details of the mechanical construction of commutators are given in Chapter IX. § 110

† A modern carbon "brush" bears no resemblance whatever to the homely article of the same name; but the old types of brushes, constructed of fine copper wire, justified the adoption of the term, as they certainly resembled brushes.

the point or brush B in Fig. 50, flow round the external circuit, and reach the brush A. When the current reaches the point A in the winding, there are two paths open to it: one being—in the special case of a 6-pole machine shown in Fig. 50—through the one-sixth A B of the winding, and the other through the remaining five-sixths A F E D C B. But it is evident that this would lead to an extremely unsymmetrical flow of current through the armature, as the smaller section of the winding would take five times as much current as the other, and this asymmetry would lead to various difficulties of a serious nature. A symmetrical flow of current may, however, be easily arranged for by providing as many brushes (or sets of brushes) as there are pole-pieces. Thus, in Fig. 50, brushes would be placed around the commutator in the positions corresponding to A, B, C, D, E and F. The brushes A, C and E—which (§ 69) are in connection with equipotential points in the winding—would then be connected together and to one terminal of the external circuit; while brushes B, D and F would also be connected together and to the remaining terminal of the external circuit. It is evident that the current coming from the external circuit will divide among the three *negative* brushes (i.e., those towards which the current flows from the external circuit), each brush receiving $\frac{1}{3}$ of the total current. Since each current which leaves a brush divides into two equal parts as it enters the winding, each section of the winding will carry $\frac{1}{6}$ of the total current, and the distribution of the current will be symmetrical, there being six parallel paths of equal resistance open to the current through the winding. The flow of current will take place as indicated by the arrows in Fig. 50. It will be noticed that the e.m.f. is that corresponding to one of the six parallel groups of conductors.

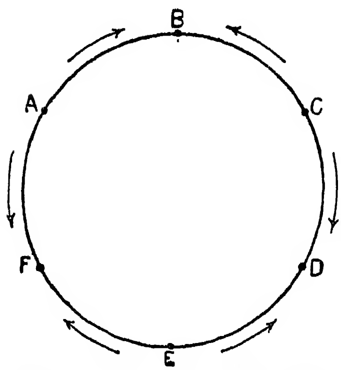


FIG. 50.—Distribution of e.m.f.'s in winding.

Considering the general case of a 2P-pole lap-wound armature, we see that such an armature requires 2P brushes (or brush sets),

P of which are connected together to form the positive, and the remaining P to form the negative brushes. The current carried by an armature conductor is $\frac{1}{2P}$th of the total current, and the e.m.f. is that corresponding to $\frac{1}{2P}$ of the total number of conductors.

§ 76. Path of Current through Wave Winding.

In the case of a wave winding, there are only two groups of conductors with oppositely-directed e.m.f.'s in them (instead of 2P groups as in a lap winding), as shown diagrammatically in Fig. 54. Hence a current coming from an external circuit can only be made to divide into two equal parts, half of it passing around each half of the winding as shown by the arrows in Fig. 54. Brushes must evidently be placed at the points corresponding to A and B, or C and D in Fig. 53, and—provided the contact area is sufficient to prevent excessive heating—two brushes (or brush sets) will be sufficient. These brushes may be placed in positions corresponding, in Fig. 53, to the points A and B, or B and C, or C and D, or D and A* (the points A and C, and B and D being, as a glance at Fig. 54 will show, equipotential, so that C may be used instead of A, and D instead of B). It is, however, also permissible to place brushes at all the points A, B, C and D in Fig. 53, and to connect the brushes at A and C to form the negative, and those at C and D to form the positive group. This only increases the number of paths by which the current enters the *commutator* (and hence reduces the current density at the

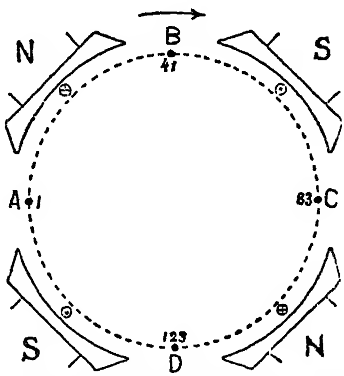


FIG. 53.—Showing direction of e.m.f.'s in different parts of winding.

* Between any two conductors in the same layer separated by a pole-pitch, only $\frac{1}{2P}$ of the total number of conductors are included in the case of a lap winding; but in a wave winding half the total number of conductors are always included.

contacts), but leaves the number of paths (2) through the *winding* unaltered (the points A and C being, as shown in Fig. 57, only separated by the conductor 42, and thus representing, for all practical purposes, the same point in the winding).

In a 2P-pole wave-wound armature, either a single pair of brushes (or brush sets), or 2P brushes may be used; in the latter case, the positive brushes consist of P alternate brushes, the remaining P alternate brushes forming the negative group. The current carried by each conductor is always half the total current, and the e.m.f. is that corresponding to half the total number of conductors.

§ 77. Calculation of Armature E.M.F.

The e.m.f. of an armature is easily calculated from the magnetic flux per pole Φ, the total number Z of external conductors, and the number n of revolutions per second. If there are 2P poles, then during one revolution each conductor cuts $2P\Phi$ lines (C. G. S.). During one second it cuts $2P\Phi n$ lines, so that the average value, in *volts*, of the e.m.f. induced in each conductor is $2P\Phi n 10^{-8}$. Now in a lap-wound armature, there are $\frac{Z}{2P}$ conductors in series between the brushes; whereas in a wave-wound armature, there are $\frac{1}{2}Z$ conductors in series. We thus have, if E stand for the e.m.f., in volts,

$$E = \Phi n Z.10^{-8} \text{ in a lap-wound armature,}$$

$$\text{and } E = P\Phi n Z.10^{-8} \text{ in a wave-wound armature.}$$

§ 78. Gramme Winding.

The lap and wave windings described consist of conductors distributed around the external surface of the armature, and are known as *drum windings*. Formerly, a winding known as a *Gramme or ring winding* was a good deal in vogue. This may be derived in a simple manner from a lap winding, by supposing each loop of the latter to be turned through 90°, so that its plane becomes radial instead of being tangential. We may suppose the rotation to be effected about the top conductor of the loop as axis, the bottom conductor thereby coming *inside* the armature core, as shown in Fig. 56. One result of the change is to render

the half-loop which is threaded through the interior of the core inactive as regards the generation of e.m.f., since the space inside the core is practically devoid of magnetic lines. The *internal* conductors are, in fact, so much "dead" copper: they add to the resistance of the armature without contributing anything towards its e.m.f. In order that the e.m.f. may be the same as that obtained with a lap winding, we must have the same number of external or active conductors in each case; and since, in the ring winding, there is a dead conductor to every active one, the winding will have twice the resistance of an equivalent lap winding. There is, however, a compensating advantage possessed by the ring winding which, in certain exceptional cases, may render its adoption advisable. It will be noticed that by removing the bottom conductor of each loop to the interior of the core, we have removed it from the immediate neighbourhood of a top conductor which is periodically at a potential—equal to the full armature e.m.f.—above and below the potential of the bottom conductor of the loop (§ 73). The removal of the bottom conductors from the immediate neighbourhood of the top ones thus results in a greatly lessened risk of breakdown, since the insulation between any two neighbouring conductors in a ring winding is never exposed to a stress exceeding that which corresponds to the maximum e.m.f. generated by a coil. *A Gramme or ring winding may therefore be advantageous in connection with machines designed for very high e.m.f.'s.*

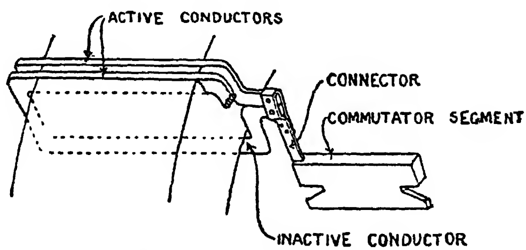


FIG. 56.—Gramme winding.

§ 79. Bar and Coil Windings.

In considering armature windings, we have assumed that each coil consists of a single turn or loop, formed by connecting two suitably bent copper conductors, each of which constitutes a half-loop. Such a winding is known as a *bar winding*. Instead

of using half-loops, entire loops, without any joint in the middle, might be used, the copper being suitably bent at the middle of the loop. As stout bars of copper are by no means easy to handle on account of their stiffness, a number of round copper wires connected in parallel may be used; this, however, is not to be generally recommended, as the valuable winding-space in the slots is thereby very imperfectly utilised.

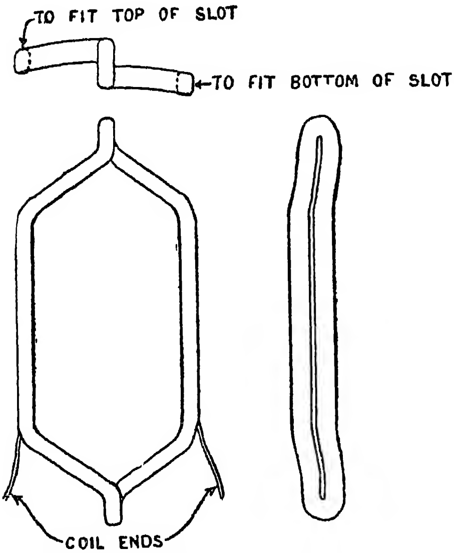


FIG. 57.—Former-wound armature coil.

Instead of having only a single turn per coil, several turns may be used if the required e.m.f. cannot be easily obtained by means of a single turn per coil without an excessive subdivision of the commutator. Such cases very frequently arise in the smaller sizes of machines. The coils are wound to the required shape on special frames or "formers," one side of the coil being arranged to fit into the bottom of a slot, and the other into the top. Such a "former-wound" coil is shown in Fig. 57.

EXAMPLES.

1. Find the e.m.f. developed by a 16-pole lap-wound armature wound with 2,560 conductors and running at 90 revolutions per minute, if the magnetic flux per pole amounts to $12 \cdot 5 \times 10^6$.

2. A four-pole wave-wound armature having 226 conductors develops an e.m.f. of 260 volts when running at 750 revolutions per minute. Find the flux per pole.

3. An eight-pole lap-wound armature having 800 conductors develops an e.m.f. of 500 volts at a speed of 250 revolutions per minute. The field cores are 15 inches in diameter, and the leakage coefficient is 1·2. Find the magnetic induction in the field cores.

4. A six-pole armature is to be provided with a simple wave winding. Find all the possible numbers of conductors lying between 500 and 550, each coil of the winding being supposed to consist of a single turn.

CHAPTER VIII.

§ 80. Armature reaction—§ 81. Periodic short-circuiting of coils by brush—§ 82. Consideration of ideal case—§ 83. Rectilinear law of current change in ideal case—§ 84. Reactance voltage—§ 85. Calculation of reactance voltage—§ 86. Rotation e.m.f. in short-circuited coil—§ 87. Prevention of sparking. Use of brushes giving large contact drop—§ 88. Reversing e.m.f. obtained by forward brush lead—§ 89. Commutating or reversing poles—§ 89a. Adjustment of interpole field—§ 89b. Flashing over and use of neutralising winding—§ 90. Care of commutator. Sparking due to purely mechanical causes—§ 91. Comparison of lap and wave windings. Equalising connections—§ 92. Method of balancing lap-wound armature—Examples.

§ 80. Armature Reaction.

When an armature is loaded, the currents circulating in its conductors give rise to a magnetic field which is superposed on the main field due to the field coil ampere-turns. The main or original field is thereby modified, and this modification is spoken of as *armature reaction*.

From the explanations given in §§ 75 and 76, it will be seen that radial planes drawn through those conductors which at a given instant are in direct connection with the brushes, divide the whole of the winding into groups or belts of conductors—each group consisting of a number of conductors equal to that comprised within a pole-pitch—conveying currents whose direction alternates from group to group as we proceed around the armature circumference. If the brushes are set so as to be in direct connection with those conductors which are exactly half-way between the pole-pieces, the armature current gives rise—as shown by the dotted letters in Fig. 58—to a series of polar surfaces along the armature circumference which are spaced exactly midway between the polar surfaces produced by the field-magnet. The directions of the currents in the consecutive groups of armature conductors are, in Fig. 58, shown in the usual manner by circles with dots and crosses. It will be seen that the armature current in this case does not give

rise to any de-magnetising effect. The superposition of the two fields due to the field-magnet and the armature current gives a resultant field which is distorted (i.e., the magnetic flux issuing from or entering a pole-shoe is not uniformly distributed over the pole-shoe surface), the distortion consisting of a weakening of the field under the left-hand half of the pole-shoe (h in Fig. 58), and a strengthening of it under the right-hand half (h' in Fig. 58). The armature current is said to give rise to a *cross-field*, or to produce a *cross-magnetising effect*.* That there is no de-magnetising effect in this case will be evident from the fact that if we take any line of induction due to the field-magnet, such as the

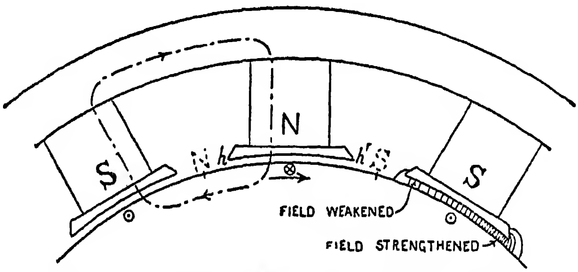


FIG. 58.—Distortion of field by armature current.

chain-dotted line in Fig. 58, then linked with it are two equal groups of armature conductors conveying currents in opposite directions, so that the resultant m.m.f. around this curve due to the armature current is zero.

If now we displace the brushes in the direction of rotation, or give them a forward lead, the points at which the current changes its direction are shifted forward in the direction of rotation, and with them also the polar surfaces due to the armature current, as shown in Fig. 59. The position of these polar surfaces relatively to the main poles is no longer symmetrical, and a glance at Fig. 59 shows that there is now a

The use of the terms "cross-field" and "cross-magnetising effect" dates back to the early days in which two-pole machines were exclusively used. In a two-pole machine, the line joining the middle points of the polar surfaces produced by the armature current is clearly at right angles to the line joining the middle points of the pole-pieces.

direct *de-magnetising* effect. This will also be seen by considering the m.m.f. around the chain-dotted line; in the belt of conductors linked with this line there is a preponderance (represented by the conductors lying to the left of the dotted N, which represents the middle of the polar surface due to the armature current, and also the point at which a change occurs in the direction of the current) of those conveying current in a direction towards the observer, and tending to set up a flux around the chain-dotted curve in a counter-clockwise direction, which is opposed to that of the main flux.

Thus when the brushes have a forward lead the magnetic effect due to the armature current is a two-fold one: a weakening

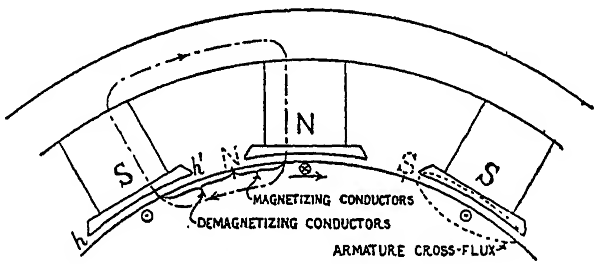


FIG. 59.—De-magnetising effect due to brush lead.

of the field as a whole (due to brush lead), and a distortion of the main field, consisting in a local weakening of the field under one polar horn (h in Fig. 59), and a corresponding strengthening of the field under the other horn (h' in Fig. 59). The amount of this weakening or strengthening may be approximately estimated as follows. The general course of the flux due to the conductors lying under cover of a pole-piece is that indicated by the dotted line marked "armature cross-flux" in Fig. 59. Since in the pole-shoe and also in the armature core the value of H would not in general be large, we may neglect the magnetic potential drops along those two portions of the path, and assume that the entire m.m.f. due to the conductors linked with the dotted curve is used up in maintaining the flux across the (double) gap and teeth. If the value of B and the B-H curve for the teeth be known, we can find the equivalent air length of the teeth by

dividing twice the depth of tooth by μ. If, therefore, g stand for the single gap length, and d for the depth of a single tooth, the approximate equivalent air length of the magnetic circuit represented by the dotted curve is $2\left(g+\frac{d}{\mu}\right)$. So that if H_a = field intensity due to armature current *alone* at the extreme edges of the polar horns, and if Z' = number of conductors under cover of pole-shoe, and i = current, in amperes, in each conductor—

$$2\left(g+\frac{d}{\mu}\right) H_a = 1{\cdot}257\ Z'i, \text{ by § 3,}$$

or

$$H_a = \frac{1{\cdot}257\ Z'i}{2\left(g+\frac{d}{\mu}\right)}$$

On subtracting H_a from the undisturbed field intensity, we obtain the resultant or weakened field close to the edge of the polar horn h in Fig. 59, and on adding H_a to the undisturbed field intensity, we obtain the strengthened field close to the edge of the horn h'. The weakening on one side and strengthening on the other steadily decrease towards the middle of the pole-piece, at which point the original field remains undisturbed.

It is easy to see that, had we displaced the brushes *backwards* instead of forwards (or given them a *backward lead*), a direct *magnetising* effect would have been superposed on the cross-magnetising effect.

In the ideal case of a perfectly symmetrical machine with a smooth-core armature, the flux distribution when the brushes are not given any lead may be represented by the curves shown in Fig. 58A, in which horizontal distances represent distances along the armature circumference, and vertical distances values of the induction. The curve marked "no-load field" shows the field obtained when the armature current is negligible. The dotted curve shows the field which would be produced by the armature current if the field-magnet were not excited. Lastly, the chain-dotted curve, obtained by the superposition of the other two, gives the actually existing or resultant field when the field-magnet is excited and the armature loaded.

If the armature were surrounded by a continuous co-axial hollow cylinder of iron not broken up into pole-shoes and interpolar spaces as in the actual machine, the armature field curve would consist of a broken straight line forming a succession of triangles with the horizontal axis, and the maximum field intensity would occur half-way between the axes of the field poles. Owing, however, to the greatly increased reluctance resulting from the existence of the interpolar spaces, the armature field drops to a low value in these regions, the curve forming a deep depression or "valley" as shown in Fig. 58A.

It must be understood that the curves of Fig. 58A refer to an

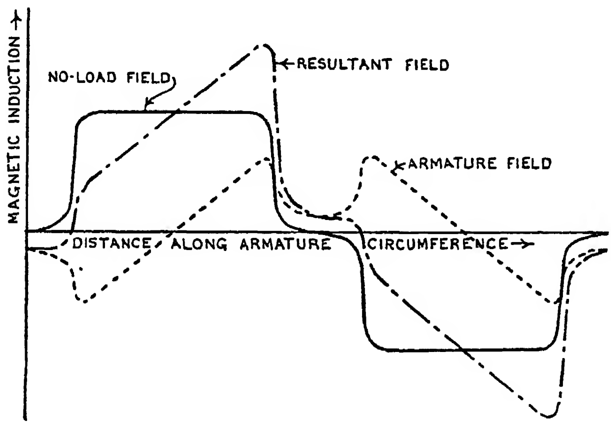


FIG. 58A.—Flux distribution curves.

ideal case. Modern machines have toothed armatures, and on account of the teeth all the curves will exhibit ripples, while lack of symmetry, possible presence of blow-holes in the castings and lack of homogeneity in the materials will all give rise to further irregularities, so that even a smooth-core machine would not give curves as smooth as those shown in Fig. 58A.

If the brushes are displaced in either direction, the dotted curve of Fig. 58A will be similarly displaced, and the effect on the resultant field may be easily studied.

§ 81. Periodic Short-circuiting of Coils by Brush.

By the action of the commutator and brushes, the alternating e.m.f.'s induced in the individual armature conductors are made to produce a uni-directional p.d. across the external circuit, as

explained in the preceding chapter. During the rotation of the armature, each brush periodically bridges across at least two commutator segments, and frequently more than two, depending on the width of the brush. While the brush is across any two segments, the coil between them has its local circuit closed through the brush—i.e., it is *short-circuited* by the brush. In Fig. 60 (*a*) the armature winding is diagrammatically represented by a straight line; the motion of the commutator is supposed to take place from left to right, as shown by the arrow. A B is a coil which is just on the point of undergoing a short-circuit by the brush. The

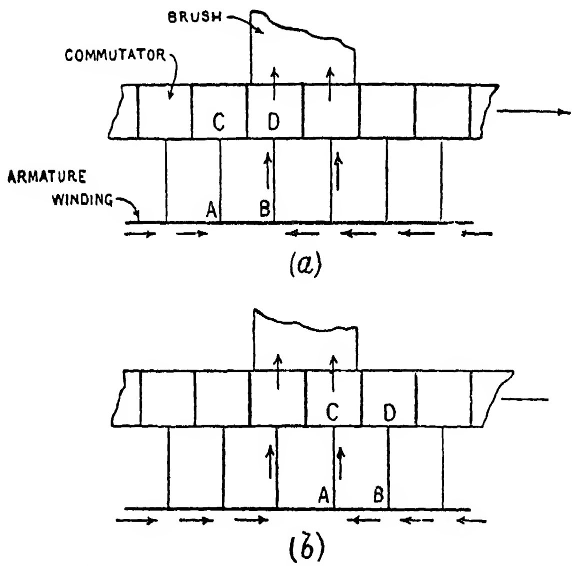


FIG. 60.—Short-circuit of a coil by a brush.

brush is receiving current from the left and right hand sections of the armature winding, the current from the left-hand section flowing from left to right, and that from the right-hand section flowing from right to left, as indicated by the arrows. Immediately before the beginning of the short-circuit, the current in the coil A B is from left to right; immediately after the end of the short-circuit, it is from right to left, as shown in Fig. 60 (*b*), where the coil A B is represented as having been just open-circuited. It is evident, therefore, that during the period of short-

circuit the current in a coil undergoes a complete reversal. This reversal is spoken of as *commutation.*

§ 82. Consideration of Ideal Case.

The circuit of the short-circuited coil has a total resistance consisting of the resistance of the coil itself, that of the two connectors between the ends of the coil and the commutator segments, the resistance of the segments, that of the two contacts between the brush and the segments, and the resistance of the brush itself. By far the greater part of the resistance is concentrated at the brush contacts. In order to make the various occurrences taking place during commutation as clear as possible, we shall in the first place consider an ideal case, assuming the resistances of the coil, connectors, &c., to be negligible in comparison with the resistances of the brush contacts, and supposing that during the period of short-circuit the coil *does not contain any induced e.m.f.'s.* The above suppositions are equivalent to the assumption that during the short-circuit the commutator segments to which the coil is connected remain at the same potential.

It is evident that the number of paths open to the current, from the winding to a brush, corresponds, at any instant, to the number of commutator segments which the brush happens to cover at that instant. Thus, in Fig. 61 the brush is shown resting on three segments, and there are three paths by which the current may pass from the winding to the brush. Now, in accordance with our assumptions, the three segments covered by the brush are practically at the same potential; the resistances of the three paths between the segments and the brush consist of the corresponding brush contact resistances, and since the resistance of a contact is inversely as its area, the current through each contact will be directly as its area. Thus our assumptions lead to the result that during the entire period of commutation *the current density* (i.e., the current per unit of area) *over the brush contact area remains uniform.*

§ 83. Rectilinear Law of Current change in Ideal Case.

Consider now, on the above assumptions, the changes which take place in the current in a coil during the period of short-

circuit. Immediately before the commencement of the short-circuit, the current in the coil A B (Fig. 60 (*a*)) is the entire current reaching the brush from the left-hand section of the winding. As soon, however, as the brush touches segment C, a new path, A C E in Fig. 61, is opened to the current. Remembering that under the conditions assumed the current-density remains uniform over the entire brush contact area, it follows that the current flowing into the brush through the portion of the brush contact area lying to the left of the point E (which separates the segments C and D) is directly proportional to the area lying to the left of E. Now, since this area increases at a constant rate during the rotation of the commutator, the current entering the brush to the left of E will also increase at a constant rate. Since, however, any increase of current along the path or paths to the left of E must be accompanied by an equal decrease of current in the coil A B, we see that immediately on the commencement of the short-circuit the current in the short-circuited coil begins to decrease *at a constant rate*. When the point E reaches the middle of the brush, the entire current flowing from the left-hand section of the winding passes into the armature entirely by the paths lying to the left of E. At this instant — the middle of the short-circuit period—the current in A B has reached a zero value. But since the current entering the brush to the left of E will go on increasing at a constant rate as before, and since the increase of current must now be supplied through A B, the current in A B changes sign, and increases at a uniform rate until the end of the short-circuit. The change

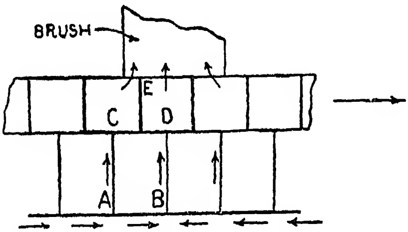


FIG. 61.—Brush covering three segments.

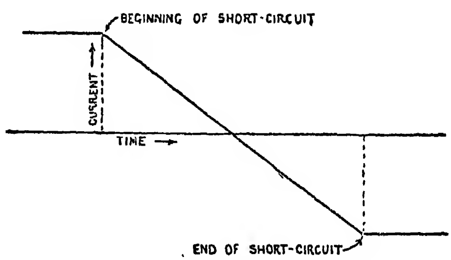


FIG. 62.—Ideal commutation.

of current in the coil A B may therefore be represented by a sloping straight line as shown in Fig. 62.

The above conditions, involving a rectilinear law of current change during the period of short-circuit, are the ideal to be aimed at, and for two reasons. In the first place, since the current flowing from a receding segment to the brush becomes vanishingly small at the instant when the brush is about to quit the segment, there can be no sparking. Secondly, the uniform distribution of current over the brush contact area results in a smaller heating loss due to contact resistance than that corresponding to any non-uniform distribution.

§ 84. Reactance Voltage.

In practice the above ideal conditions are, unfortunately, difficult to realise. This is due to the fact that in general the short-circuited coil does not form a negligible dead resistance, as we have supposed it to do, but is the seat of various induced e.m.f.'s. These e.m.f.'s and their effect on commutation we now proceed to study. The current flowing through any coil gives rise to a magnetic flux linked with it, and as soon as the current begins to change, the magnetic flux due to it also changes, and induces an e.m.f. in the coil which opposes the change (§ 5). This e.m.f. is spoken of as the *e.m.f. of self-inductance* (or self-induction) or the *reactance voltage*. Its primary effect is to retard

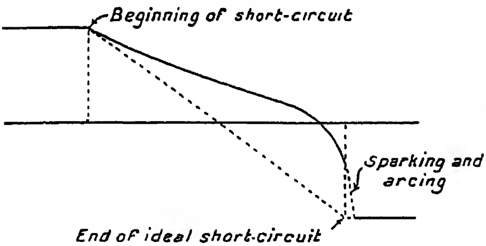


FIG. 63.—Imperfect commutation.

In order to show that any want of uniformity increases the heating loss at the brush contact, we may consider the case of a brush over one-half of whose contact area there is a current $i + e$, and over whose other half the current is $i - e$. If $r =$ resistance of each half-area of contact, the total loss is $r[(i + e)^2 + (i - e)^2] = 2r(i^2 + e^2)$; but with a uniformly distributed current of the same total value the loss would have been $2ri^2$, which is less than that corresponding to a non-uniform distribution.

the changes which would otherwise take place in the current, and the results of such retardation are non-uniformity of current distribution (with consequent increase of heating loss) and increased rate of current change during the last stages of commutation, which is liable to lead to sparking.

When the e.m.f. of self-inductance is small, the evil effects just mentioned may not be serious. But with a large e.m.f., the current curve may be so much distorted that instead of the straight line shown in Fig. 62 we have a curve of the shape shown in Fig. 63. The current changes very slowly at first, and the current density over the forward part of the brush greatly increases, causing intense local heating and, in extreme cases, surface fusion of the receding segment. During the final stages of commutation, the resistance of the contact area between the tip of the brush and the receding segment is rapidly increasing (owing to the rapid reduction of area), hence the current must change at a very rapid rate, as shown by the steepness of the curve in Fig. 63. The accompanying rapid rate of change of flux induces a very high e.m.f. in the coil, sufficiently high to cause a spark to jump across from the brush to the segment as the latter leaves the brush. The current follows the spark for a short time in the form of an arc, causing further fusion of the already intensely heated metal, and prolonging the short-circuit beyond the instant at which the segment has left the brush (this is indicated by the dotted part of the curve in Fig. 63). The fusion of the segments results in an irregular surface which causes mechanical vibration and further aggravates the evil. Under such conditions, the life of the commutator will be very short, and the cost of maintenance very high. Hence the problem of sparkless commutation has always formed one of the leading problems in dynamo design.

§ 85. Calculation of Reactance Voltage.

The mean e.m.f. induced in a short-circuited coil by the reversal of the current is given by the quotient $\dfrac{\text{change of flux linked with coil}}{\text{duration of short-circuit}}$

divided by 10^8 in order to reduce the C. G. S. units of e.m.f. to volts. It is evident that, the flux being approximately proportional to the current, the self-inductance e.m.f., or reactance voltage, is proportional to the current to be reversed. It is therefore convenient to determine, in the first place, the change of flux due to the reversal of one ampere of current. This may be done by the following method, which is due to Hobart.[1]

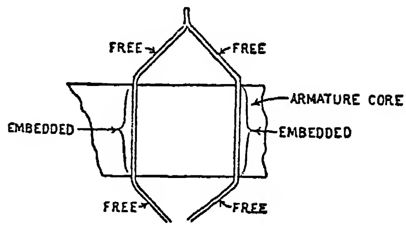


FIG 64.—To explain meaning of "embedded and "free" lengths.

Each coil may be regarded as consisting of an "embedded" portion or length, which corresponds to the portions of the conductors embedded in the core slots, and a "free" length, represented by their projecting ends (Fig. 64). Hobart finds that a flux of 4 C. G. S. lines may be assumed per ampere per cm. of embedded length, and a flux of ·8 C. G. S. line per ampere per cm. of free length. Now if we consider the top conductor of a loop which is short-circuited, then the bottom conductor lying in the same slot belongs to a loop which is simultaneously short-circuited by a neighbouring brush. Similarly, corresponding to the bottom conductor of a short-circuited loop, there is a top conductor in the same slot belonging to a loop short-circuited by a neighbouring brush. Thus the fluxes due to the embedded portions of the short-circuited loops change simultaneously, and the effect is the same as if there were 8 C. G. S. lines ($= 2 \times 4$) per ampere per cm. of embedded length. Let us consider, in the first place, the simple case in which only one loop is short-circuited at a time. If l_e denote the embedded length of the loop, and l_f its free length, and if $i =$ current (amperes) in loop, the change of flux when the current changes from $+i$ to $-i$ is given (in accordance with the above explanations) by—

$$2i\,(8l_e + \cdot 8l_f) = 16\,(l_e + \cdot 1l_f)\,i.$$

[1] *Journal of the Institution of Electrical Engineers*, vol. xxxi., pp. 185–207 (1901).

If each coil consists of S turns instead of a single one, the corresponding change of flux will be S^2 times as great. For with a given current through the coil, increasing the turns S-fold will increase the magnetic flux through *each* turn S-fold; and since the total flux is the sum of the fluxes through all the turns, the total flux will be increased S × S or S^2-fold. Thus the change of flux taking place during reversal when each coil consists of S turns and when only one coil at a time is short-circuited by each brush is given by—

$$16\,(l_e + \cdot 1 l_f)\,S^2\,i,$$

where, it must be remembered, l_e and l_f denote respectively the embedded and the free portion of a *single* turn or loop of the coil.

If, as is generally the case, there are several coils simultaneously short-circuited by a brush, then since these coils overlap more or less perfectly, the change of flux in each coil will be increased in proportion to the number of coils simultaneously short-circuited by each brush. If s denote this number, then the change of flux during reversal is

$$16\,(l_e + \cdot 1 l_f)\,S^2\,s\,i.$$

In order to find the mean value of the reactance voltage induced in the coil during the period of short-circuit, we have to divide the above change of flux by the duration of the short-circuit, and by 10^8 (in order to reduce C. G. S. units to volts). Hence, T denoting the time of the short-circuit, we have

$$\text{mean value of reactance voltage} = \frac{16\,(l_e + \cdot 1 l_f)\,S^2\,s}{T \times 10^8}\,i \quad . \; . \; . \quad (1).$$

T is easily found from the brush width w_b, the radius r of the commutator, and the revolutions per second n. A glance at Fig. 65, where the positions of the two segments 1, 2 are indicated by full lines at the commencement of the short-circuit, and by dotted lines at the end of the short-circuit, shows that during the short-circuit the coil moves through an angular distance (in radians) $\frac{w_b}{r}$

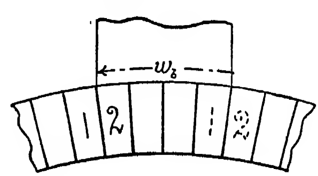


FIG. 65.—To illustrate duration of short-circuit.

which is a fraction $\frac{w_b}{2\pi r}$ of the circumference. Now since the time of a revolution is $\frac{1}{n}$, that of the short-circuit is

$$T = \frac{w_b}{2\pi r n}.$$

According to Hobart, the *mean value of the reactance voltage as given by formula* (1) *should not exceed* 2 *volts*. For very low-speed machines, however (80 to 120 revolutions per minute), it may be as high as 2·5 volts.†

§ 86. Rotation E.M.F. in Short-circuited Coil.

The reactance voltage is not in general the only induced e.m.f. which exists in the short-circuited coil. Owing to armature reaction (§ 80), the short-circuited coil will generally be moving across a magnetic field which is the resultant of the field due to the field and armature ampere-turns, and a corresponding induced e.m.f. will be generated by the coil. Since this e.m.f. is due solely to the rotation of the armature, we may, in order to distinguish it from the reactance voltage, conveniently term it the *rotation e.m.f.*

If we suppose the brushes to have no lead (§ 80), so that they are in direct connection with those conductors which occupy the middle portions of the interpolar spaces, then the only field cut by the short-circuited coil during its motion is the armature cross-field, and a reference to Fig. 58, together with an application of Lenz's law (§ 5), will easily enable the reader to ascertain

T generally lies between ·001 and ·003 second.

† Numerous formulæ have been proposed from time to time by different investigators as criteria regarding the performance of a machine from the sparking point of view. One of these is due to Professor S. P. Thompson [*Howard Lectures*, *Journal of the Society of Arts*, vol. liv., p. 1020 (1906)]. The higher the value of $j = \frac{10^8}{l.\,q.\,v.\,z.}$, the better the commutation. In this formula, l = net length, in inches, of iron in armature core, measured in a direction parallel to the shaft; q = ampere-conductors per inch length of armature circumference; v = peripheral speed of commutator, in feet per minute; z = ratio of number of armature conductors to number of commutator segments. The value of j should never fall below unity. It varies from 1·25 to about 6 in good machines.

that the direction of the rotation e.m.f. (due to the armature cross-field) is the same as that of the reactance e.m.f. Hence *all the evil effects arising from the existence of the reactance voltage will be aggravated by the rotation e.m.f. due to the armature cross-field.*

Now if by any suitable method we could succeed not only in locally wiping out the armature cross-field, but in establishing a field having the opposite direction, then the sign of the rotation e.m.f. would be reversed, and this e.m.f. could be made to balance, more or less completely, the reactance e.m.f. The ideal conditions for sparkless running would then be approximated to more or less perfectly.

Two methods of attaining this object are available. One of these, which is now only of historical interest, consists in giving the brushes a *forward lead.* The other method, which is invariably used nowadays, depends on the use of special auxiliary *reversing poles* or *interpoles.* Before studying these two methods in detail, however, we shall consider what other means are available for minimising the effects of the total e.m.f. induced in the short-circuited coil when this e.m.f. cannot be rendered negligible.

§ 87. Prevention of Sparking. Use of Brushes giving large Contact Drop.

One method of dealing with the difficulties consequent on the presence of an induced e.m.f. in the short-circuited coil is to reduce this e.m.f. to the lowest practicable amount. This may be done by subdividing the armature winding as much as possible, the limit being reached when each coil consists of a single turn.† It is not always, however, possible to carry the

The reactance e.m.f. is, for a given current, proportional to the *square* of the number of turns in the coil. For if we compare the total linkage of magnetic flux with a coil containing S turns with the total linkage of flux in a coil consisting of a single turn, both coils being supposed to convey the same current, then since the flux linked with *each* turn of the coil of S turns is S times that linked with the coil of a single turn, and since the former coil contains S turns, the total linkage of flux is S^2 times as great as in the latter coil. The reactance e.m.f. being proportional to $\frac{\text{change of flux}}{\text{period of short-circuit}}$, will therefore also be S^2 times as great.

† By the use of the double commutator—one on each side of the armature—commutation may be confined to a *single conductor* at a time.

subdivision so far, especially in the case of armatures of small diameter, where the number of commutator segments is limited by the minimum permissible thickness of a segment (if the thickness is reduced below a certain limit, the commutator becomes too weak mechanically). The highest permissible values of the reactance e.m.f., and the method of calculating this e.m.f., are dealt with in § 85.

If the drop of potential over the brush contact is large in comparison with the reactance e.m.f., the latter will have relatively little effect in disturbing the ideal current distribution, since the percentage difference between the p.d.s existing between the segments and the brush will not be abnormally high, and the current distribution will depart but little from the rectilinear law (§ 83) corresponding to the ideal case. For this reason, a large drop of potential over the brush contacts is beneficial in suppressing sparking, and it is for this reason that carbon brushes have, except in special cases, entirely superseded brushes made of copper gauze, which were so extensively used at one time.

The resistance of a brush contact is found *not to obey Ohm's law*, i.e., the ratio $\frac{\text{p.d.}}{\text{current}}$ is found to vary with the current, *decreasing as the current increases.* As a result, the drop of potential over the contact increases less rapidly than in proportion to the current. This effect is much more marked with carbon than with copper brushes. In order to prevent excessive heating at the brush contacts, the current density must not be allowed to exceed a certain limit, depending on the material of the brushes. In the case of copper gauze brushes, the normal limit is about 160 amperes per square inch of contact area, and at this current density the potential drop per brush contact amounts to about ·035 volt (or a combined drop of ·07 volt for positive and negative brushes). The permissible current density in the case of carbon brushes depends largely on the particular brand of carbon used, being greater for the softer and less for the harder grades of carbon. It varies from about 60 amperes per square inch with very soft carbons to about 25 amperes per square inch with very hard carbons. The highest potential drop over the brush contact occurs with the hardest grades of carbons, and may reach as high a value as 1·5 volts per contact (corresponding to a total drop of 3 volts). In most cases, however, the drop per contact lies between 1 and

$1\frac{1}{4}$ volts (total drop of 2 to 2·5 volts). For current densities exceeding about 20 amperes per square inch, the drop in the case of carbon brushes increases so very slowly that it may be regarded as approximately independent of the current. The contact drop is almost independent of the peripheral speed of the commutator, but varies with the mechanical pressure, decreasing rapidly at first, then more and more slowly as the pressure is increased. It also depends on the direction of the current across the contact, the drop in most cases (but not always) being *greater* at the positive brush, where the current flows from metal to carbon.

From the data given above the great superiority of the carbon over the copper gauze brush in maintaining sparkless running is amply evident. But this superiority is obtained at the expense of efficiency, since a large contact drop involves a considerable expenditure of power at the contacts. The use of carbon brushes also involves larger and more expensive commutators than would be required with copper gauze brushes, which allow of a much higher current density at the contacts.

Since sparking results from a non-uniform current distribution over the brush contact area, any device which tends to maintain a uniform current density over the contact area will also tend to suppress sparking. Now if the current density is not uniform, the current on entering the brush will flow sideways or across the brush until, higher up the brush, a sensibly uniform distribution is reached. Hence, if the cross-flow be prevented the current will tend to distribute itself more uniformly over the brush contact. In a type of carbon brush introduced by the Morgan Crucible Co., Ltd., and known as the *morganite* brush, the resistivity of the brush in a transverse direction is much higher than in a longitudinal one. This brush is also remarkable on account of the very high current densities (up to 100 amperes per square inch) which may be used with it.

§ 88. Reversing E.M.F. obtained by Forward Brush Lead.

In addition to the method involving the use of brushes giving a high contact resistance, and a design of winding giving a low

With very soft carbon brushes, the drop may be much less.

reactance voltage, another method is available for suppressing sparking. Since the trouble is caused entirely by the presence of the induced e.m.f. in the short-circuited coil, it is obvious that a remedy may be found in the introduction into the short-circuited coil of a rotation e.m.f. equal in amount, but opposite in direction, to the reactance voltage. By means of such an e.m.f., known as a *reversing e.m.f.*, the resultant e.m.f. in the coil may be reduced to zero, and the machine will run sparklessly.

Two methods are available for obtaining a reversing e.m.f., viz., the method of *brush lead* and that of *reversing poles*. The first of these—which, in the early days of dynamos, was the sole method in use for suppressing sparking—consists in displacing the brushes in the direction of rotation, or giving them a *forward lead*. The result of such a forward displacement is that the

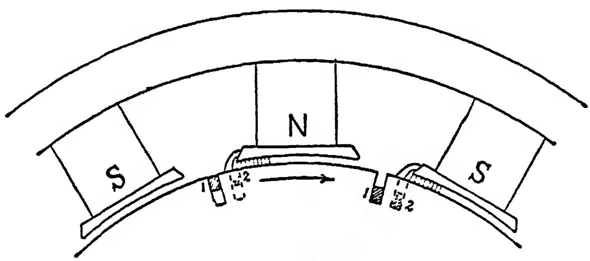


FIG. 66.—Effect of brush lead.

short-circuit, instead of being allowed to take place when the two sides of a loop or coil are half-way between the pole-pieces, as in the position 1, 1 in Fig. 66, is delayed until the sides of the loop have entered magnetic fields of opposite polarity to those across which they had previously been moving, so that they occupy the position 22 in Fig. 66. A reversing e.m.f. will thus be obtained, and if equal in amount to the reactance e.m.f., will neutralise this latter.

Since the field intensity gradually increases as we approach the edge of a pole-shoe, it follows that the value of the reversing e.m.f. may be adjusted by varying the amount of the brush "lead." Further, since the reactance e.m.f. will increase with the current to be reversed, the lead must be increased with increase of load in order that the reversing e.m.f. may keep pace with the increasing reactance e.m.f. Hence in the days when

brushes of copper gauze were used, which have a very low contact resistance, and when the presence of a suitable reversing e.m.f. was absolutely essential in order to secure sparkless running, the brush lead had to be varied to suit each particular load. The necessity of frequent adjustments of the brush lead was a serious inconvenience. With the advent of the carbon brush, this necessity ceased to exist. Provision was, however, made for altering the position of the brushes, which were given a forward lead so as to aid the effect due to the high contact resistance by a partial neutralisation of the reactance e.m.f. by the reversing e.m.f. The position of the brushes was adjusted once for all by trial so as to give the most favourable results as regards sparkless running over the entire range of load, and the machine was run with the brushes fixed in this position at all loads. It is clear that with such a fixed brush position there was only one particular value of the load current for which exact neutralisation of the reactance voltage was obtained; but the high brush contact resistance was sufficient to effect practically sparkless commutation in spite of the imperfect neutralisation of the reactance by the reversing e.m.f.

§ 89. Commutating or Reversing Poles.

The second method of injecting into the short-circuited coil a reversing e.m.f. is that now invariably used. In this method the brushes have no lead, but a reversing e.m.f. is induced in each short - circuited coil by the field produced by an auxiliary pole. One such *reversing* or *commutating pole* (sometimes also termed *interpole*) is placed between each pair of main poles. The arrangement is shown in Fig. 67. The auxiliary pole carries a winding traversed either by the entire armature current or by a convenient fraction of it. Thus the field due to the reversing pole increases

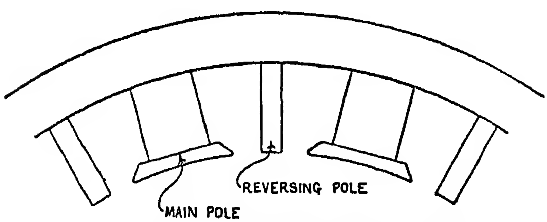


FIG. 67.—Commutating or reversing poles.

Except in the case of very small machines, where the sparking difficulty is never serious.

nearly in proportion to the current—which is the precise condition required for sparkless running. The polarity of each reversing pole is, of course, the same as that of the next main pole in the direction of rotation, so as to make the e.m.f. induced in the short-circuited coil by the rotation a reversing one. In order to reduce the amount of copper on the auxiliary poles, the length of these in a direction parallel to the shaft is generally much less than that of the main poles, the auxiliary pole cross-section being frequently either circular or approximating to a square. Not only does this result in a saving of copper, but it has the further advantage of not blocking up the entire space between the main poles (which would interfere seriously with the ventilation of the machine).

In machines as formerly constructed the limit of output was practically fixed by considerations of sparking; but in machines fitted with reversing poles, the limit of output is fixed by the safe temperature rise. This latter limit may be extended very considerably by providing numerous ventilating apertures. Hence although the extra cost of the reversing poles may be of importance, yet by reason of the increased output the cost of a machine with commutating poles is, for a given output, less than that of a machine not fitted with such poles, except in the case of small machines.

The pole-arc of the reversing poles is generally about 15 per cent. of the pole-pitch; it should in any case be of sufficient width to cover at least two teeth and two slots, in order to prevent excessive fluctuations in the reversing flux and mechanical vibration of the reversing pole; such vibration was found to be troublesome in the early days of interpole machines, and it is now common practice to anchor the reversing pole-shoe to the main pole-shoes by means of suitable strips of copper or brass.

§ 89a. Adjustment of Interpole Field.

Two methods may be used for adjusting the interpole field; in the first, the interpole ampere-turns are varied, and in the case of large machines the only practicable way of doing this consists in providing an adjustable shunt across the interpole winding; in the second method, the interpole ampere-turns are kept constant and the interpole air-gap is varied, generally by the insertion or withdrawal of thin iron liners between the yoke and the interpole.

L 2

In order to determine the best value of the interpole field, one or other of the following methods may be employed, a low-reading voltmeter being required for the purpose of making the adjustment.

(1) The voltmeter is connected to two points on the brush, close to the commutator surface, one on each side of the brush ; these are marked 1 and 2 in Fig. 68. In the case of ideal commutation, the current density over the brush contact surface being uniform (§ 83), the current is everywhere normal to the surface, and there is no tangential component of current, and hence no drop of potential in a tangential direction ; so that a voltmeter connected across 1 and 2 would give a zero reading. The voltmeter being so connected, the interpole field is adjusted until a *minimum* reading is obtained.

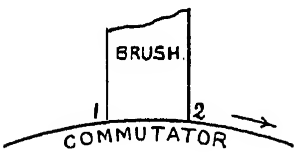


FIG. 68.—To illustrate adjustment of interpole field.

(2) One terminal of the voltmeter is connected to any point on the brush, while the other is connected in succession to a number of points on the *commutator* surface comprised within the arc 1—2. The voltmeter readings are plotted against distances along the arc, the best condition corresponding to the curve which forms the nearest approach to a horizontal line (the latter would be obtained with uniform current density over the brush contact surface).

(3) One voltmeter terminal being in connection with the brush, the other is connected to a point on the *commutator* surface just ahead of the brush in the direction of rotation (just ahead of 2 in Fig. 68), and the interpole field is adjusted to give a minimum voltmeter reading (a high reading would indicate high current density near the leading edge 2 of the brush).

§ 89b. Flashing Over and Use of Neutralising Winding.

A trouble which, under certain conditions, is very liable to occur, particularly in the case of high-voltage machines, is that known as *flashing over*. This consists in the momentary formation of an arc which flashes or spreads over the entire commutator surface from positive to negative brush sets, and may in some cases result in damage to the machine ; it in any case results in a temporary stoppage of the machine.

A flash-over is primarily caused by an abnormally high e.m.f. in

armature coils—high enough to maintain an arc. If while a coil is in a position where such an e.m.f. is being generated by it the commutator segments to which it is connected happen to be bridged across by a particle of conducting matter, and the particle is vapourised by the momentary short-circuit current in the coil, the arc thus started between the segments will be maintained. The arc vapour spreads to neighbouring segments, and the arc may rapidly extend or flash across from brush set to brush set. A flash-over is generally accompanied by a very loud report.

The mean e.m.f. per armature coil (or, neglecting the resistance drop in the coil, the mean p.d. between two neighbouring segments) does not ordinarily exceed about 12 or 15 volts,* and this would be somewhat lower than the *maximum* e.m.f. per coil even in the case of an unloaded machine (in which there is no field distortion), owing to the fact that the greater part of the total e.m.f. is generated by the coils under cover of the poles. When, however, the armature is loaded and field distortion takes place (Fig. 58A), the e.m.f. in the coil which is moving across the strongest part of the field will greatly exceed the mean e.m.f. per coil, and a flash-over is much more liable to occur. In the case of high-voltage machines, in which the highest permissible value of the mean e.m.f. per coil is used, a very heavy load or short-circuit is apt to cause a flash-over. This danger may be guarded against by providing the machine with a *neutralising* or *compensating* winding, which consists of coils embedded in the pole-shoes (the latter being slotted to receive the coils, as shown in Fig. 68A), and provides a number of ampere-turns equal and opposite to the armature ampere-turns, thereby *neutralising* or wiping out the armature field and preventing the field-magnet flux from suffering distortion. Automatic adjustment of the neutralising ampere-turns to the armature current is obtained by connecting the neutralising winding in series with the armature.

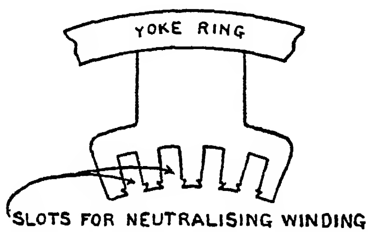


FIG. 68A.—Field pole for machine with neutralising winding.

* In the case of machines provided with a neutralising winding, this limit may be pushed up to about 20 volts.

Since a neutralising winding adds considerably to the cost of the machine, it is only used in very exceptional cases, where operation without it would be extremely unsatisfactory or even impracticable. Incidentally, a neutralising winding, by wiping out the armature field, greatly facilitates sparkless running; it also—by preventing distortion—does away with the additional hysteresis and eddy-current losses which would arise in a loaded machine not provided with such a winding.

§ 90. Care of Commutator. Sparking due to Purely Mechanical Causes.

It is important that the commutator should be kept scrupulously clean, and that it should present an absolutely true surface to the brushes. If these conditions are not fulfilled, trouble due to vibration of the brushes and partial fusion of the surface of the segments is likely to arise. As soon as sparking begins, the commutator surface is quickly roughened, increasing the brush vibration and aggravating the evil. If the roughening of the surface is only slight, fine sandpaper may be used for restoring the surface to its original smoothness. It is best to run the commutator without lubricant of any kind. On no account should oil be allowed to reach the commutator surface, as it is found to have a disintegrating effect on the mica between the segments. It is for this reason that oil-throwers (Fig. 82) are provided on the shaft.

A frequent and troublesome cause of sparking is the tendency of the mica to rise above the general surface of the commutator, due to the more rapid wear of the copper segments. A very slight amount of projection is sufficient to cause serious sparking. This trouble may be dealt with very effectively by grooving out the mica slightly so as to depress it below the copper surface, and various special tools for effecting this longitudinal grooving have been devised.

§ 91. Comparison of Lap and Wave Windings. Equalising Connections.

The reactance voltage increasing, in accordance with formula (1) of § 85, in proportion to the current to be reversed, it follows that *with machines of ordinary construction*, not provided with

reversing poles, there is a definite limit to the current in each armature path. This limit may be taken at about 100 amperes (or 200 amperes per brush set). Now, since in the case of a simple wave winding there are only two paths open to the current through the armature, it follows that this winding becomes impracticable when the current output of the machine exceeds about 200 amperes (unless reversing poles be resorted to). For larger currents, the lap winding must be used.

The wave winding has the important advantage over the lap winding of perfect symmetry of the paths through the armature. For since each path includes conductors under cover of *all* the pole-pieces (§ 71), the e.m.f.'s induced in the two paths must necessarily be equal, even if the flux varies from pole to pole (on account of an eccentric position of the armature relatively to the field, or other irregularities). Such, however, is by no means the case with a lap winding (§ 69). For the conductors forming the different paths are under cover of different pole-pieces, and if there is magnetic asymmetry (i.e., if the flux from different poles is not the same), the e.m.f.'s in the various paths may differ, causing local armature currents on open circuit and unequal division of the current (likely to result in sparking at the overloaded brushes) among the different paths when the armature is loaded. In machines with lap windings, therefore, it is important to have the armature very carefully centred, and all the poles as nearly alike as possible. Even when every precaution has been taken, however, it is difficult to secure perfect symmetry and obtain electrical balance of the various paths—especially in large machines, where the paths are numerous and of very low resistance. Hence in such machines it is usual to provide a number † of *equalising connections* in order to prevent any set of brushes from becoming overloaded and hence sparking. These connections consist of stout copper strips connecting points in the winding which normally are equipotential. They are frequently arranged in the form of copper rings carried on insulating supports attached to the arms of the armature spider on the side away from the commutator, each ring being in connection with a number of (normally) equipotential armature

In the case of very large slow-speed machines, this limit may be exceeded; the permissible limit being determined by the reactance voltage.

† One for every 6 to 12 commutator segments.

conductors. In Fig. 69 is shown diagrammatically an equaliser 1, 2, 3, connecting three equipotential points of a 6-pole lap winding (these points would be distant by twice the pole-pitch); B_1, B_2 and B_3 denoting the positive brushes. The coils *a, b, c* are just about to undergo a short-circuit. Now in these coils there will be no appreciable e.m.f., and they may be regarded as forming three equal parallel resistances between the equaliser and the brushes. If the equaliser is of sufficiently low resistance, the points 1, 2 and 3 which it connects will not differ appreciably in potential. Hence even if the currents in the coils *d, e* and *f* be unequal, those in *a, b* and *c* will be practically equalised, and each brush set will receive appreciably the same current. It must, however, be clearly understood that such equalisers do not reduce the increased heating loss (but rather increase it) due to unequal division of the current: the currents are only equalised in the *last few coils* at each end of a path—those which are practically out of the magnetic field. In the active coils, the

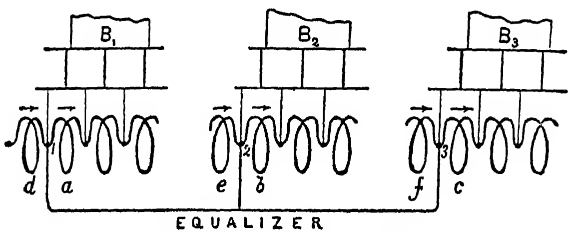


FIG. 69.—Connections of equaliser ring.

current would still be unequally distributed, and the equalisers quite inoperative. The equalisers only come into play, and equalisation of current gradually takes place, in those coils only which are either approaching or receding from a brush. The only function performed by equalisers is the *prevention of sparking* which would occur with unequally loaded brushes.

It may be mentioned, however, that in the case of lap windings, even when not provided with equalising rings, any serious unbalancing of the armature is to some extent automatically checked by increased armature reaction under the more heavily loaded poles.

The troubles arising from the difficulty of obtaining perfect electrical balance of the different paths through a lap-wound

armature render it advisable to use the wave winding wherever possible. Since the introduction of reversing poles, the restrictions formerly imposed on this latter type of winding by considerations regarding sparking have been practically removed, and the winding is now being used for currents (up to 1,000 amperes) greatly in excess of those formerly thought practicable.

§ 92. Method of Balancing Lap-wound Armature.

Two methods of adjusting the position of a lap-wound armature in its field so as to secure a sufficiently accurate balance of its various circuits are available. One of these is a purely mechanical method, the other an electrical one. In the first method, an accurately concentric position of the armature in its field is secured by the aid of a tapered or wedge-shaped gauge which may be inserted between the armature and the various pole-shoes. It is assumed that geometrical symmetry will result in electrical balance of the armature. This method is capable of general application. The second method can only be used conveniently in the case of armatures whose equalising connections are easily removable (Fig. 90). The cables connecting the different sets of brushes of like polarity are removed, as are also the equalising connections, and by means of a voltmeter readings across the consecutive pairs of neighbouring brushes are taken when the machine is running on open circuit. Any want of balance will be indicated by differences in the consecutive voltmeter readings. By displacing the armature relatively to the field in such a direction as to reduce the air-gaps on the side corresponding to the lower readings, the voltmeter readings may be equalised to a greater or less extent. This procedure is repeated until the greatest difference between any two readings does not exceed the desired amount (say 1 per cent.). Instead of stepping round the armature with a voltmeter which is connected to consecutive pairs of *neighbouring* brushes, a differential method of testing may be used, in which a low-reading voltmeter is connected in succession across alternate brushes, which are of the same polarity. In a perfectly balanced armature, a zero reading would be obtained

for each position; in any actual case, the voltmeter reads the out-of-balance voltage between the different groups of armature conductors.

EXAMPLES.

1. During the short-circuit of a certain armature coil, the flux linked with it changes by a total amount equal to 180,000 C. G. S. lines. If the time of the short-circuit is ·001 second, what is the reactance voltage of the coil?

2. The armature of a certain dynamo is running at 800 revolutions per minute; the commutator consists of 123 segments, and the thickness of each brush is such that the brush spans 3 segments. Find the time during which a coil remains short-circuited.

3. A commutator consisting of 450 segments has a diameter of 600 mm., and runs at a speed of 400 revolutions per minute. If the thickness of each brush is 12 mm., what is the time of short-circuit of a coil?

4. In a six-pole lap-wound generator, the total contact area per brush set is 6 square inches, and the current-density over the contact area is 35 amperes per square inch. If the total voltage drop over the two brush contacts in series is 2 volts, what is the power lost at the brush contacts?

5. A twelve-pole lap-wound armature having 1,326 conductors gives a current of 1,200 amperes. The equivalent single air-gap length is ·35 inch, and the pole-arc is 70 per cent. of the pole-pitch. If the average gap induction is 7,000, what is the induction close to the commutating edge of a pole?

6. A ten-pole armature (lap-wound) has 1,440 conductors and delivers a current of 1,000 amperes. The brushes have a forward lead such that a coil is short-circuited when it has advanced 15 per cent. of the pole-pitch beyond the neutral position (i.e., the position half-way between two pole-pieces). Find the de-magnetising ampere-turns per magnetic circuit.

CHAPTER IX.

§ 93. Dynamo used as motor—§ 94. Calculation of torque—§ 95. Effect of changes in brush p.d. and field flux on speed of motor—§ 96. Arrangement of field winding. Characteristics—§ 97. Series-wound generator—§ 98. Series-wound motor—§ 99. Shunt-wound generator—§ 100. Shunt-wound motor—§ 101. Compound-wound generator—§ 102. Compound-wound motor—§ 102a. Conditions for self-excitation of generator—§ 103. Motor-starting switches—§ 104. Methods of speed control—Examples.

§ 93. Dynamo used as Motor.

A DYNAMO is a reversible machine; when driven mechanically, it is capable of supplying electric power; when supplied with electric power, it is capable of running as a motor, converting a large proportion of the electric power supplied to it into mechanical power.

The possibility of using a dynamo as a motor immediately follows from the principle explained in § 2, according to which any conductor conveying a current when placed with its length at right angles to a magnetic field experiences a force which is perpendicular to its own length and to the direction of the field. In modern machines, with toothed-core armatures, the pull is mainly exerted on the teeth; but the total tangential pull is, for a given flux per pole and given armature current, the same as that which would be obtained if the core had a smooth surface and the conductors were placed on the surface instead of being embedded in slots.

If a current be sent through the armature from some external source, the field being excited, the direction of the current is the same in all the conductors which are under cover of a pole-piece, and in all the groups of conductors under cover of pole-pieces of like polarity. In passing from a group under cover of a north pole to one under cover of a south pole, the current changes its sign: hence the pulls contributed by the different conductors are all in the same direction. A reversal of the current through the armature, or a reversal of the field polarity, will reverse the torque and hence the direction of rotation; but a simultaneous

reversal of armature and field currents will leave the direction of torque and rotation unaltered.

§ 94. Calculation of Torque.

The torque exerted by the armature of a motor is easily calculated from the flux per pole and the armature current. By Lenz's law (§ 5), the e.m.f. induced in the conductors of a motor is opposed to the current. Hence the e.m.f. of a motor is frequently spoken of as its *counter-e.m.f.* or *back-e.m.f.* Let E stand for this e.m.f., and let V = brush p.d., r_a = armature resistance, and i_a = total armature current. Then we have

$$i_a = \frac{V - E}{r_a}, \text{ or } V = E + r_a i_a \quad . \quad . \quad . \quad . \quad (1).$$

Now the total power supplied to the armature is Vi_a, or, using (1),

$$Ei_a + r_a i_a^2.$$

The expression for the power is thus seen to consist of two terms. The second term, $r_a i_a^2$, represents the power lost by heating of the armature coils. The first term gives the remainder of the power, which undergoes transformation into mechanical power. Thus

$$\text{mechanical power} = Ei_a \quad . \quad . \quad . \quad . \quad (2).$$

We can, however, easily obtain an expression for the mechanical power in terms of the torque and speed. Let T stand for the torque in *lb. feet*, and n for the revolutions per second. Then the mechanical power is equal to $2\pi Tn$ foot lbs. per second. Expressing it in h.p., we have, since 1 h.p. equals 550 foot lbs. per second,

$$\text{mechanical power} = \frac{2\pi n}{550} T \text{ h.p.}$$

$$\text{,,} \qquad \text{,,} \qquad = \frac{746}{550} \cdot 2\pi n T \text{ watts.}$$

$$\text{,,} \qquad \text{,,} \qquad = 8{\cdot}52\, nT \text{ watts.}$$

Equating this expression to the one given by (2), we find

$$T = \frac{Ei_a}{8{\cdot}52 n} = {\cdot}1174 \frac{E}{n} i_a \quad . \quad . \quad . \quad . \quad (3).$$

But from the expression for the e.m.f. given in § 77, we find $\frac{E}{n} = \frac{\Phi ZP^*}{b \times 10^8}$, and on substituting this in (3) we find

$$T = \cdot 1174 \frac{ZP}{b \times 10^8} \cdot \Phi i_a \text{lb. feet.} \quad . \quad . \quad . \quad . \quad (4).$$

Since in the above expression Z, P and b are constants for a given machine, we see that the torque is proportional to the product of flux per pole into armature current—a result which is also obvious from a consideration of the fact that the pull acting on any individual conductor is proportional to the current and the field. It must be carefully noted that the torque given by (4) is the *total* torque, including that required to overcome the resistances due to mechanical friction, hysteresis and eddy-currents.

§ 95. Effect of Changes in Brush P.D. and Field Flux on Speed of Motor.

We shall next consider the effect on the speed of the motor due to changes of brush p.d. and field flux.

In the expression for the brush p.d. given by (1), the first term, E, which represents the motor e.m.f., is under normal conditions of operation by far the more important, the resistance drop $r_a\, i_a$ being always a small fraction (some 2 per cent. at full load in fairly large motors) of V. Hence V may be regarded as approximately equal to E. Now for a given field flux, or flux per pole, E is proportional to n. Hence the speed n of a motor will vary *nearly in proportion to the brush p.d.* if the flux per pole is maintained constant.

Suppose next that the brush p.d.—and hence, also, approximately, E—remains constant, and that the flux is varied. The constancy of E involves the constancy of the product Nn. Thus with a constant brush p.d., the speed of the motor will vary nearly *inversely as the flux per pole.*

§ 96 Arrangement of Field Winding. Characteristics.

There are various ways of arranging the field windings of dynamos, and the behaviour of a machine depends very largely

* Where $2b$ = number of parallel paths through armature.

on the mode of connecting up the field coils. The three possible modes of connection are shown diagrammatically in Fig. 70, and are known as the series (*a*), the shunt (*b*), and the compound (*c*) field winding. In the series winding, the field coils carry the same current as the armature; in the shunt winding, the field coil has a very high resistance in comparison with that of the armature, and under full-load conditions carries a current which is only a small fraction of the armature current. In the compound winding, both kinds of coils are used: a series coil traversed by the main current supplied to or coming from the external circuit, and a shunt coil connected across the brushes (or across the terminals, as shown by the dotted line in Fig. 70) which takes only a small fraction of the full-load current. The shunt and series coils of dynamos are closely analogous to the voltmeter and ammeter circuits respectively of measuring instruments, the first having a very high resistance and the second a very low one.

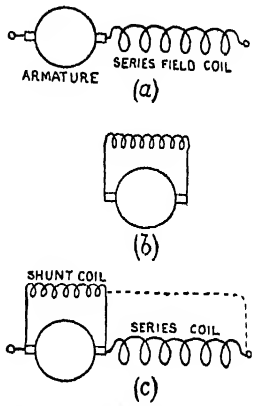


FIG. 70.—Methods of connecting field coils.

In studying the effects due to the different field windings, we assume that the machine, if run as a generator, is driven at a constant speed; and that if run as a motor, it is supplied at a constant p.d. In the case of a generator, the most important relation is that connecting the terminal p.d. with the load current (or current in external circuit). A curve showing this relation is known as a *characteristic*. In the case of a motor, the most important relation is that connecting speed and torque; and the corresponding curve is known as a *mechanical characteristic*.

§ 97. Series-wound Generator.

In the case of a series-wound generator, the exciting current is the same as the load current, so that the e.m.f., and hence also the p.d., will increase (up to a certain limit) with increasing load, and the characteristic will be a curve rising from a point A near the origin, as shown in Fig. 71. The p.d. increases rapidly and almost in proportion to the current during the early stages

(represented by A C), while the field cores are well below saturation, and the reluctance of the iron part of the magnetic circuit is negligible. But as the permeability of the iron decreases, the p.d. begins to increase less rapidly, the curve becomes flatter, and ultimately begins to bend down as shown in the figure. The downward bend is caused partly by armature reaction (§ 80), partly by armature resistance drop. The e.m f. O A obtained on open circuit is due to the residual field (without which the machine could not start).

If we take any point P on the characteristic, and join it to the origin, then the external resistance is given by $\frac{\text{p.d.}}{\text{current}} = \frac{BP}{OB} = \tan \widehat{BOP}$. Now this ratio remains nearly constant along the initial rising portion of the characteristic, so that any small accidental change in the external resistance will cause very large changes in the p.d. and current. The part A C of the characteristic therefore represents unstable conditions of working. Stability can only be obtained by working in the region which lies beyond C.

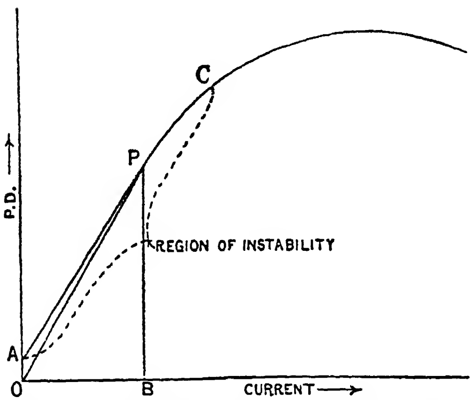


FIG. 71.—Characteristic of series-wound dynamo.

Series-wound *generators* are very little used; their only applications are in series systems of arc lighting, and in a peculiar system of high-voltage constant-current power transmission known as the Thury system.

It may be noted that if the external circuit of a series-wound generator contains a counter-e.m.f., and if this counter-e.m.f. becomes accidentally strong enough to overpower the generator e.m.f., reversing the current, then the polarity of the generator field, and hence also its e.m.f., will be reversed, resulting in a heavy short-circuit. For this reason, a series-wound dynamo would be totally unsuitable for charging accumulators.

In cases where series-wound generators are used, they are arranged to regulate for approximately constant current. This regulation is effected partly by designing the machine so that it has a strong armature reaction, partly by means of a special regulator which shunts more or less of the current through the field, thereby keeping the main current nearly constant.

§ 98. Series-wound Motor.

A series wound motor is essentially a variable speed motor, the speed decreasing with increase of torque, as shown by the mechanical characteristic of Fig. 72. The reason for this behaviour of a series motor is not far to seek. The torque of a motor is, as we have seen (§ 94), proportional to the product Φi_a. Hence an increase of torque involves an increase of this product. Now since Φ increases with increase of i_a, an increased torque involves an increase in Φ. Again, the e.m.f. of the motor remaining approximately constant (since the p.d. of supply is assumed to be constant), it follows that Φn must also remain nearly constant (§ 77). Thus an increase of torque, which involves an *increase* of Φ, is accompanied by a *decrease* of n. Further, so long as the torque is small, and the current correspondingly small, the iron is well below saturation, and Φ changes rapidly with the current and torque; the speed accordingly changes rapidly with the torque. But as saturation is approached, the change in Φ due to a given change in the current and torque is relatively much less, with a correspondingly smaller change in n; thus for large values of the torque the speed changes much less rapidly with the torque than for small values. This accounts for the steady decrease in the steepness of the mechanical characteristic of Fig. 72.

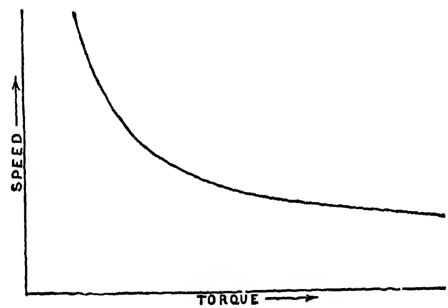


FIG. 72.—Mechanical characteristic of series motor.

A series motor exactly fulfils the conditions required in the case of tramcar motors: it exerts the largest torque at the lowest speeds, and the speed automatically changes with change of load. Hence almost every traction motor is a series-wound motor. Series motors are also used in cases where motors are required to run under constant loads—as in the case of fan motors, and where a variable speed is suitable—as in crane motors.

If the load on a series motor be reduced below a certain limit, the motor will run up to a dangerous speed; if entirely unloaded, the armature will race until it flies to pieces. This source of danger is indicated by the extreme steepness of the mechanical characteristic (Fig. 72) at small loads. Hence motors with a series winding should never be used in cases where the motor is liable to be entirely unloaded.*

§ 99. Shunt-wound Generator.

A shunt-wound generator has a characteristic of the form shown in Fig. 73. The shunt circuit being always closed, the machine builds up a strong field, and develops a high e.m.f., on open circuit. If the external circuit be now closed, the armature current increases, causing an increased armature resistance drop and reducing the p.d. This results in a weakening of the exciting current, which by lowering the e.m.f. further reduces the p.d. A still further weakening of the field and drop of p.d. is brought about by armature reaction (§ 80). Since all these effects increase with increasing armature current, there is a steady decrease in the p.d. with increasing load. This decrease is not, however, excessive within normal limits of working, so that a shunt-wound machine may be regarded as giving a roughly constant p.d.

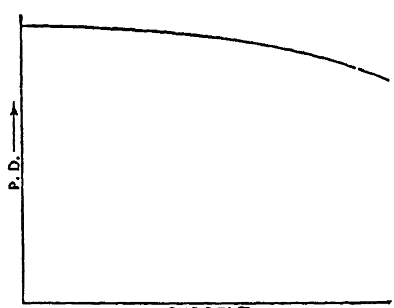


FIG. 73.—Characteristic of shunt-wound dynamo.

The shunt winding is more generally used than any other form of field winding for *generators*, and is the only suitable winding

* A minimum load equal to about 25 per cent. of full load is required in most cases to prevent the speed of the motor from rising above the limit of safety.

for machines intended to charge secondary batteries. In order to enable the p.d. to be adjusted to the desired value, it is usual to provide a *field rheostat*, or regulating resistance connected in series with the field winding, as shown in Fig. 74. The normal direction of the current through the armature, shunt winding and external circuit is indicated by the full-line arrows in Fig. 74. If the machine is charging a secondary battery, and if by reason of a slight decrease of speed or through any other cause its e.m.f. momentarily drops below that of the battery, a reversal of current takes place in the main circuit, as shown by the dotted arrows in Fig. 74. It will, however, be noticed that while the armature current is reversed, no reversal takes place in the shunt winding, and the e.m.f. retains its original direction. The machine is simply momentarily converted into a motor, and as soon as its speed increases again, the e.m.f. rises, and charging is resumed. There is thus no serious disturbance or heavy short-circuit such as would occur with a series-wound generator.

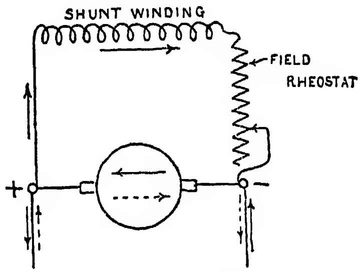


FIG. 74.—Showing effect of current reversal through shunt dynamo.

The bulk of the large generators in central stations supplying current to lighting loads are provided with a shunt winding and field rheostat.

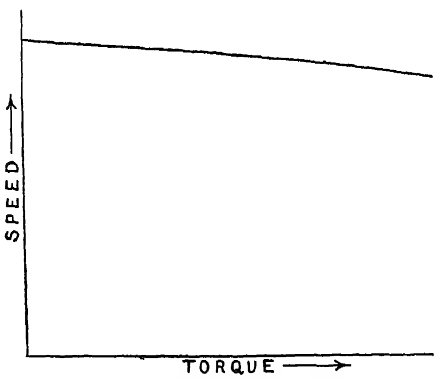


FIG. 75.—Mechanical characteristic of shunt motor.

§ 100. Shunt-wound Motor.

The mechanical characteristic of a shunt-wound motor is shown in Fig. 75. Since the p.d. of supply is constant, the exciting current and Φ will also be practically constant. The armature e.m.f. is also nearly constant, decreasing

but slightly with increase of load. Hence there will be only a slight decrease in the product Φn, or, Φ remaining nearly constant, a slight drop in the speed. A shunt-wound motor is therefore a motor whose speed remains nearly constant at all loads.

Most stationary motors, such as those driving machine-tools and other machinery, are shunt-wound.

§ 101. Compound-wound Generator.

A series-wound generator gives, as we have seen, a rising characteristic, while a shunt-wound generator gives a slightly drooping one. By combining the two windings, i.e., by using both a shunt winding connected across the brushes and a series winding placed in the main circuit, as shown in Fig. 70 (*c*), we are able to obtain a generator whose characteristic is neither a rising nor a falling one, i.e., a generator giving a constant p.d. at all loads. The characteristic of such a compound-wound generator is shown in Fig. 76. Owing to the concavity in the magnetisation curve of iron, it is impossible to obtain an absolutely straight line for the characteristic, which is always slightly concave downwards. Frequently (as in traction generators) it is advisable to have a rising characteristic, such as the dotted one shown in Fig. 76. This may be obtained by increasing the turns in the series coil. The machine is then said to be *over-compounded*.

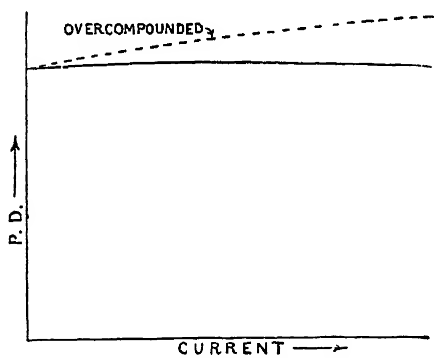


FIG. 76.—Characteristic of compound-wound dynamo.

The compound winding is used in connection with machines intended for incandescent lighting, especially when carried out on a small scale (ship lighting), and in traction generators.

* $V = E + r\,i_a$. Thus, V being constant, E must slightly decrease as i_a is increased.

§ 102. Compound-wound Motor.

Although there are two possible varieties of the compound winding for motors, only one of these, known as the cumulative compound winding, in which the magnetic effects of the series and shunt coils are additive, is of practical interest. This winding is employed in cases where a decrease of speed is desired with increase of load, and where the danger connected with a plain series winding, the liability of the motor to race when unloaded (§ 98), must be avoided. The mechanical characteristic of a motor with a cumulative compound winding is shown in Fig. 77. The shunt winding may here be regarded as imposing a safe limit on the speed of the motor. Compound-wound motors are used in cases where there are very heavy sudden overloads, as in rolling mills. If a shunt-wound motor were used in such cases, the sudden demand for power would have to be supplied entirely by the generator. But by allowing the motor speed to decrease with increase of load, and fitting the motor shaft with a heavy fly-wheel, the momentary demand for power will be largely supplied by the kinetic energy given out by the fly-wheel as its speed decreases, and the generator may be made of smaller size than would otherwise be necessary.†

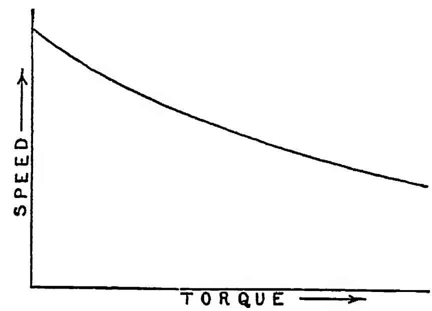


FIG. 77.—Mechanical characteristic of compound-wound motor.

The other form is known as the *differential* compound winding, and in it the series coil opposes the shunt coil. In a simple shunt-wound motor, the speed would drop slightly with increase of load; the series winding, by weakening the field, causes a slight increase of speed (§ 95), thus compensating for the effect due to armature resistance drop.

† In some instances, *shunt-wound* motors have been fitted with fly-wheels. The absurdity of this practice will at once become evident when it is considered that the kinetic energy of a fly-wheel during a momentary overload can only be utilised if the speed *decreases* with increase of load; but this decrease of speed in the case of a shunt-wound motor is so slight that unless an abnormally heavy and costly fly-wheel be used the desired effect cannot be obtained.

§ 102a. Conditions for Self-excitation of Generator.

Consider the case of a series-wound generator (§ 97) which is running at a constant speed on open circuit, so that there is no current flowing in either armature or field. Owing to the residual magnetism of the field, the armature will develop a small e.m.f. Let the field winding be disconnected from the armature and separately excited, the direction of the current being such as to increase the initial magnetic flux; let the exciting current be increased by a number of steps, and for each value of the current let the e.m.f. of the armature be found by means of a voltmeter connected across the brushes. Finally, let the e.m.f. be plotted against the exciting current as in Fig. 77A. We thus obtain a curve which is generally referred to as the *magnetisation* or *excitation* curve or characteristic (also variously termed *no-load* or *open-circuit* characteristic) of the machine.

Let us next suppose that the field winding is re-connected to the armature as usual (the two being in series), and that the circuit of the machine is closed through an external resistance. At the instant of closing the circuit, the armature is generating the small e.m.f. corresponding to the residual flux of the machine; when, therefore, the circuit is closed, this e.m.f. will cause a small exciting current to flow through the field winding which will increase the flux; but the increased flux will cause the e.m.f. to increase, which will cause a further increase of flux and a further increase of e.m.f., and so on, the action being a cumulative one. It would thus at first sight appear as if the e.m.f. of the machine should go on increasing indefinitely. So far from this being the case, however, we find that the e.m.f. never increases beyond a certain limit depending on the total resistance of the circuit, and that when this resistance exceeds a certain limit, the rise of e.m.f. above the residual value is so slight as to be practically negligible; we then say that the machine refuses to excite itself.

We shall now investigate the conditions under which a machine is capable of exciting itself, *i.e.*, of generating an e.m.f. greatly in excess of that corresponding to its residual magnetism, and consider the reason why in every case the e.m.f. cannot exceed a definite value.

Let in Fig. 77A the curve P Q R S represent the magnetisation curve of the machine. If the total resistance of the circuit remains

constant, the current will increase in direct proportion to the e.m.f., and in the diagram of Fig. 77A the relation connecting e.m.f. and current for a given constant value of the total resistance of the entire circuit would be represented by a straight line, making an angle θ with the axis of current such that $\tan \theta =$ total resistance of circuit.

Whatever the values of the e.m.f. and current, these values must be such that the point corresponding to them lies on the curve P Q R S. Again, assuming the current to have reached a *steady* value, this value must be such that the current and the corresponding e.m.f. are represented by a point lying on the resistance line corresponding to the given total resistance of the circuit. Hence, under

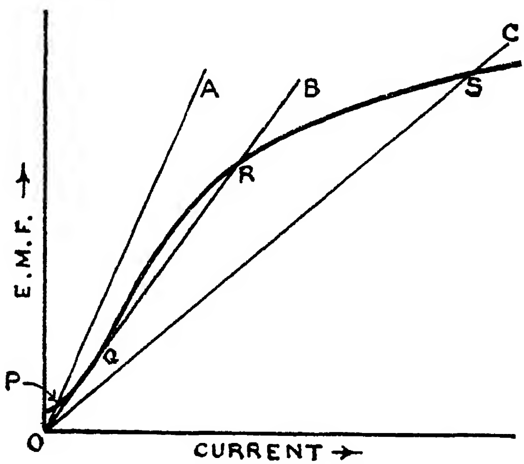


FIG. 77A.—To illustrate conditions for self-excitation.

steady conditions the current and e.m.f. are given by the intersection of the total resistance line with the magnetisation curve.

Suppose now that the total resistance is represented by the line OA, which intersects the characteristic in P, and suppose that the switch was originally open while the machine was running, and is suddenly closed. The e.m.f. due to residual flux causes a current to rise in the circuit, and the point connecting current with total resistance drop travels along the straight line OA; but the point connecting current with generator e.m.f. travels along the characteristic. It will be noticed that, so long as the current is below the value defined by P, the generator e.m.f. exceeds the resistance drop,

the balance causing the current to increase. This increase goes on until the point P is reached, where the e.m.f. is just equal to the resistance drop, and there is no balance to cause any further increase of current.. The current therefore becomes *steady* at P. It will be noticed that at the point P the generator e.m.f. increases with the current at a lower rate than the resistance drop. From this it follows that if, owing to a slight momentary rise of speed or decrease of resistance, the current begins to rise, the generator e.m.f. will increase less rapidly than the resistance drop as the current rises, and the rise will be checked when the two become equal. Similarly it follows that a small momentary decrease of speed or increase of resistance will result in only a small decrease of current. Hence the point P represents a *stable* condition of working, and it will be seen that the resistance corresponding to OA is such that the machine is prevented from "building up" its field, its flux at P being only slightly greater than the residual flux.

Next suppose that the total resistance is represented by the line OB, which *touches* the characteristic at Q. Then Q will be a stable point for a momentary *decrease* of current, but *unstable* for an increase of current, since in the latter case the generator e.m.f. will increase more rapidly than the resistance drop, so that there will be nothing to check the increase of current. The current will therefore *go on* increasing until equality of generator e.m.f. and total resistance drop is reached, and this will not happen until the point R is reached, which again corresponds to a point of *stable* working. Thus any slight momentary increase of current will cause the machine to pass from Q to R.

It will now be seen that the line OB, which is tangential to the characteristic at its lower bend or knee, corresponds to the *maximum* resistance with which the generator is able to build up its field, and any resistance in excess of this value, which may be termed the *critical resistance*, will prevent the machine from exciting itself. On the other hand, the machine will build up its field for all values of the total resistance *less* than the critical resistance.

Returning to the consideration of the resistance corresponding to OA, which is above the critical resistance, let us suppose that the speed of the generator is gradually raised. Since for a given value of

This balance is used up in overcoming the counter e.m.f. which appears in the circuit by reason of the varying magnetic flux due to the rising current.

the current or flux the e.m.f. is proportional to the speed, an increase of speed will have the effect of lengthening all the ordinates of the characteristic in the same ratio, that of the speeds. As this lengthening takes place, the characteristic will ultimately, at a certain speed, become tangential at its lower bend to OA, and at any speed in excess of this the machine will build up its field. We may express this result by saying that, for a given value of the total resistance, there is a *critical speed* above which the machine will excite itself.

A shunt-wound generator running on open circuit (without an external load but having its field connected across the armature) may be regarded as a series machine whose circuit is closed through its field winding, and similar considerations apply in this case. A shunt-wound machine will refuse to excite if its speed is below the critical value.

§ 103. Motor-starting Switches.

The armature of a motor at rest not containing any counter-e.m.f., it is evident that on account of the low armature resistance it would not be permissible to start the motor by simply closing a switch which connects the motor terminals directly to the supply mains. The enormous current so produced would simply blow the fuses, and would be equivalent to a short-circuit of the mains. Let us, for example, consider a 240-volt shunt-wound motor whose full-load armature current is 30 amperes, and whose armature r sistance is ·3 ohm. If the full p.d. of 240 volts were applied to the armature when at rest, the current would be $\frac{240}{\cdot 3} = 800$ amperes! In order to prevent this enormous rush of current at starting, it becomes necessary to reduce the p.d. across the armature by means of a

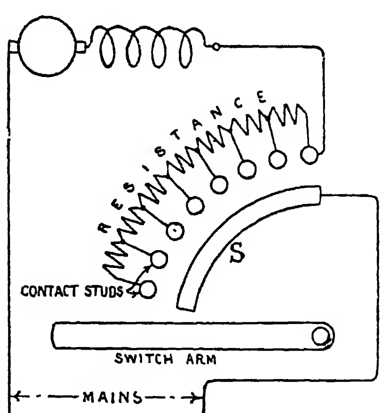


FIG. 78.—Starting switch for series motor.

suitable resistance connected in series with it, the resistance being gradually cut out as the armature speed and e.m.f. increase. Such a starting resistance with its multiple-contact switch is termed a *motor starter* or starting switch.

In the case of a series-wound motor, a simple starter of the form shown in Fig. 78 may be used. With the switch arm on the first contact, the whole of the starting resistance is included; by gradually moving the contact-arm over the consecutive contact-studs as the motor gains speed, the resistance is cut out.

When a series-wound motor is switched off, the magnetic flux through the field cores disappears more or less suddenly, and in so doing induces a momentary e.m.f. in the field winding. But since this winding consists of comparatively few turns, the total e.m.f. is never sufficiently high to endanger the insulation of the coils.

With a shunt-wound motor, however, a sudden interruption of the shunt circuit would give rise to very high e.m.f.'s in the field coils, as these consist of a large number of turns of comparatively fine wire. The e.m.f. so induced might be sufficient to break down the insulation, and would in any case subject it to severe and unnecessary strains. For this reason, the shunt circuit is arranged so as to remain always closed through the armature circuit, whether the motor is at rest or running. In Fig. 79, which shows a typical form of motor-starter for a shunt-wound motor, the circuit A B C D E F, it will be noticed, always remains closed. When the current is switched off, the magnetic flux through the field begins to disappear, but since the armature is still running, it maintains the excitation, so that the flux dies away comparatively slowly. Even if the switch were placed on the first contact and then suddenly pulled back before the armature had had time to start, no harm would be done; for as soon as the flux begins to disappear the e.m.f. induced in the field winding gives rise to a current which tends to maintain the flux, and only allows it to disappear slowly instead of suddenly, as would be the case if the field circuit were broken.

In Fig. 79 are shown two devices which are almost invariably used in connection with motor starters, and which are known as the *no-voltage* and *overload* magnets. The switch arm is provided with a spiral spring which normally keeps it in the "off" position. Attached to the arm is a rectangular piece of soft iron which

forms the armature of the no-voltage magnet, and which is held against its poles when the switch arm has been brought into the full "on" position. So long as the no-voltage magnet is excited, the switch arm is held in this position. But if the current is switched off, or if there should occur a break in the shunt circuit, the no-voltage magnet loses its magnetism, and by the action of the spring the switch arm is thrown into the "off" position. The overload magnet is provided with an armature pivoted at one end.

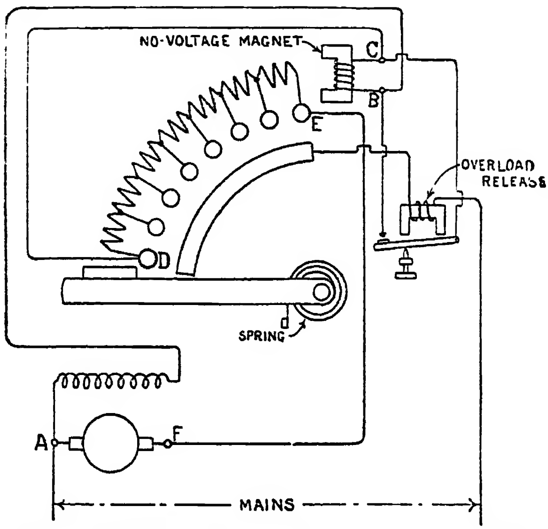


FIG. 79.—Starting switch for shunt or compound motor.

So long as the armature current does not exceed a safe limit, the magnet is not strong enough to attract its armature. But when this limit is exceeded, the armature is attracted, short-circuiting the no-voltage magnet and throwing the switch arm into the "off" position. The point of cut-off may generally be adjusted to any desired value by raising or lowering the armature, so as to decrease or increase its distance from the poles of the magnet.

The form of motor starter just described may also be used in connection with a compound-wound motor, in which case the series winding comes between the points E and F in Fig. 79.

If a motor is required to run both ways, a reversing switch is introduced into the armature circuit, so that by reversing the armature current the direction of rotation may be reversed. In order to prevent a heavy short-circuit in case an attempt is made to reverse the current through the armature before the starting switch has been brought back to its "off" position, the starting and reversing switches should be interlocked, either mechanically or electrically, so as to render it impossible to throw over the reversing switch until the starting switch has been brought into the "off" position. The most satisfactory arrangement is to combine the starting and reversing portions into a single switch. Such a combined switch is shown diagrammatically in Fig. 80. The switch arm carries two triple contacts, shown by the dotted lines in the figure. The bottom contact is arranged to move over the three contact segments, and the top contact to move over two contact segments and the row of contact studs in connection with the starting resistance. Each contact stud lying to the right of the vertical centre-line is connected to the corresponding stud lying to the left of the centre-

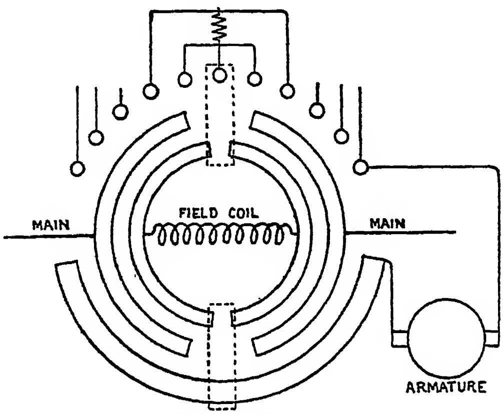


Fig. 80.—Combined reversing and starting switch.

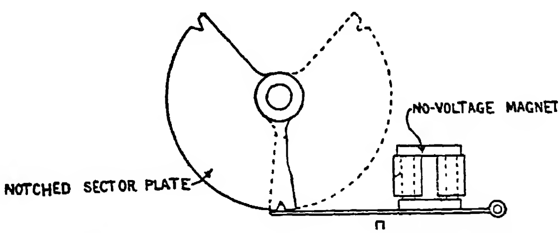


Fig. 81.—Magnetic detent for keeping starting switch in "on" position.

line. There is only one set of starting resistances (only the first section of the starting-resistance is shown in Fig. 80). It will be seen, by tracing out the connections, that while the armature current flows one way or the other, according to the direction in which the switch arm is moved from its "off" position, the direction of the field current remains unaltered. In the "off" position shown by the dotted lines in the figure, the field is short-circuited by the switch contacts, which bridge across the gaps between the two inner segments to which the field coil is connected. The switch arm is held in the full "on" position by the magnetic detent shown in Fig. 81; the notched sector being mounted on the spindle of the switch arm. If the switch is intended for speed regulation as well (§ 104) the sector has as many notches on it as there are contact studs, so that it may be left permanently on any contact stud.

The resistances used in motor-starting switches may take various forms. For comparatively small currents, they consist of spirals of wire which are either contained in a cast-iron box completely filled with sand (the sand giving the spirals the necessary mechanical support and preventing their getting out of shape and causing short-circuits) or are wound on metal tubes covered with asbestos or some other insulating material, or on grooved cylinders of porcelain. For larger currents, metal plates or grids are used, which may be immersed in an oil bath to facilitate cooling.

If the circuit supplying a motor is provided with a main switch in addition to the starting switch, the motor may be stopped by opening the main switch. The starting switch does not immediately fly back into the "off" position, as the no-voltage magnet will continue to receive current from the motor armature, and will keep its armature attracted until the speed falls below a certain limit. Frequently, however, it is convenient to be able to stop the motor without having to open the main switch, and for this purpose a small push-button switch is provided which, when pressed, short-circuits the no-voltage magnet and releases the switch arm. If the contacts of this releasing switch are dirty, it may fail to act. In an arrangement patented by F. Broadbent* this difficulty is overcome. The no-voltage magnet coil

* *Electrical Review*, vol. 64, p. 118 (1909).

is normally connected in parallel with a resistance, and the act of pressing the button disconnects the magnet coil (at one end) while leaving the resistance in circuit, which now takes the whole of the field current.

§ 104. Methods of Speed Control.

A problem of great importance in connection with motors is that of speed control. The speed of a motor may be altered by altering either the brush p.d. or the field flux. Two methods of speed control are in general use. These may be described as the rheostatic and the field control method. In the first, the brush p.d. is altered by connecting a resistance in series with the armature. The great disadvantage of the method lies in its wastefulness. If, for example, we wish to reduce the armature speed to half its original value, the brush p.d. must be approximately halved, which means that about half the total power supplied to the armature circuit is wasted in the regulating resistance. The field control method is incomparably more economical. In this, the field is weakened or strengthened until the desired speed is reached. The difficulty connected with this second method is that the motor must be made larger and is therefore more expensive than would be the case if rheostatic control in the armature circuit were used. This is due to the fact that if the armature is to be capable of developing its normal torque when running at the highest speed, i.e., in the weakest field, it must take a much larger current than when running in the strongest field, which gives the lowest speed. Thus in the field control method increased efficiency of working is obtained at the expense of greater first cost of motor.

In a series-wound motor, the weakening of the field is effected by placing a shunting resistance across the field winding. In a shunt-wound motor, the same end is attained by using a field rheostat in the shunt circuit.

Not infrequently, a combination of both methods of speed control is used in connection with the same motor.

A modification of the field control method has also been used

which may be described as the brush displacement method of speed control. This can only be used satisfactorily with machines having reversing poles.

We have seen (§ 88) that in the case of a generator a reversing e.m.f. may be obtained in the short-circuited coils by giving the brushes a *forward* displacement, and that such a forward displacement results in a weakening of the field by the de-magnetising belt of ampere-turns which is formed on the armature. If we now consider the case of a motor, and suppose that the directions of field flux and armature rotation are similar to those for the same machine when used as a generator, then it is obvious that the armature currents must have opposite directions in the two cases. What was a reversing e.m.f. in the generator will, therefore, no longer be such in the motor, and in order to obtain a reversing e.m.f. in a motor, the brushes must be given a *backward* displacement relatively to the direction of rotation, or a *backward lead.* The armature current having now the opposite direction to that in a generator, a backward displacement of the brushes in a motor has the same effect as a forward displacement in a generator—we again obtain a de-magnetising belt of ampere-turns. In motors required to run both ways (such as traction motors) the brushes are not displaced from their position of symmetry; but in the case of non-interpole motors whose direction of rotation remains unaltered, sparkless running may be facilitated by giving the brushes a backward lead.

If instead of displacing the brushes backwards in a motor we were to displace them forwards, we should obtain a magnetising belt of conductors on the armature instead of a de-magnetising one, and the field would be strengthened. In motors of ordinary construction, this could not be done to any large extent owing to sparking troubles. But in the case of motors provided with suitably designed reversing poles, the sparkless commutation zone may be widened to such an extent that a considerable forward or backward brush displacement becomes possible without causing sparking. Now, since a backward displacement weakens the field, and a forward one strengthens it, we have a means of varying the field intensity, and hence controlling the speed of the motor, by simply shifting the brushes one way or the other. This method of brush displacement has in practice been found capable of giving an enormous range of speed variation.

A method of speed control which has been used in connection with special applications, but which, owing to the heavy cost of the plant required, is not suitable for general use, is the *Ward Leonard* system of control. In this, the speed of the motor is varied by varying the voltage across its armature while keeping its field excitation constant, but instead of obtaining the voltage variation by the wasteful method of a series resistance, a special motor-generator set is used, and the excitation of the generator is varied as required. The method thus involves the use of three machines in place of a single motor—the motor driving the generator, the variable voltage generator, and the variable speed motor connected to the load. In some cases, as when the supply is an alternating current one, a small exciter set is required in addition. In spite of the heavy capital outlay which the system involves, its efficiency, ease of control and enormous range of speed variation have secured for it recognition as one of the standard methods of speed control.

EXAMPLES.

1. The core of a four-pole wave-wound motor armature is provided with 47 slots, each slot containing 24 conductors. If the flux per pole is $1{\cdot}33 \times 10^6$, and the armature current is 17 amperes, what is the total torque, in lb. feet, exerted by the armature?

2. The wave-wound armature of a four-pole motor contains 990 conductors, and exerts a total torque of 150 lb. feet when taking a current of 33 amperes. If the armature runs at 700 revolutions per minute, what is the p.d. across its brushes? The resistance of the armature winding is ·065 ohm, and the total brush contact drop is 2·3 volts.

3. In a certain motor, the flux per pole is $2{\cdot}35 \times 10^6$, and the armature runs at a speed of 950 revolutions per minute when taking a current of 129 amperes, the brush p.d. being 220 volts. The resistance of the armature winding is ·0295 ohm, and the total brush contact drop is 2·2 volts. If the flux per pole be reduced to 2×10^6, and the brush p.d. to 200 volts, the load on the motor (i.e., the resisting torque) remaining unaltered, what will be the new speed of the motor armature?

4. The flux per pole in a four-pole tramcar motor amounts to $3{\cdot}65 \times 10^6$. The wave-wound armature contains 758 conductors, and is geared to the car axle by means of spur gearing whose speed reduction ratio is 4·78. If the armature takes a current of 50 amperes, what pull will it exert at the circumference of the car wheels, assuming the wheels to be 30 inches in diameter, and the friction losses (including hysteresis and eddies in armature) to amount to 12 per cent. of the total mechanical power developed by the armature?

CHAPTER X.

§ 105. Shafts and journals—§ 106. Armature cores and spiders. Equalising rings—§ 107. Slot insulation and binding wires—§ 108. Relation connecting dimensions and speed with output of armature—§ 109. Usual values of armature speed—§ 110. Construction of commutators—§ 111. Construction of yoke and poles—§ 112. Brushes and brush gear—§ 113. Bearings—§ 114. Weight and cost of dynamos. Examples.

§ 105. Shafts and Journals.

ARMATURE shafts are made of the best mild steel (generally Bessemer steel). In addition to the torsional stress arising from the transmission of the driving torque through the shaft, the shaft is subjected to a bending stress due to the weight of the armature and any possible asymmetry of the field, such as might arise from a slightly eccentric position of the armature relatively to the field, caused by imperfect initial adjustment of the bearings, or by wear of the bearings after prolonged running. Further, in the case of some machines—such as traction generators or motors—the machine may be liable to sudden heavy overloads, and the stresses in the shaft will then greatly exceed the normal stresses. In order to allow for all such contingencies, armature shafts are generally designed with a very ample margin of safety.

The diameter of the *thickest* part of the shaft may be determined by means of the formula

$$d = C \sqrt[3]{\frac{\text{kilowatts}}{\text{revolutions per minute}}},$$

d being the diameter in *inches*, and C having a value which ranges from 7 to 9, the higher values of C being used in connection with machines of smaller output.

Within the bearings the shaft may be considerably reduced in diameter, the diameter of the journals being made about 10 per cent. less than that of the thickest part of the shaft. In small machines a much greater reduction (up to 30 per cent.) may be made. The peripheral speed of the journals does not generally exceed 600 feet per minute.

The length of a journal is generally from $2\frac{1}{2}$ to $3\frac{1}{2}$ times its diameter, the length of journal increasing with the speed.

In order to prevent the oil from creeping along the shaft and reaching the commutator and winding—which would endanger the insulation of the machine—it is usual to provide the shaft with *oil-throwers* on the inner ends of the journals. Such an oil-thrower may consist either—as shown in Fig. 82—of a ridge turned on the shaft, or of a sharp-edged ring shrunk on to the shaft.

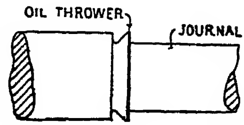


FIG. 82.—Oil-thrower on dynamo shaft.

§ 106. Armature Cores and Spiders. Equalising Rings.

The armature core is built up of either Swedish iron or mild steel stampings, of a thickness not exceeding ·025 inch. The insulation between the stampings consists of a coating of insulating varnish (in some cases, very thin paper insulation is used). The aggregate thickness of the insulation amounts to about 12 per cent. of the gross length of the laminations.

In the case of very small armatures, the armature stampings consist of discs threaded directly on the shaft, and held in place by means of two gun-metal end-plates, one of which abuts against

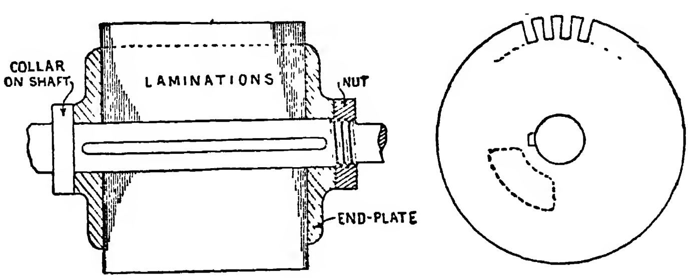


FIG. 83.—Method of mounting small armature core.

a collar on the shaft, while the other is tightened up by means of a nut, as shown in Fig. 83. Each disc is stamped out with a key-way which fits over the feather key provided on the shaft. If the armature is of somewhat larger size, the radial depth of

the laminations becomes unnecessarily great, and the core may be lightened by punching out large openings in the core plates, as shown by the dotted line in the left-hand bottom quadrant of the end view in Fig. 83, the core discs becoming wheel-shaped, with a number of radial arms connecting the boss to the periphery. Besides lightening the core, this plan has the advantage of improving its ventilation.

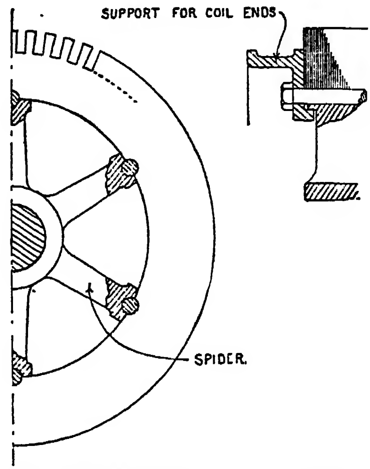


FIG. 84.—Method of mounting large core.

In all large machines the laminated ring which forms the armature core is supported by a cast-iron frame, technically known as a *spider*. When the core is built up of whole rings, which are obtainable up to 4 feet in diameter, the spider consists of a central boss fitting over the shaft, and a number of arms projecting radially outwards, as shown in Fig. 84. The core plates are clamped between two end-plates, provided with flanges which support the ends of the coils.

In very large machines the armature core ring is built up of segments, of the form shown in Fig. 85. The spider is in this case provided with a rim, and the stampings are dovetailed—as shown in Fig. 86 — into the spider rim. By splitting the rim, the spider casting is relieved of the stresses which might otherwise arise by reason of unequal cooling after casting. The number of spider arms is generally from 6 to 10. The cross-section of the arms varies in different designs, being frequently elliptic (ratio of axes 2 : 1 or 3 : 1), with the arms solid or hollow, and sometimes +-shaped or H-shaped. The armature stampings in alternate layers are arranged to break joint with each other, the butt joints of

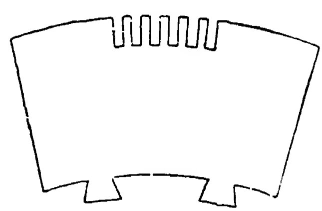


FIG. 85.—Core stamping.

one layer coming opposite the middle lines of the segments forming the next layer (this is indicated by the dotted radial lines in Fig. 86). The stampings are bolted between two clamping rings in a manner similar to that shown in Fig. 84, the clamping rings being as before provided with flanges for supporting the coil ends.

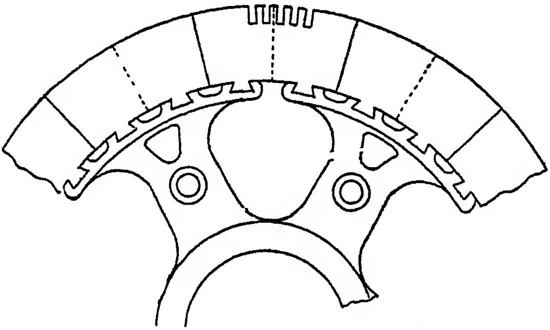


FIG. 86.—Armature spider and core stampings.

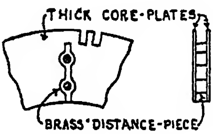


FIG. 87.—Spacing disc for armature core.

In order to secure good ventilation the core is provided, at intervals of about 4 inches, with ventilating gaps about half an inch wide. These gaps between the consecutive groups of core plates are formed by the interposition of special distance-pieces. Various designs of such distance-pieces or ventilating space-blocks have been used. One form is shown in Fig. 87. The core plates on either side of the ventilating aperture are made thicker than the others—the thickness amounting to about one-sixteenth of an inch—and they are riveted to brass blocks of the shape shown. Another arrangement is illustrated in Fig. 88. The ventilating space-blocks in this case consist of thick plates similar to the core-plates, but having

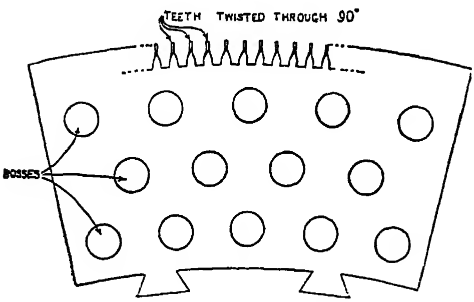


FIG. 88.—Ventilating space-block.

In large machines these rings would not be continuous, but would consist of a number of segments.

their teeth twisted through 90 degrees so as to make the top edges parallel to the shaft; the plates are further provided with large cup-shaped bosses which project the requisite distance outside the plane of the plate on each side.

The radial depth of the armature core-discs is generally from ·3 to ·4 of the pole-pitch; the pole-pitch being the distance, measured along the armature circumference, between the centre-lines of two neighbouring pole-pieces, so that pole-pitch $= \frac{\pi \times \text{armature diameter}}{\text{number of poles}}$. The ratio of the core length, measured in a direction parallel to the shaft, to the core diameter is, in machines of ordinary construction, approximately given by

$$\frac{\text{core length}}{\text{core diameter}} = \frac{2 \cdot 5}{\text{number of poles}}.$$

The teeth of the core-plates are either straight or are expanded at the top so as to partially close the slots. The armature conductors are conveniently held in place by means of hard-wood (beech) wedges,* as shown in Fig. 92. The minimum width of a tooth—*i.e.*, its width at the base—should not be below about 4 mm., or ·16 inch. The ratio of width of tooth to width of slot varies from ·5 to 1. The depth of a slot does not as a rule exceed four times its width, and is generally less than ·2 of the pole-pitch.

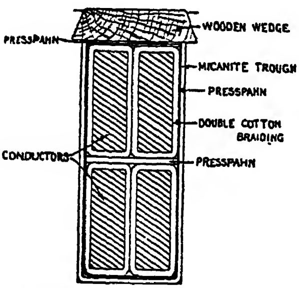


FIG. 89.—Arrangement of conductors in slot.

In old machines not provided with interpoles, the slots in the armature core were generally of relatively large width and few in number, and each slot contained from 4 to 6 coil-sides. In the case of modern interpole machines, however, in order to secure good commutation, it is usual to have only two coil-sides per slot, so that the slots are numerous and narrow, and each coil moves through exactly the same region of the field during its short-circuit.

In order to avoid excessive hysteresis and eddy-current losses in the pole-shoes, the width of slot is generally less than ¾ inch.

Equalising rings (§ 91) are generally attached to the extension

* In high-speed machines these wedges would consist of aluminium or bronze.

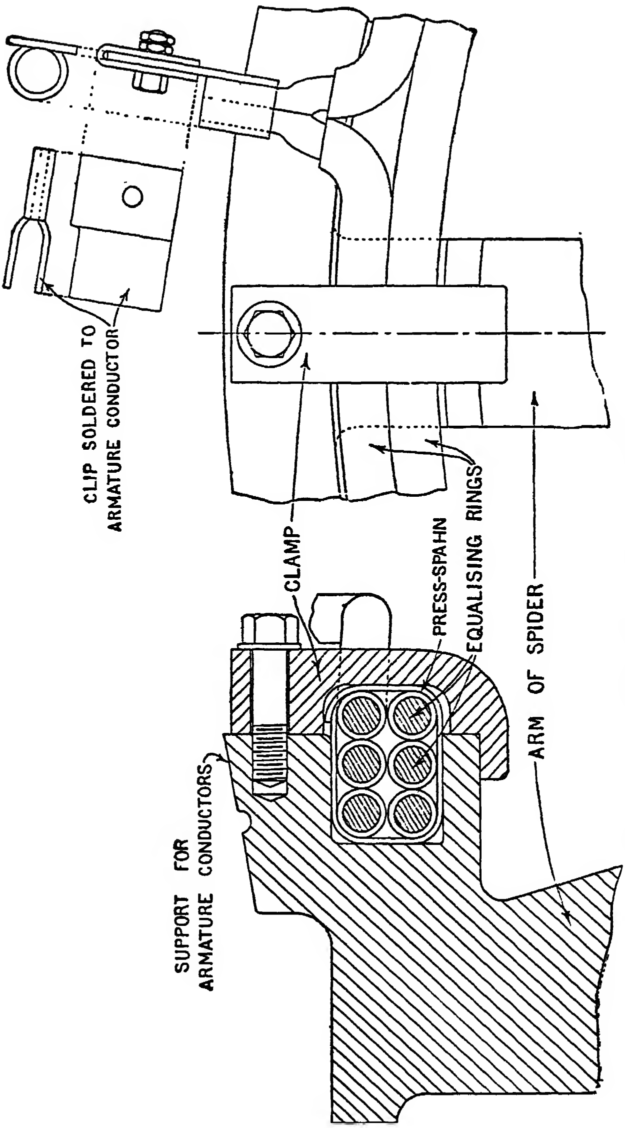


FIG. 90.—Detachable equalising rings (Crompton & Co., Ltd.).

of the armature spider, which serves as a support for the projecting ends of the conductors. For the purpose of balancing the armature (§ 92), it is convenient to make these rings easily

removable. A form of construction used by Messrs. Crompton & Co., Ltd., is shown in Fig. 90. The equalising rings consist of insulated cables held in a recess in the extension of the spider (or the "winding pulley") by means of a number of equally spaced clamps. At the points where connection is to be established with the armature winding, the cables are brought out and soldered to a socket provided with an extension having the form of a flat plate, as shown in the figure. This plate is bolted to a saddle forming part of a clip which is soldered to the armature conductor. It is evident that by undoing the bolted joints and bending back the cables the equalising rings may be easily disconnected.

§ 107. Slot Insulation and Binding Wires.

The slots are lined with a suitable insulating material before the conductors are placed in them. This lining in some cases consists of press-spahn; in others of micanite about ·03 inch thick. Sometimes a double lining, consisting of an outer layer of ·02 inch micanite and an inner layer of ·01 inch press-spahn, is used. The conductors themselves are insulated by a cotton braiding, whose thickness may be taken as ·018 inch. In the case of small armatures round wire, with a double cotton covering which increases the diameter of the wire by about ·012 inch, is used.

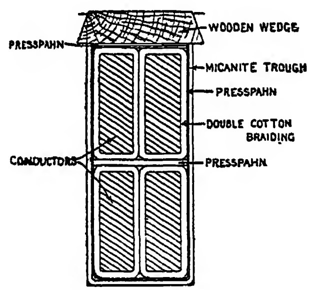


Fig. 89.—Arrangement of conductors in slot.

The end portions of the conductors, and in the case of open slots the portions lying in the slots as well, are held in place by external bands of wire. The binding-wire consists of phosphor-bronze or steel, and has a diameter of from ·02 to ·064 inch. The wire is wound over a band of micanite cloth or similar material, which protects the conductors against any possibility of a short-circuit through

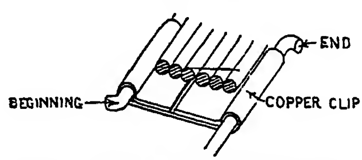


Fig. 91.—Method of securing ends of binding wire.

the band of wire. The width of any one band should not exceed three-quarters of an inch, in order to prevent excessive eddy-current loss. For the purpose of to some extent binding together the individual wires forming a band, sheet-copper strips are placed around it at intervals, and soldered to the wires. The beginning and end of each band are secured by means of a copper clip, as shown in Fig. 91.

§ 108. Relation Connecting Dimensions and Speed with Output of Armature.

We shall now consider the relation of the dimensions and speed of an armature to its output. It is evident that for a given diameter, length of core and air-gap induction, the e.m.f. and hence the output, will vary in simple proportion to the speed. For a given diameter, air-gap induction and speed, the output will vary in simple proportion to the length of core: since by increasing the length of core in any ratio we increase the length of each conductor, and hence the e.m.f. of the machine, in the same ratio. If we next suppose the diameter of the core to be increased, the length, air-gap induction and speed remaining constant, then, supposing the winding to be carried out with conductors of the same size as before, we see that the total number of conductors, and the number of lines cut by each during one revolution, will both be increased in the ratio in which the diameter has been increased. Hence the e.m.f., which is proportional to the product of the total number of conductors into the lines cut by each during one revolution, will be increased in proportion to the *square* of the diameter. Thus—assuming that the maximum permissible current through each armature conductor remains the same—the output would be proportional to the product of length of core, square of diameter, and revolutions per minute.

It must be remembered, however, that with increase of peripheral velocity the ventilation of the armature is considerably improved, and hence a higher current-density may (for a given rise of temperature) be allowed in the armature conductors.

For this reason, a given increase in the diameter will bring about an increase of output which is *greater* than in proportion to the square of the diameter. Similarly, an increase of speed will increase the output in a somewhat greater ratio than that in which the speed is increased.* The relation connecting the speed and dimensions of an armature with its output is thus a somewhat complicated one. A *rough* first approximation may be obtained by means of the formula—

$$\text{kilowatts} = K\, d^2\, l\, m,\dagger$$

where d = diameter of armature, in inches; l = length of core, in inches; m = revolutions per *minute;* and K is a coefficient, frequently termed the *output coefficient,* whose value depends on the output, in the manner represented by the following table:‡

Output, in kw. =	25	50	100	250	500	1,000	2,000
$K.10^6$ =	16·5	21	26	33	39·5	46	52·5

In the case of small machines, the output coefficient also depends on the *voltage* for which the machine is wound. This is due to the fact that for a higher voltage a larger number of smaller wires will have to be used, and as a consequence the amount of space taken up by the insulation will increase, and the net copper winding space will decrease.

§ 109. Usual Values of Armature Speed.

The peripheral speed of armatures varies from about 3,500 feet per minute in small armatures to about 5,000 feet per minute in large ones.

The peripheral speed of commutators generally ranges from 1,500 to 4,000 feet per minute.

As regards the angular velocity of the armature, this is very largely determined by that of the prime mover. If we exclude the class of dynamos intended for direct coupling to steam turbines, the speeds most commonly employed at the present time may be taken to be as follows:—

* Provided the output is limited by the permissible temperature rise and not by sparking.

† A formula of the type "kilowatts $= K\, d^2\, l\, m$" was first given by Esson, *Journal of the Institution of Electrical Engineers*, vol. xx., p. 272 (1891), and later by Kapp; see the latter's remarks in the *Journal* quoted, vol. xxxi., p. 230 (1901).

‡ *Elektrotechnische Zeitschrift*, vol. xxvi., p. 803 (1905).

Output in kilowatts	5	10	20	50	100	200	500	1000	2000	3000
Revs. per minute	1100	1000	800	550	400	300	200	100	80	75

§ 110. Construction of Commutators.

Commutators are nowadays invariably built up of hard-drawn copper segments, separated by mica insulation (·03 inch to ·04 inch thick). The mica used for this purpose is chosen so as to wear away at about the same rate as the copper of the segments; with very hard mica, ridges are apt to be formed between the segments, causing vibration of the brushes and consequent sparking.

The commutators of small machines are supported by a special sleeve, which is keyed directly to the shaft. In large machines, a supporting spider of cast-iron is used. This is either bolted to the arms of the armature spider or mounted on an extension of the boss of the armature spider, so as to be entirely clear of the shaft. A typical form of construction is shown in Fig. 92. The segments are provided with dove-tail extensions on their inner surfaces, which rest on an insulating cylinder (about one-tenth of an inch thick) of mica. The segments are clamped at their ends by means of clamping rings, V-shaped moulded mica insulating rings (one-tenth of an inch thick) being interposed between the segments and the clamping rings. The pro-

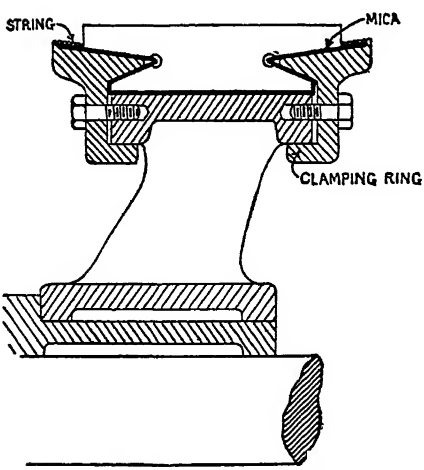


FIG. 92.—Type of commutator construction

jecting ends of the mica V-rings are mechanically protected by bindings of shellaced string, which prevent the flaking away of the mica. In very large commutators, the clamping rings are divided into a convenient number of segments. For the sake of greater security against centrifugal stress, they are provided with projections along their inner edges, which overlap the inner surface of the spider-rim.

The radial depth of the commutator segments at their ends (i.e., their minimum radial depth) is generally between 1 inch and $1\frac{1}{4}$ inches, while the radial depth of the dovetail is about $\frac{3}{4}$ inch to 1 inch, giving for the maximum radial depth of the segments about $2\frac{1}{4}$ inches. The minimum axial length of a segment—i.e., the distance between the points of the two V's, or the minimum width of the dovetail—is about one-third of the total length. The thickness of a commutator segment should not be less than ·1 inch at the top and ·04 inch at the bottom.

When the commutator is running, each bar or segment is, owing to centrifugal action, subject to a bending stress, and represents a uniformly loaded beam fixed at its ends. Hence, with the two V-rings method of construction, there is a limit to the axial length of commutator beyond which it would not be safe to go. For commutator surface speeds of about 4,000 ft./min., this limit lies in the neighbourhood of 2 feet.

In order to minimise the risk of flashing over (§ 89B), the distance between consecutive brush sets should not be below a certain limit depending on the voltage of the machine, about 1 inch of commutator circumference being allowed for every 80 volts.

§ 111. Construction of Yoke and Poles.

The yoke ring may be either of cast iron or cast steel—depending on which of the two materials is the cheaper to use.

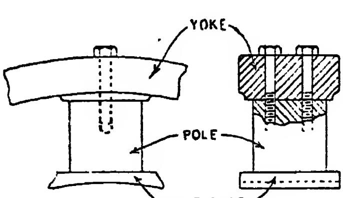


FIG. 93.—Field pole construction.

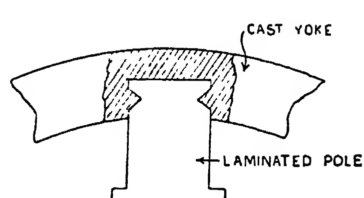


FIG. 94.—Laminated pole cast into yoke.

The field poles are of cast steel, or are built up of steel laminations, and the pole-shoes may be either cast in one with the poles or built up separately of sheet steel laminations, and fixed to the poles by means of screws. In large machines, the field yoke is divided into halves by a horizontal plane, and in the largest sizes it may become necessary to sub-divide it still further. There are various methods of attaching the poles to the yoke. One of the commonest is to use two bolts, as shown in Fig. 93. Another method, which gives a very good joint, is to cast the poles into the yoke (Figs. 94 and 95). In Fig. 94 the pole is a laminated one, while in Fig. 95 it is of cast steel, of circular cross-section; the grooves in the upper part of the pole, which enters

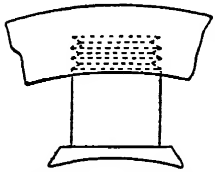


FIG. 95.—Solid steel pole cast into yoke.

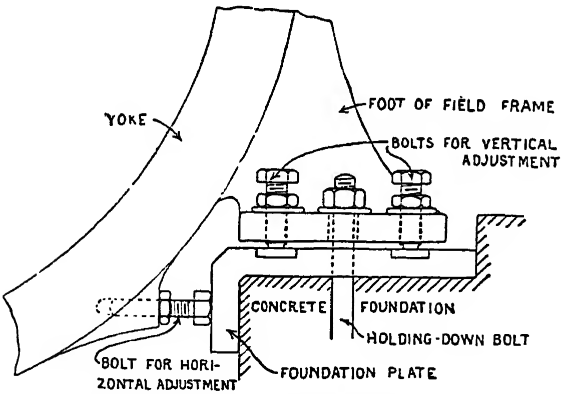


FIG. 96.—Method of supporting field ring.

the yoke, give a large area of contact with the cast metal, and the joint is a very perfect one.

In large machines provision should be made for exact centreing of the field relatively to the armature. One method of doing this is illustrated in Fig. 96. It will be seen that the field is capable of displacement in either a vertical or a horizontal direction.

§ 112. Brushes and Brush Gear.

The carbon brushes almost universally employed in modern dynamos are from $\frac{3}{8}$ inch to 1 inch thick, $\frac{5}{8}$ inch to $1\frac{1}{2}$ inches wide, and 1 inch to $1\frac{1}{2}$ inches long. The contact surface of ordinary carbon brushes should not be less than $\cdot 9i/\sqrt{v}$, where i = current in amperes, and v = peripheral velocity of commutator, in feet per minute. Numerous types of brush-holders have been devised. A simple form is shown in Fig. 97. Instead of using a single large brush, it is cus-

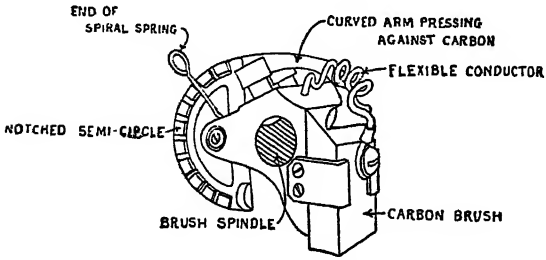


FIG. 97.—Type of brush-holder.

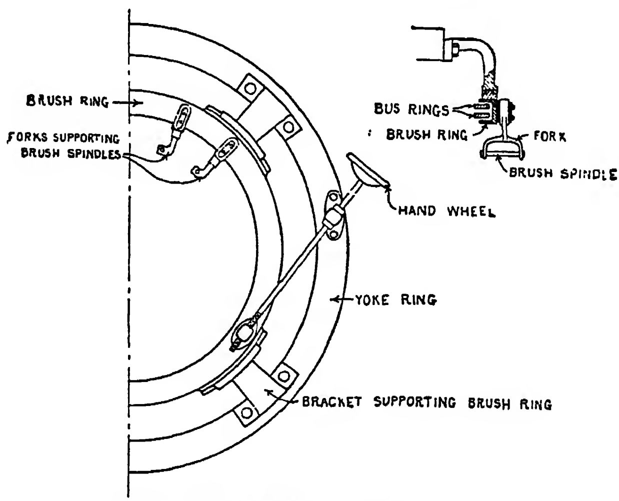


FIG. 98.—Method of supporting brush ring.

* *Electrical World*, vol. 55, p. 1139 (1910).

tomary to use a number of smaller ones, as a small brush is much more likely to give good contact with the commutator over its entire surface than a large one. The requisite number of brush-holders is mounted side by side on a rod or spindle known as the brush spindle. The brush spindles are supported from a metal ring known as the brush ring, by means of suitable forks bolted to the ring, as shown in Fig. 98. The brush forks are heavily insulated from the ring, and brush-holders of the same polarity are connected, by means of flexible cables, to a 'bus ring of copper. These 'bus rings, which are in connection with the generator terminals, are shielded by the brush ring as shown, and are mechanically attached to it with the interposition of suitable insulating supports. In designs where the 'bus rings are exposed they are heavily taped with insulating tape. In order to allow of exact adjustment of the brushes, the brush ring is made movable by providing it with projections which fit into corresponding guides in the arms by means of which it is supported from the yoke ring (Fig. 98). The displacement of the brushes is generally accomplished by the aid

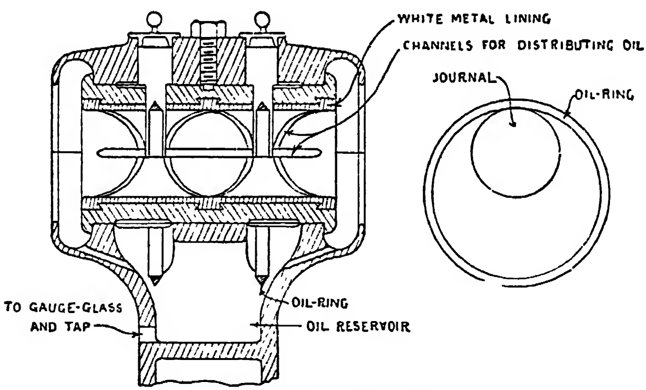


FIG. 99.—Self-oiling bearing.

of a long screw fitted with a hand-wheel, as shown in the illustration; both the bearing at the hand-wheel end, which is attached to the yoke ring, and the nut fitted to the brush ring, are capable of rotation about axes parallel to the shaft of the machine, thereby giving the screw perfect freedom of motion.

§ 113. Bearings.

In small machines, the bearings, which are always of the self-oiling type, and are provided with either a cylindrical (Fig. 99) or a spherical (Fig. 100) seating, are either supported by pedestals arising from the bed-plate of the machine, or—in the case of machines of the enclosed type—are contained in the end-shields, which are bolted to the yoke ring on each side. In large machines, special foundation plates are provided for the support of the yoke ring and the bearings.

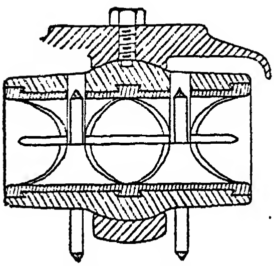


FIG. 100.—Self-oiling bearing with spherical seating.

§ 114. Weight and Cost of Dynamos.

Since by doubling the speed of a given machine we may approximately double its output, it is clear that the size, weight and price of the machine are not determined by its output only, but by the *work done per revolution*. We may conveniently express this latter in watt-minutes per revolution, or (since $\frac{\text{watt-minutes}}{\text{revolutions}} = \frac{\text{watts}}{\text{revolutions per minute}}$) in watts per revolution per minute. The following table gives some idea of the prices and weights of dynamos of modern construction :—

$\frac{\text{Watts}}{\text{revs. per min.}}$ =	1	2	3	4	5	7	10	15	20	30	40	50
Price in £ .	22	28	34	40	45	54	63	76	88	110	130	148
Weight in cwt.	3·2	4·4	5·6	6·8	8·0	10·4	13·4	17·6	21·4	28	34	40

$\frac{\text{Watts}}{\text{revs. per min.}}$ =	70	100	200	300	400	500	700	1000	1500	2000
Price, in £ .	180	230	360	470	560	640	770	950	1150	1300
Weight, in cwt.	51	67	110	145	170	193	235	295	380	455

The prices and weights in the above table refer to machines complete with bed-plate, two bearings, pulley and slide-rails. In the case of machines for direct coupling to prime movers, supplied with only a single bearing and a coupling, but no bed, the prices would be from 15 per cent. to 10 per cent. less,

and the weights from 50 per cent. to 20 per cent. less than those given above, the difference in each case decreasing as the size of the machine increases. The prices given represent the selling prices; the cost of manufacture being from 60 per cent. to 80 per cent. of the selling price. As the prices vary according to the state of the metal market, and as, of course, different prices are charged by different firms, the above table must not be regarded as giving very precise information, and variations of some 15 per cent. on either side of the figures given above may be expected.

As an example of the use of the above table, we shall roughly determine the price and weight of a 100-kilowatt generator, running at 400 revolutions per minute (mounted on bed-plate, complete with pulley). Here $\dfrac{\text{watts}}{\text{revolutions per minute}} = 250$, and by reference to the table we find that the approximate price would be £420, and the weight 128 cwt., or 6·4 tons. If supplied for direct driving, with only one bearing and no bed-plate, the price would be about £370 (about 12 per cent. less than £420), and the weight about 100 cwt., or 5 tons.

The weight of the armature is from 20 to 30 per cent. of the total weight of the machine, while its cost is about half the total cost of the machine.

If we consider the weight of effective material in a dynamo—i.e., material which forms part of either the electric or the magnetic circuit of the machine—then the copper is found to form about 30 per cent., and the iron about 70 per cent., of the total.

In the case of very small ("fractional horse-power") motors, the approximate prices and weights may be ascertained from the following table:

$\dfrac{\text{Horse-power.}}{\text{revs. per min.}}$	10^{-5}	5×10^{-5}	10×10^{-5}	15×10^{-5}	20×10^{-5}	30×10^{-5}	40×10^{-5}	50×10^{-5}
Price, £ .	3	4·25	5·5	7·3	8	10·4	12·8	15
Weight, lb. .	7·5	18	27	36	44	64	84	104

Examples.

1. Find approximately the diameter of the thickest part of the shaft of a 600-kilowatt generator designed for a speed of 55 revolutions per minute.

2. The armature core of a 14-pole generator has a diameter of 100 inches. Find approximately the radial depth of the core stampings.

3. A 75-kilowatt dynamo designed for a speed of 395 revolutions per minute has an armature whose diameter is 29 inches, and whose core length is 12½ inches. Find its output coefficient.

4. Find the approximate cost and weight of a 110-kilowatt dynamo intended to run at a speed of 735 revolutions per minute.

CHAPTER XI.

§ **115**. Usual values of induction in dynamos. Ampere-conductors per unit length of armature circumference — § **116**. Calculation of field ampere-turns— § **117**. Potential drop over teeth—§ **118**. Calculation of air-gap reluctance— § **119**. Potential drop in field core and yoke—§ **120**. Field ampere-turns required to balance demagnetising effect of armature—§ **121**. Field ampere-turns required to compensate field distortion—§ **122**. Numerical example. Examples.

§ 115. Usual Values of Induction in Dynamos. Ampere-conductors per Unit Length of Armature Circumference.

THE following are the values of the magnetic induction commonly used in the various parts of the magnetic circuit of a modern generator:—

Armature core	10,000 to 12,000
Armature teeth	18,000 to 22,000
Air gap	7,000 to 10,000
Field cores (cast steel)	12,000 to 15,000
Yoke { cast steel	10,000 to 12,000
Yoke { cast iron	4,000 to 6,000

The product of the number of armature conductors into the current carried by each gives us the total *ampere-conductors* or *ampere-wires* of the armature. The ampere-conductors per inch of armature circumference vary from about 300 in small machines to about 900 in large ones (this corresponds to 120 to 350 ampere-wires per cm. length of armature circumference). The following table may be taken as representing approximately the relation connecting the output of a machine with the ampere-conductors per inch of armature circumference:—

Output in kw.	5	10	15	20	30	40	60	80	100	200	500	1000
Ampere-conductors/inch ...	300	350	380	410	450	490	560	620	660	750	850	900

The length of (single) air-gap varies from about one-tenth of an inch (or, say, ·25 cm.) in small machines to about three-eighths of an inch (or, say 1 cm.) in large ones. The current-density in the

armature conductors ranges from about 1,500 to 2,500 amperes per square inch (230 to 390 amperes per square cm.) of cross-section, while in the field coils it varies from 750 to 1,000 amperes per square inch (125 to 155 amperes per square cm.).

§ 116. Calculation of Field Ampere-turns.

The ampere-turns required to produce a given flux per pole are calculated in a manner similar to that explained in § 33. There are, however, certain difficulties connected with some parts of the problem which we now proceed to consider in detail.

A reference to Fig. 101 shows that each magnetic circuit may be divided into two symmetrical halves by a radial plane (shown chain-dotted in Fig. 101) drawn half-way between two neighbouring poles. Since the total m.m.f. maintaining the flux in any circuit is contributed by two neighbouring coils, the simplest plan is to consider one half of a circuit, such as that marked O A B C D E in Fig. 101, and to determine the total fall of magnetic potential along it. This fall of magnetic potential is equal to the m.m.f. to be provided by a *single* field coil.

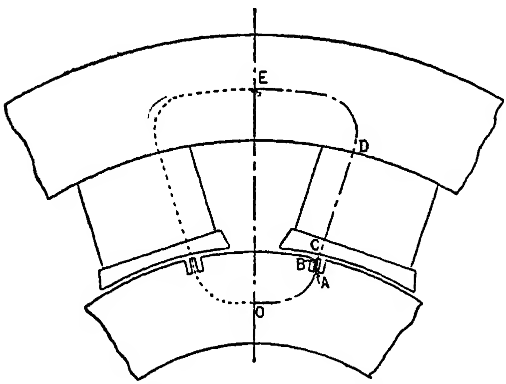


FIG 101.—Magnetic circuit of dynamo.

The line O A B C D E is intended to represent the mean path of the flux along one half of a magnetic circuit. Although no doubt exists regarding the exact lengths of A B, B C and C D, the lengths O A and D E are somewhat uncertain; but owing to the fact that the fall of magnetic potential along O A and D E is generally a small fraction of the total fall along O A B C D E, an error in estimating the lengths of O A and D E will not introduce any serious percentage error into the final result.

The line O A B C D E passes through a number of media

having different permeabilities, and hence, in accordance with the principles explained in § 33, we divide it into as many parts as there are regions of different permeability. We determine the drop of magnetic potential over each part, and the sum of all such drops gives us the total fall of potential from O to E—i.e., the m.m.f. to be provided by each field coil.

Let Φ be the total flux per pole. Then $\frac{1}{2}\Phi$ is the flux per magnetic circuit. Dividing this by the radial cross-section of iron below the teeth in the armature core, we obtain the value of B in the body of the core. A reference to the B-H curve (§ 24) for armature stampings gives us the corresponding value of H. If we denote the length O A, measured in cms., by l_a, the fall of magnetic potential along O A is given by $l_a H_a$, where H_a is the value of H in the armature core.

§ 117. Potential Drop over Teeth.

The calculation of the magnetic potential drop over the armature teeth (i.e., over A B in Fig. 101) is attended with difficulty on account of the fact that the teeth taper, and hence the value of the induction varies from point to point of A B. We commence by determining the value of the induction at the top of a tooth, i.e., at the point B in Fig. 101, on the assumption that the entire flux leaving the polar surface enters the *tops* of the teeth, none of it going into the slots. This assumption is not correct, as in reality some of the flux passes down the slot and then enters the flanks of the teeth, as shown in Fig. 102, which gives

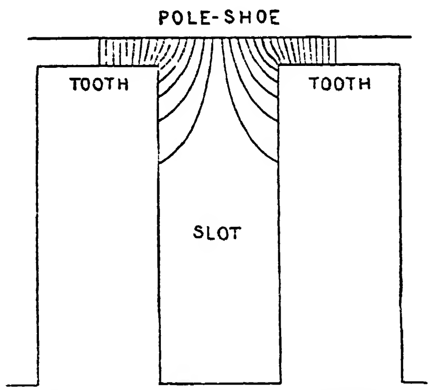


FIG. 102.—Flux distribution in air-gap.

The above is a *very rough* approximation—fortunately sufficient for practical purposes, owing to the smallness of the magnetic potential drop along O A. It is easy to see that the cross-section over which the flux is spread between O and A is by no means constant, but varies from point to point; hence B will also vary along O A. instead of—as assumed—remaining constant.

the approximate distribution of the flux in the air-gap and slots. Neglecting this effect, however, we obtain the flux per tooth by dividing the total polar flux Φ by the number of teeth which receive the flux—a number which is somewhat greater than that included within the polar arc, owing to the lateral spreading of the flux. The spreading or "fringing" will be the greater the longer the air-gap. As an average, we may take about 10 per cent. as the amount to be added to the actual number of teeth under cover of the pole-shoe in order to take the "fringing" into account. The flux per tooth having been thus obtained, we determine the induction at the top of a tooth by dividing the flux per tooth by the area of *iron* at the top of the tooth. Part of the total area of the top of a tooth is occupied by insulation between the core stampings, and the space so occupied generally forms from 10 to 15 per cent. of the total area. If $l =$ gross length of *laminations* (ventilating ducts not included), and $t_1 =$ width of tooth at the top, both expressed in cms., we may take $\cdot 88\, lt_1$ to represent the nett area of iron at the top of a tooth. The induction B_{t_1} is then given by $B_{t_1} = \dfrac{\text{Flux per tooth}}{\cdot 88\, lt_1}$

Having found the induction at the top of a tooth, we proceed to determine it at the bottom of the tooth and at a number of intermediate sections, on the assumption that the flux nowhere leaves the tooth (passing into the slots). The induction will then vary inversely as the width of tooth at the given cross-section. If $t_2 =$ width of tooth at bottom, the induction at the bottom of the tooth will be $B_{t_1} \times \dfrac{t_1}{t_2}$. Let us choose three intermediate cross-sections equally spaced; the corresponding widths of tooth will then be $t_1 - \frac{1}{4}(t_1 - t_2) = \frac{3}{4}t_1 + \frac{1}{4}t_2$; $t_1 - \frac{1}{2}(t_1 - t_2) = \frac{1}{2}(t_1 + t_2)$; and $t_1 - \frac{3}{4}(t_1 - t_2) = \frac{1}{4}t_1 + \frac{3}{4}t_2$. The corresponding values of the induction will be

$$B_{t_1} \times \frac{t_1}{\frac{3}{4}t_1 + \frac{1}{4}t_2} = B_{t_1} \times \frac{4}{3 + \frac{t_2}{t_1}};\quad B_{t_1} \times \frac{2}{1 + \frac{t_2}{t_1}};\ \text{and } B_{t_1} \times \frac{4}{1 + 3\frac{t_2}{t_1}}$$

The inductions at the various cross-sections having been found, we next refer to a B-H curve, and find the corresponding

values of H. In the Table on p. 23, § 24, values of B and H are given for armature stampings up to B = 18,300. Since, however, the induction in armature teeth is frequently carried beyond this limit, we require an extension of the Table. This is given below:—

H =	170	275	400	480	580	690	840	1,020	1,240
B =	18,000	19,000	20,000	20,500	21,000	21,500	22,000	22,500	23,000

Having determined the values of H at the different cross-sections of the tooth, we next plot a curve—such as that shown in Fig. 103—having distances down the tooth for ordinates and values of H for abscissæ. It is evident that the mean ordinate of this curve represents the mean value of H along the tooth, and its area the product of the mean H into the length of tooth —i.e., the area gives us the fall of magnetic potential, in C. G. S. units, down the tooth.

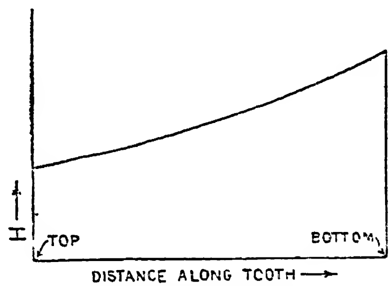


FIG. 103.—Variation of H along tooth.

§ 118. Calculation of Air-gap Reluctance.

In calculating the fall of magnetic potential along the air-gap, we have to make an allowance (1) for the equivalent increase of area due to the "fringing" effect, or lateral spreading of the flux, and (2) for the equivalent increase in the length of gap due to the fact that the flux is not uniformly distributed, but is crowded into the regions lying immediately above the teeth, as shown in Fig. 102. Both these problems have been investigated mathematically by Mr. F. W. Carter,* and his results have been confirmed by independent experimental investigations.

In order to allow for the fringing effect, we may, in estimating the effective cross-sectional area of the gap, add to the length of

* *Electrical World and Engineer*, vol. xxxviii., p. 884 (1901); also *Journal of the Institution of Electrical Engineers*, vol. xxxiv., p. 48 (1904).

the polar arc an amount cg, where g is the gap length and c a coefficient whose value depends on the ratio $r = \frac{\text{interpolar arc.}}{\text{gap length}}$ The following Table gives the values of c corresponding to various values of the ratio r:—

$r =$	10	12	14	16	18	20	22	24	26	28	30	35	40	45	50	60
$c =$	2·43	2·65	2·84	3·00	3·15	3·28	3·40	3·51	3·61	3·70	3·78	3·98	4·14	4·28	4·40	4·66

On adding the amount cg to the length of the polar arc, and multiplying the sum by the length of the pole-shoe measured in a direction parallel to the shaft, we obtain the equivalent cross-sectional area of the gap. The quotient of Φ by this area gives us the mean value of B or H in the gap.

The virtual lengthening of the gap due to the crowding of the lines into the spaces immediately above the teeth may be taken into account by multiplying the actual gap length by a certain "correction coefficient" k_1. The value of this coefficient may be expressed with a sufficient degree of accuracy for all practical purposes by the following very simple formula due to T. C. Baillie*:—

$$k_1 = \frac{t_1 + s}{t_1 + \dfrac{5gs}{5g + s}}$$

where g = length of gap; s = width of slot; t_1 = width of tooth at top.

A further increase in the equivalent gap length must be made if the core is—as is generally the case—provided with ventilating ducts. If in the formula given above for k_1 we substitute for s the width of the ventilating gap, and for t_1 the length of each continuous section of laminations, we obtain a second correction coefficient k_2, which enables us to make an allowance for the increase in the gap reluctance due to the presence of the ventilating ducts. The final corrected gap length is given by k_1k_2g.

* *The Electrician*, vol. 62, p. 494 (1909). This formula is a simple modification of that originally given by F. W. Carter (*loc. cit.*).

Multiplying the mean value of H in the gap, found, as already explained, by $k_1 k_2 g$, we obtain the fall of magnetic potential, in C. G. S. units, across the gap.

§ 119. Potential Drop in Field Core and Yoke.

In considering the drop of magnetic potential along the field core or pole and the yoke, we have to take into account the fact that the flux in these parts is greater than Φ, owing to magnetic leakage. The total flux in the field core is $\nu\Phi$, ν being the leakage coefficient (§ 34). The value of ν varies within certain limits, and depends on the size of the cores. For multipolar machines of ordinary construction, we may take $\nu = 1{\cdot}2$ as a safe average value.* Dividing νN by the cross-sectional area of the field core, we obtain the value of B in the core, and by reference to the B-H curve the corresponding value of H. The product of this into the length of core (C D in Fig. 101) gives the drop of potential over the core.†

Similarly, on dividing $\frac{1}{2}\nu\Phi$ by the yoke cross-section, we obtain B in the yoke, and the corresponding value of H when multiplied by the length of path in the yoke (D E in Fig. 101) gives us the potential drop in the yoke part of one-half of the magnetic circuit considered.

The sum of the various magnetic potential drops so found, when multiplied by ·795, or, approximately, by ·8 (§ 18), gives us the *ampere-turns per pole*. Instead of using the C. G. S. unit of magnetic force, finding the magnetic potential drop in C. G. S. units, and then reducing to ampere-turns, we may conveniently adopt the ampere-turn per cm. (or the ampere-turn per inch) as our unit of magnetic force (§ 18), plot curves connecting B with this unit for the various materials used, and so find the magnetic potential drops directly in ampere-turns. The sum of these gives us the ampere-turns per pole. The size of wire to be used in winding the coil is then found as explained in § 35, and the

* In small machines, ν may reach a value of 1·3.

† Strictly speaking, the pole-shoe should be considered separately, as its area is in general different from that of the field core; but the potential drop over the pole-shoe is so small that this would be a needless refinement.

‡ Since only *half* the flux per pole passes through the yoke section.

amount of wire, which depends on the permissible temperature rise, is determined in accordance with the principles explained in § 128.

§ 120. Field Ampere-turns Required to Balance Demagnetising Effect of Armature.

In determining the ampere-turns per coil, we have hitherto supposed that the field ampere-turns are the only m.m.f. acting around the path of the main flux. So long as the brushes are not displaced from their neutral position, this assumption holds good. But the forward displacement given to the brushes of ordinary machines for the purpose of securing sparkless commutation (§ 88) calls into play a demagnetising belt of armature conductors which directly opposes the field ampere-turns (§ 80). If $\frac{Z\, i_s}{2\, P}$ represents the number of ampere-conductors per pole-pitch (i_s being the current in each conductor, Z the total number of conductors, and P the number of pairs of poles), the demagnetising ampere-turns per pole called into play by giving the brushes a forward displacement, represented by a fraction f of the pole-pitch, are given by $\frac{f\, Z\, i_s}{2\, P}$. Although the exact amount of brush displacement is not easy to estimate, it is advisable, in order to use a safe value, to suppose that the displacement amounts to half the interpolar arc, so that the conductors forming the short-circuited coil are just under the edge of the pole-shoe.

§ 121. Field Ampere-turns Required to Compensate Field Distortion.

Besides the direct demagnetising effect which is due to the forward brush lead, a further effect is exerted by the armature current, which consists in an increase of the reluctance of the main magnetic circuit due to the crowding of the flux to one side of the pole-shoe. This increase of reluctance is mainly brought about by the large decrease in the permeability of the teeth lying under cover of that side of the pole-shoe towards which the flux is crowded, the teeth being very strongly magnetised even before

distortion takes place. The increased reluctance of the path followed by the main flux necessitates an increase in the field ampere-turns if the required value of the flux is to be maintained. The amount to be added to the field ampere-turns will depend on the number of ampere-conductors under cover of a pole-shoe, and on the field ampere-turns required to maintain the given flux on open circuit. The relation connecting these quantities has been investigated experimentally by Hobart,* and the results of his researches may be approximately expressed by the very simple formula

$$a_d = \frac{a_o Z_p i_s}{26{,}000 + 3a_o},$$

where a_d is the addition to be made to the field ampere-turns per pole to compensate for the increased reluctance due to distortion; a_o = field ampere-turns per pole required to maintain the given flux when the machine is running on open circuit; Z_p = number of conductors under cover of a pole-shoe; and i_s = current in each armature conductor.

§ 122. Numerical Example.

We shall apply the method explained above of calculating the field ampere-turns to the case of a 400 kilowatt, 480 volt, shunt-wound generator designed for a speed of 110 revolutions per minute.† The field is a 10-pole one, and the armature is provided with a simple lap winding. The drop of voltage in the armature and brush contacts at full load is estimated to amount to about 10 volts, so that the full-load e.m.f. is 490 volts. The corresponding flux per pole entering the armature is given by (§ 77)

$$\Phi = \frac{490 \times 10^8 \times 60}{110 Z.}$$

There being 240 slots in the armature core, each containing six conductors, Z = 240 × 6 = 1440. Hence

$$\Phi = \frac{490 \times 10^8 \times 60}{110 \times 1{,}440} = 18{\cdot}57 \times 10^6.$$

Referring to Fig. 101, the lengths of the various parts of the magnetic half-circuit are as follows:—

* *Elementary Principles of Continuous Current Dynamo Design*, p. 72.

† Full details of this generator, which was exhibited at Liège in 1905 by the *Société Anonyme des Ateliers de Constructions Électriques de Charleroi*, are given in *L'Eclairage Électrique*, vol. xlv., p. 11 (1905).

O A = 26 cms.; A B = 4·4 cms.; B C = ·85 cm.; C D = 40 cms.; D E = 50 cms.

We now proceed to find the magnetic potential drops over the different paths.

1. *Armature core.* The gross length of the core is 43 cms. There are four ventilating ducts, each 1·2 cm. wide. The length of laminations is thus $43 - 4 \times 1{\cdot}2 = 38{\cdot}2$ cms. Allowing 12 per cent. for insulation between the stampings, we find that the nett length of iron in the core is $38{\cdot}2 \times {\cdot}88 = 33{\cdot}6$ cms. The radial depth below the teeth is 25·6 cms. Hence cross-section of iron in core $= 33{\cdot}6 \times 25{\cdot}6 = 860$ square cms. This gives for the induction $\dfrac{18{\cdot}57 \times 10^6}{2 \times 860} = 10{,}800$, say. The corresponding H (§ 24) is 10·7. Hence fall of magnetic potential along the path O A in Fig. 101 is $10{\cdot}7 \times 26 = 278$.

2. *Armature teeth.* The number of teeth per pole-pitch is 24. The pole-pitch is 62·83 cms., and the polar arc 47 cms. long. Hence the number of teeth directly under cover of the pole-shoe is $\dfrac{24 \times 47}{62{\cdot}83} = 17{\cdot}97$. Allowing 10 per cent. for fringing, we assume the number of teeth actually carrying the flux to be 19·8. Hence the flux per tooth is $\dfrac{18{\cdot}57 \times 10^6}{19{\cdot}8} = {\cdot}938 \times 10^6$. The width t_1 of a tooth at the top is 1·32, and its width at the bottom t_2 is 1·20. The area of iron along the top is $t_1 \times$ nett length of iron in core $= 1{\cdot}32 \times 33{\cdot}6 = 44{\cdot}35$ square cms. We thus get B_{t_1} (induction at top) equal to $\dfrac{{\cdot}938 \times 10^6}{44{\cdot}35} = 21{,}140$. Proceeding as explained in § 117, we find the following values of the induction at equidistant points, starting from the top:—

21,140	21,640	22,150	22,700	23,260

The corresponding values of H are (§ 117)—

610	720	900	1,110	1,380

The depth of slot or tooth is 4·4 cms. On plotting a curve similar to that shown in Fig. 103, we find for the magnetic potential drop along the teeth the value 4,114 (the mean H is 935).

3. *Air-gap.* The interpolar arc being $62{\cdot}83 - 47 = 15{\cdot}83$ cms.,

we have r (§ 118) $= \frac{15\cdot83}{\cdot85} = 18\cdot6$, the length of air-gap g being $\cdot85$ cm. Referring to the Table given in § 118, we find $c = 3\cdot19$, so that $cg = 2\cdot7$. The corrected length of pole-arc is thus $47 + 2\cdot7 = 49\cdot7$ cms., and the effective area of gap is $49\cdot7 \times 43 = 2,137$ square cms. The mean value of H in the gap is thus $\frac{18\cdot57 \times 10^6}{2,137} = 8,690$. We next calculate the correction coefficients k_1 and k_2. The width of slot s is $1\cdot3$ cm., so that $t_1 + s = 1\cdot32 + 1\cdot3 = 2\cdot62$, and $t_1 + \frac{5gs}{5g + s} = 2\cdot32$. Hence $k_1 = \frac{2\cdot62}{2\cdot32} = 1\cdot13$. The length of a continuous section of the laminations is $\frac{38\cdot2}{5} = 7\cdot64$ cms., and since the width of each ventilating duct is $1\cdot2$ cm., we find $k_2 = \frac{1 + 2\cdot84}{\cdot86 + 2\cdot84} = \frac{3\cdot84}{3\cdot7} = 1\cdot04$.

The corrected air-gap length is $k_1k_2g = 1\cdot16 \times 1\cdot04 \times \cdot85 = \cdot998$, and the drop of potential across the gap is $8,690 \times \cdot998 = 8,670$, say.

4. *Field core and yoke.* Assuming the leakage coefficient to be $1\cdot2$, we find for the flux through the pole or field core $18\cdot57 \times 10^6 \times 1\cdot2 = 22\cdot3 \times 10^6$. The flux through the yoke, which is half of this, is $11\cdot15 \times 10^6$. The cross-sectional area of the pole is 1,385 square cms., and that of the yoke 1,750 square cms.

Hence the induction in the field core is $\frac{22\cdot3 \times 10^6}{1,385} = 16,100$. and that in the yoke $\frac{11\cdot15 \times 10^6}{1,750} = 6,370$. The field cores are of forged iron. For this material (for which no data are given in § 24) the value of H corresponding to $B = 16,100$ is about 40. The yoke is of cast-iron, and on referring to the Table given in § 24 we find $H = 28$. Hence the potential drop over the core is $40 \times 40 = 1,600$, and that over the yoke is $28 \times 50 = 1,400$.

The total drop of magnetic potential over one-half of the magnetic circuit is thus $278 + 4,114 + 8,670 + 1,600 + 1,400 = 16,100$, say. The corresponding ampere-turns are $\cdot8 \times 16,100 = 21,850$.

5. *Ampere-turns to balance armature reaction.* The full-load

external current is $\frac{400,000}{480} = 834$ amperes. We shall assume, as an approximation, that the shunt current is about 2 per cent. of the full-load current, so that the total armature current is $834 \times 1{\cdot}02 = 850$. The current per conductor is 85 amperes, since there are 10 poles. The ampere-conductors per pole-pitch are $850 \times 144 = 12{,}240$. We shall assume the brush lead to amount to half the interpolar arc, so that f (§ 120) $= \frac{7{\cdot}9}{62{\cdot}8} = {\cdot}126$. This gives for the demagnetising ampere-turns per pole $12{,}240 \times {\cdot}126 = 1{,}543$.

6. *Ampere-turns to compensate distortion of field.* Using the formula given in § 121, we have, since $a_o = 13{,}200$, $Z_p = \frac{Z}{2\,P} \times \frac{\text{pole-arc}}{\text{pole-pitch}} = 144 \times \frac{47}{62{\cdot}83} = 108$, say, $i = 85$,

$$a_d = \frac{13{,}200 \times 108 \times 85}{26{,}000 + 39{,}600} = 1{,}847.$$

7. *Total ampere-turns at full load.* These are given by

$$12{,}850 + 1{,}543 + 1{,}847 = 16{,}200, \text{ say.}$$

8. *Size of wire to be used in winding shunt coils.* Assuming all the field coils to be connected in series with each other, we have for the p.d. across each coil $\frac{480}{10} = 48$ volts. Hence (§ 35) the resistance of the mean turn of a coil $= \frac{48}{16{,}200} = {\cdot}00296$ ohm, at the working temperature, which, assuming a mean temperature rise of 50° C., and an engine-room temperature of 20° C., would be 70° C. We may assume the depth of the winding to be about $2\frac{1}{2}$ inches. The diameter of the field core is $16\frac{1}{4}$ inches, so that the length of the mean turn is $\pi\,(16{\cdot}25 + 2{\cdot}5) = 59$ inches. Hence the size of wire must be such that at a temperature of 70° C. 59 inches of it have a resistance of ·00296 ohm, or 1,000 yards have a resistance of 1·81 ohms. The temperature coefficient of copper being ·004, the resistance of 1,000 yards of the wire at 15° C. would be $\frac{1{\cdot}81 \times 1{\cdot}06}{1{\cdot}28} = 1{\cdot}50$ ohms. On referring to a wire table in which the resistances, at 15° C., of 1,000 yards of the various sizes

are given, we find that 1,000 yards of No. 9 S. W. G. have a resistance of 1·48 ohms, which is sufficiently near the value required by us. This, as a matter of fact, was the size actually adopted by the makers of the machine.

Examples.

1. The number of slots in the core of a six-pole lap-wound armature is 210, and each slot contains four conductors. The armature diameter is 30 inches, and the full-load armature current 360 amperes. Find the number of ampere-conductors per inch length of armature circumference.

2. A four-pole armature has an external diameter of 745 mm. and a gross core length of 330 mm. There are 151 slots, each $7·3 \times 35$ mm., and two ventilating ducts, each 15 mm. wide. The insulation between the laminations occupies 14 per cent. of the gross space taken up by the laminations. If the flux per pole is $12·1 \times 10^6$, and if this flux is carried by twenty-eight teeth, what is the drop of magnetic potential, in ampere-turns, over a single tooth?

3. The field of a ten-pole generator has a bore of $96\frac{3}{4}$ inches, the armature diameter being 96 inches. The polar arc extends over 75 per cent. of the pole-pitch. There are 300 slots in the armature, each ·525 inch wide. The gross length of core is 20 inches, and there are six ventilating ducts, each $\frac{3}{8}$ inch wide. If the flux per pole is 20×10^6, what are the ampere-turns required to maintain this flux across a single air-gap?

4. A lap-wound six-pole armature contains 900 conductors. The full-load armature current is 540 amperes, and the forward brush lead corresponds to 11 per cent. of the pole-pitch. Find the field ampere-turns per pole required to balance the demagnetising effect of the armature.

5. In a certain eight-pole machine the number of ampere-conductors per inch length of armature circumference is 520 at full load, the armature diameter is 43 inches, and the length of each polar arc is 12·3 inches. The number of field ampere-turns per pole required to produce the full-load e.m.f. on open circuit has been determined to be 7,000. How many additional ampere-turns per pole will be required to balance (*a*) the demagnetising and (*b*) the distorting effect of the armature current?

CHAPTER XII.

§ **123**. Losses occurring in dynamo—§ **124**. Losses due to resistances of windings and brush contacts—§ **125**. Hysteresis and eddy-current losses in armature core—§ **126**. Hysteresis and eddy-current loss in pole-shoes, and eddy-current loss in armature conductors—§ **127**. Additional losses at full load. Mechanical friction losses. Efficiency—§ **128**. Calculation of temperature rise—§ **129** Experimental determination of efficiency. Soames testing brake—§ **130**. Eddy-current brake—§ **131**. Efficiency tests of small generators—§ **132**. Field's efficiency test for tramway motors—§ **133**. Hopkinson's efficiency test—§ **134**. Hopkinson's efficiency test as applied to series-wound machines—§ **135**. Swinburne's efficiency test—§ **136**. Determination of temperature rise. Heating tests. Examples.

§ 123. Losses occurring in Dynamo.

Of the total mechanical power supplied to a dynamo, a certain fraction is converted into electrical power which is available in the external circuit, while the remainder is wasted in the machine itself. Similarly, of the total electrical power supplied to a motor, part is usefully converted into mechanical power available at the motor pulley, and the remainder is lost in the motor. We shall now study the various sources of loss in a dynamo or motor. These losses may be divided into

(1) Losses due to heating of the field and armature coils by the currents flowing through them.

(2) Brush contact losses due to electrical heating of the contacts.

(3) Hysteresis and eddy-current losses in the armature core and pole-pieces.

(4) Eddy-current losses in the armature conductors.

(5) Mechanical friction losses.

§ 124. Losses due to Resistances of Windings and Brush Contacts.

The losses due to heating of the armature and field coils are easily calculated from the resistances of the windings and the currents. If r be the resistance of the winding in question

(armature or field, as the case may be), and i the current flowing through it, the watts lost in producing heat in the winding are given by $r\,i^2$. The resistance to be used in this formula is that of the *hot* winding. The temperature rise of the coils above their surroundings is generally from 35° to 45° C.

The watts lost by *electrical* heating of the brush contacts are also given by $r_c\,i^2_a$, where r_c is the resistance of the contacts, and i_a the armature current. As has already been mentioned (§ 84), r_c is not a constant quantity, but varies nearly inversely as the current, so that $r_c\,i_a$ is nearly constant, and equals from 2 to 2·5 volts. The brush contact loss by electrical heating is thus between 2 and 2·5 times the armature current, and hence

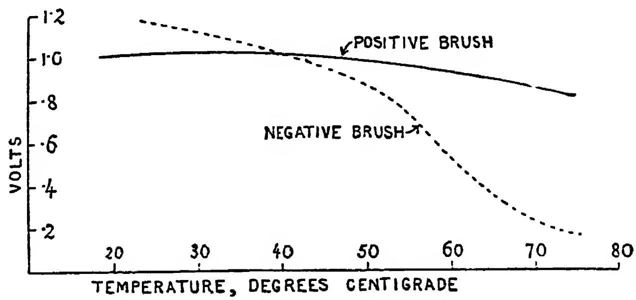


FIG. 104.—Effect of temperature on brush contact drop.

increases approximately in simple proportion to the current, unlike the losses in the coils, which increase in proportion to the square of the current.

Investigations by E. Arnold and E. Pfiffner* have brought to light some very remarkable results as regards the effect of *temperature* on the brush contact resistance. The voltage drop depends, among other things, on the *direction* of the current across the contact. At moderately high temperatures, the drop at the positive brushes may be either greater or less than that at the negative brushes. As the temperature rises, however, the drop at the negative brushes decreases much more rapidly than that at the positive brushes, so that at high temperatures the negative brush drop is in most cases considerably less than the

* *Elektrotechnische Zeitschrift*, vol. xxviii., p. 263 (1907). The effect considered varies a good deal according to the grade of brush, and occasionally a grade of brush is found for which the positive and negative contact drops are equal; see paper by P. D. Manbeck in the *Electrical World*, vol. lxxxix, p. 237 (1927).

positive one. A typical case is shown in Fig. 104, which refers to "le Carbone Z" brushes at a current density of about 65 amperes per square inch. The *negative brush is thus seen to be much more sensitive to temperature changes than the positive one.* These results readily account for some, at first sight, rather puzzling effects. Thus, it has frequently been noticed that sparking generally commences at brushes of one polarity before it appears at those of the other; in view of the fact that the voltage drop is in general different at the positive and negative brushes, and that the higher voltage drop is more favourable to good commutation (§ 87), this behaviour of a machine receives a ready explanation. Again, it has been noticed that sometimes a machine which gives no trouble when cool begins to spark at the *negative* brushes after it has been kept running for a sufficient time to raise the temperature of the commutator beyond a certain limit. This, again, becomes perfectly intelligible in view of the curves of Fig. 104, since the great drop in the brush contact voltage deprives the negative brush of the good commutating properties which it possessed at a lower temperature.

125. Hysteresis and Eddy-current Losses in Armature Core.

The calculation of the hysteresis and eddy-current losses in the armature core presents formidable difficulties, and can only be carried out roughly. The main difficulty lies in the large variation in the value of B from the inner surface of the core outwards. It is usual, in dealing with the armature core loss, to divide this into the loss in the teeth and that in the body of the core.

The induction in the teeth has a very high value, and for such high values of the induction Steinmetz's 1·6th power law no longer holds.* The following Table may be used in calculating the hysteresis loss in the teeth :—

B =	18,000	19,000	20,000	21,000	22,000	23,000	24,000
Watt-seconds (joules) per c.c. per cycle =	·0023	·0025	·0027	·0028	·0029	·003	·003
Joules per lb. per cycle =	·134	·146	·158	·164	·17	·175	·175

The teeth being tapered downwards, the value of B increases from the top to the base of a tooth. We may use the mean value

* *The Electrician*, vol. xxxvi., p. 116 (1895).

of B along the tooth in calculating the hysteresis loss by the aid of the above Table.

For the calculation of the hysteresis loss in the core, the following Table may be used:—

B =	10,000	11,000	12,000	13,000	14,000	15,000
Joules per c.c. per cycle =	·00082	·00096	·00112	·00127	·00142	·0016
Joules per lb. per cycle =	·048	·056	·065	·074	·083	·093

The eddy current losses in both teeth and core may be approximately calculated by the aid of the formula

$$\text{watts lost by eddy-currents, per c.c.} = 7{\cdot}5\ B^2 f^2 t^2\ 10^{-11},$$

where f = cycles per second (= revolutions per second × number of pairs of poles),‡ and t = thickness of core discs, in cm. The above may also be written

$$\text{watts lost by eddy-currents, per lb.} = 5\ B^2 f^2 t^2\ 10^{-10}.$$

From a large number of tests of machines of different sizes and types carried out by Esterline and Reid,* and by Parshall and Hobart,† it appears that within the range of values of B commonly in use the total core loss (teeth and body of core, hysteresis and eddies) may be approximately represented by

$$\text{watts per lb. of core} = 7x + 10x^2,$$

where $x = Bf\ 10^{-6}$.

This last formula may be used as a rough check on the approximate value obtained for the core loss by means of the preceding Tables and formulæ.

§ 126. Hysteresis and Eddy-current Loss in Pole-shoes, and Eddy-current Loss in Armature Conductors.

Owing to the crowding of the lines into the tops of the teeth, the distribution of the flux over the polar surface is not uniform, there being alternate maxima and minima of the induction as we proceed along the polar arc. Further, on account of the rotation of the armature, the distribution of the flux over the pole-face will vary periodically with great rapidity, the period of the disturbance being equal to the time which the armature takes to advance a distance corresponding to the sum of the widths of a

* *Transactions of the American Institute of Electrical Engineers*, vol. xx., p. 1323 (1903).

† *Electric Machine Design*, p. 118 (1906).

‡ The value of f generally lies between 20 and 30.

tooth and slot. We thus have a very rapid swaying of the lines within the pole-shoe, which gives rise to hysteresis and eddy-current losses. This kind of disturbance does not penetrate to any great depth, as the flux distribution again becomes practically uniform within the pole-shoe at a small distance from the surface; but owing to the extreme rapidity with which the lines are kept vibrating, the loss close to the polar surface may become appreciable. This loss is greatest with short air-gaps, wide slots, and feebly saturated teeth. The eddy-current loss may in such cases be largely suppressed by laminating the pole-shoes.

So long as the induction in the teeth does not exceed about 20,000, there are very few lines passing through the conductors lying in the slots, and there is no appreciable eddy-current loss in them. If, however, the teeth be strongly saturated, a considerable number of lines will enter the slot, and with conductors of large cross-section the non-uniformity of the field within the slot will, as the slot moves away from the edge of the pole-shoe during its passage through the "fringe" of the field, give rise to eddy-currents in the conductors. It is impossible to calculate, and difficult to estimate, the extent of this loss, which, however, only becomes important in the case of armatures with very strongly saturated teeth ($B > 20,000$).

§ 127. Additional Losses at Full Load. Mechanical Friction Losses. Efficiency.

When the armature is loaded, armature reaction causes (§ 80) a distortion of the field, and a consequent increase in the maximum value of B in the teeth and other portions of the core, and in the eddy-current losses in core and conductors. Thus the "full-load" core losses will generally be greater than the open-circuit or "no-load" losses. Here, again, it is impossible to pre-determine the exact increase of loss due to field distortion.*

The mechanical friction losses consist of (*a*) brush friction; (*b*) bearing friction; and (*c*) air friction or "windage."

The co-efficient of friction for carbon brushes is about ·3, and since the pressure is generally about 1·25 lb. per square inch of contact area, the tangential resisting force per square inch of contact

* See paper by Epstein in *Journal of the Institution of Electrical Engineers*, vol. 38, p. 54 (1906).

area is ·375 lb. Hence if v_c = peripheral velocity of commutator, in feet per minute, and a = sum of contact areas of all the brushes, the brush friction loss, in watts, is given by

$$\frac{\cdot 375\, a\, v_c \times 746}{33{,}000} = \cdot 0085\, a\, v_c.$$

The loss due to brush friction, it will be noticed, increases in simple proportion to the speed of the commutator.

The bearing friction loss is, however, found to increase as the 1·5th power of the speed. This is due to the fact that we here have to a large extent to deal with the fluid friction of the lubricant. If d = diameter, and l = length of journal, both expressed in inches, and if v = peripheral velocity, in feet per minute, the friction loss in the bearing, in watts, is found to be given approximately by

$$\text{watts lost in bearing} = 10^{-3}.\, d\, l\, v^{1 \cdot 5}.$$

The air friction loss is very difficult to estimate. A rough allowance for it may be made by assuming it to be equal to from 10 to 15 per cent. of the bearing friction loss. This would only apply to machines of ordinary construction. In turbine-driven machines, the air-friction loss becomes extremely important, and may even exceed the bearing friction.

The ratio of the useful power developed by a generator or motor to the total power absorbed by it is defined to be the *efficiency** of the machine. The efficiency, of course, varies with the load; when used without any qualification, the term "efficiency" is understood to refer to the full-load efficiency.

§ 128. Calculation of Temperature Rise.

The output of a machine is limited by either sparking or heating. The sparking limit may be estimated from the reactance voltage (§ 85). The permissible rise of temperature in any part of a machine is usually from 35° to 45° C. The probable temperature rise of the various parts may be approximately estimated

* By some writers, a distinction is made between *commercial* efficiency and *electrical* efficiency. The former corresponds to what we have defined as the efficiency simply; by the latter is meant, in the case of a dynamo, the ratio $\frac{\text{useful electrical power}}{\text{total electrical power}}$; and in the case of a motor, the ratio $\frac{\text{electrical power transformed into mechanical power}}{\text{total electrical power supplied to motor}}$.

from the rate of heat generation and the area of the cooling surface. Considering first the case of a field coil, the area of whose curved outer surface* is a square inches, and in which w watts are dissipated, we can approximately calculate the temperature rise of the *outer surface* of the coil by means of the formula

$$t = \frac{65\,w}{a},$$

where t is the temperature rise in degrees C. The temperature rise of the inner layers may be considerably (from 10 to 50 per cent.) higher, depending on the depth of the winding. The average temperature rise t_m as deduced from the change in the resistance of the coil is found to be given approximately by †

$$t_m = (1 + \cdot 4d)t,$$

d denoting the depth of winding, in inches ‡.

The pre-determination of the temperature rise in the armature is a much more uncertain matter. In the case of large well-ventilated armatures, the surface temperature rise t_a is approximately given by

$$t_a = \frac{65\,w}{(1 + \cdot 0005\,v)\,a},$$

w being the total watts dissipated in the armature winding and core, v the peripheral velocity in feet per minute, and a the cylindrical surface of the winding (square inches).

The temperature rise t_c of the commutator may be taken as

$$t_c = \frac{50\,w}{(1 + \cdot 0005\,v)\,a},$$

w denoting the watts dissipated by heating of the brush contacts electrically and by friction, v the peripheral velocity of the commutator in feet per minute, and a the cylindrical surface of the

* The end surfaces of the coil (cheeks of bobbin) are not to be included in estimating the cooling surface.

† Niethammer, *Moderne Gesichtspunkte für den Entwurf Elektrischer Maschinen und Apparate*, p. 12 (1903).

‡ In a paper published in *The Electrician*, vol. lviii., p. 448 (1907), Mr. G. A. Lister states, as the result of a number of experiments on the heating of field coils, that in the case of machines of ordinary construction, of the "open" type, the probable mean temperature rise, *as measured by the increase of resistance method* (§ 136), is given by $t = Cw/A$, where A is the *total* surface of the field coil in square inches, including the end surfaces and the interior surface in contact with the field core; and C is a coefficient whose value ranges from about 190 for a 50 kilowatt machine to about 230 for a 500 kilowatt one.

commutator (sq. inches). This formula gives safe values so long as the machine runs sparklessly.

In the case of machines of the semi-enclosed and totally enclosed types, the circulation of the air inside the machine tends to equalise the temperatures of the various parts, and the average temperature rise t may be approximately determined by a formula of the type $t = K\frac{w}{a}$, where w = total watts lost in the machine, and a = total external surface of machine. K is a constant whose value may be taken as 100 for semi-enclosed and 130 for totally enclosed machines. In the case of totally enclosed machines, the cooling surface is frequently greatly increased by providing the case of the machine with a number of ribs similar to those used in radiators, as shown in Fig. 105, which illustrates a type of motor made by Messrs. Mather and Platt, of Manchester.

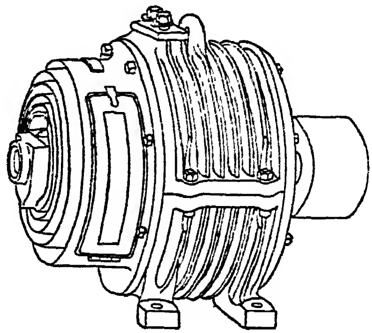


FIG. 105.—Totally enclosed motor in ribbed casing (Mather & Platt).

§ 129. Experimental Determination of Efficiency. Soames' Testing Brake.

The formulæ given in the preceding sections enable us to calculate approximately the efficiency and probable temperature rise of a machine from its constructional data, and may be regarded as forming useful guides to the designer. We shall now explain how the efficiency of a finished machine may be determined experimentally.

The method to be adopted will depend to a large extent on the size of the machine. We shall first consider the determination of the efficiency of a motor of small or moderate size.

For motors up to 5 kilowatts, a form of brake devised by Soames, and illustrated in Fig. 106, may be conveniently employed. A band of braided hemp passes around the motor

pulley, the ends of the band being attached to a steel lever free to swing on knife-edges supported by hard steel plates which are fitted into the lower forked end of a screwed shank. The shank passes through a nut fitted with a hand-wheel. This nut is supported by a tripod stand. The tripod head contains a sleeve which fits the shank loosely, and which is provided with two inwardly projecting pins acting as guides to the shank, which is grooved to receive them. These pins prevent the shank from rotating when the hand-wheel is turned in tightening or slackening the brake. In using the brake, the tripod is carefully adjusted so as to bring the shank vertically above the motor shaft, and the lever into the middle plane of the pulley. The sliders which support the brake band are adjustable, and are fixed at points on each side of the knife-edge distant from the latter by an amount equal to the radius of the pulley *plus* half the thickness of the brake band. The portions of the band not in contact with the pulley will then be vertical, and with the lever in a horizontal position

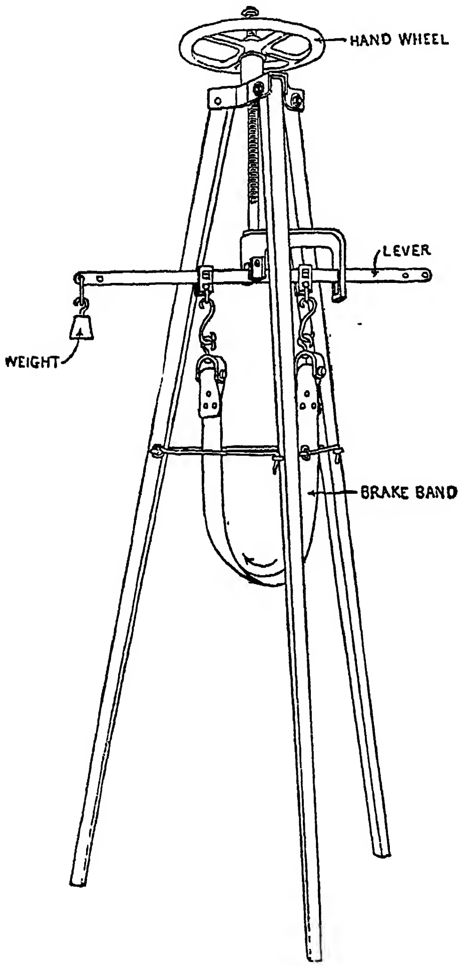


FIG. 106.—Soames' testing brake.

the torque at the motor pulley will be accurately transmitted to the lever, and may be measured by hanging a weight from one end of it so as to balance the torque. If W = weight suspended from lever in lbs., and d = its distance from the knife-edge, in feet, the torque, in lb. feet, is Wd. The work done per revolution is $2\pi d W$ (or $2\pi \times$ torque), so that if m = revolutions per minute, the power in foot lbs. per minute is $2\pi m d W$, and in h.p., $\frac{2\pi m d W}{33,000}$. Expressing this power in watts, we have

$$\text{brake-power of motor in watts} = \frac{2\pi m d W}{33,000} \times 746.$$

On dividing this latter quantity by the total electrical power, in watts, supplied to the motor (as measured by an ammeter and voltmeter) we find its efficiency.

Since the factor $\frac{2\pi d . 746}{33,000}$ in the expression for the brake-power is constant if the weight is always hung from the same point—as is actually the case—we need merely, in order to find the power in watts, take the product $W m$ and multiply it by the value of this constant, which is marked on the lever.

§ 130. Eddy-current Brake.

An extremely convenient and accurate form of testing brake for motors is that known as the eddy-current brake. It consists essentially of a disc or discs of copper which may be mounted on the shaft of the motor to be tested, and which are made to revolve between the poles of a magnet or system of magnets so arranged that powerful induced currents, or eddy-currents, are made to circulate in the copper discs. By Lenz's law (§ 5), the motion of the discs is resisted, and they tend to carry the inducing magnets round with them. By attaching to the magnet or magnets a suitable lever, the torque due to the eddy-currents, which is also the useful torque exerted by the motor, may be measured.

One of the best known forms of eddy-current brake is that devised by Messrs. Morris and Lister. This is shown in

To these gentlemen the author is indebted for the drawings and detailed information regarding their brake.

Figs. 107 and 108. Two flat rings of high-conductivity hard-rolled copper, with thickened inner and outer edges, are screwed to the arms of light supporting spiders of aluminium which are

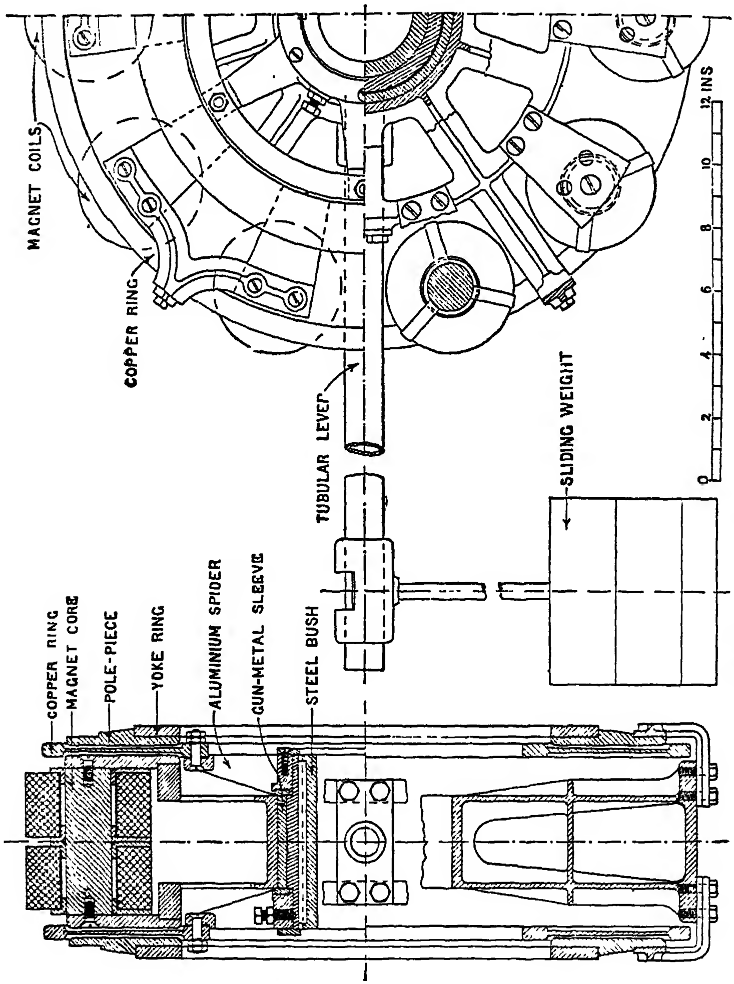


FIG. 107. -Morris and Lister's 30 h.-p. eddy-current brake.

mounted on a cast-iron bush. This bush revolves freely inside a gun-metal sleeve, mounted on which is an aluminium casting which carries the magnetising coils of the series of electro-magnets arranged between the copper rings. This casting also

carries a long tubular lever projecting both ways. One arm of the lever is shorter than the other, and carries a counterpoise which serves to balance the torque due to the sliding weight when this latter is at the zero of the scale engraved along the longer arm of the lever. The motion of the end of this arm is limited by two stops. Immediately outside each copper ring are a series of pole-pieces facing the ends of the magnet cores, and connected to a common yoke-ring which serves to complete the magnetic circuits. Each yoke-ring with its system of pole-pieces is rigidly supported by a number of forked aluminium castings (each

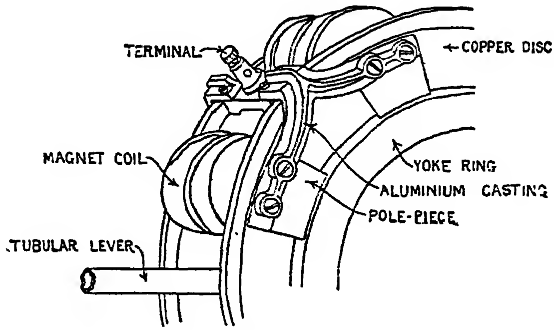


FIG. 108.—Morris-Lister eddy-current brake.

prong being screwed to a pole-piece) which curve round the outer edge of the copper ring and are secured to projections on the central aluminium casting supporting the magnetising coils. The terminals of the coils are mounted on the same projections. In order that the brake may be made to fit motor shafts of different diameters, a series of steel bushes are provided, which fit inside the cast-iron bush supporting the copper rings, and are secured to it by a feather key. The steel bushes are provided with key-ways, so that they may be keyed to the motor shaft. The value of the torque is adjusted to the required amount by suitably varying the exciting current of the magnets. The brake-power is calculated as already explained in § 129.

§ 131. Efficiency Tests of Small Generators.

The efficiency of a small dynamo may be conveniently determined by loading it as a motor, and finding the efficiency by

means of a brake test as already described. Since the losses taking place in the machine with a given p.d. across its terminals and given current through its armature are the same, whether the machine be running as generator or motor, the efficiency corresponding to a given load may be calculated independently of whether the machine is loaded one way or the other (i.e., as motor or generator). Another very convenient method of testing small generators consists in coupling them to a standardised motor—i.e., one whose efficiency at various loads has previously been determined by brake tests—and measuring the power w_1 supplied to the motor and the useful power w_2 developed by the generator. If η_1 is the efficiency of the motor when absorbing an amount of power w_1, its brake-power is $\eta_1 w_1$, and this also represents the mechanical power supplied to the generator. The efficiency of the generator is thus $\dfrac{w_2}{\eta_1 w_1}$.

If two exactly similar machines are available, they may be mechanically coupled, and one loaded as a generator while the other is driven as a motor. If w_1 is the power absorbed by the motor, and w_2 the useful power developed by the generator, the joint efficiency of the two machines (which is the product of their individual efficiencies) is w_2/w_1; and since the machines are similar, and therefore have equal efficiencies, the efficiency of each is $\sqrt{w_2/w_1}$.

§ 132. Field's Efficiency Test for Tramway Motors.

The method last mentioned, which can only be carried out if two absolutely similar machines are available, has been used, with very satisfactory results, by M. B. Field† for testing tramway motors. Such motors are provided with series windings, and each motor drives a car axle through single-reduction spur gearing, consisting of a pinion on the motor shaft and a spur-wheel on the car axle. Owing to the use of gearing, the efficiency which is important from a practical point of view is the *combined efficiency* of motor and gearing. In order to find this combined

* This result is only approximate, since the load on the motor is necessarily greater than that on the generator, and hence the losses and efficiencies are different in the two machines.

† *Journal of the Institution of Electrical Engineers*, vol. 31, p. 1,283 (1902).

efficiency, the motors are placed on a special testing frame, and geared to each other by means of one of the gear wheels actually used on the car axles. The electrical connections are as shown in Fig. 109, the letters VM denoting a voltmeter, and AM an ammeter. One of the machines runs as a motor, the other as a generator, and *the field of the generator is connected in series with the motor.* Since, therefore, the field flux is the same in each case, and since the two armatures are constrained to run at the same speed, it follows that the losses taking place by hysteresis, eddies and friction in the two machines are equal. Let V_1 be the p.d. across the motor terminals, and i_1 the motor current; let V_2 be the p.d. across the generator armature, and i_2 the generator current; then since $V_2 i_2$ is the useful power developed by the generator, and $V_1 i_1$ the power absorbed by the motor, it follows that $V_1 i_1 - V_2 i_2$ represents the total loss in the motor, *plus* the loss in the generator armature. Now if h denote the losses due to resistances of motor field and armature (including brush contacts) and to resistance of generator armature and brush contacts, $V_1 i_1 - V_2 i_2 - h$ gives the total loss in the two machines due to hysteresis, eddies and friction (including gear friction), and half this amount gives the loss in question occurring in *each* machine. If to this be added the resistance loss in the motor, we get the total loss w_m in the motor and its gear, and from this its combined efficiency $\dfrac{V_1 i_1 - w_m}{V_1 i_1}$.

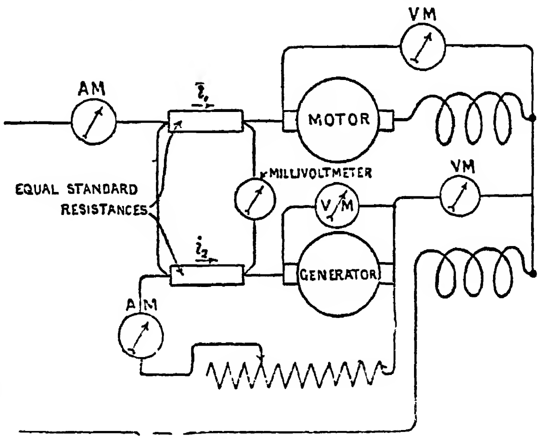


FIG. 109.—Field's efficiency test for tramway motors.

A refinement due to E. Wilson enables the difference $V_1 i_1$ $V_2 i_2$ to be determined with a higher degree of accuracy than is

Electrical Review, vol. 53., p. 490 (1903).

attainable by a direct measurement of V_2 and i_2. This consists in introducing into the motor and generator circuits two equal standard resistances as shown, connecting them directly on one side, and through a milli-voltmeter on the other. From the milli-voltmeter reading and the known value of each resistance the difference i_d of the currents $i_1 - i_2$ is easily determined, and $i_1 - i_d$ gives a relatively much more accurate value of i_2 than that obtained by a direct reading. In the same way, an additional voltmeter is connected as shown between one of the generator brushes and the junction of the field coils, and if v_d is the reading of this voltmeter, $V_1 - v_d$ is used instead of the direct reading V_2. Thus $V_1 i_1 - (V_1 - v_d)(i_1 - i_d)$ is used instead of $V_1 i_1 - V_2 i_2$.

§ 133. Hopkinson's Efficiency Test.

When dealing with machines above, say, 20 kilowatts the most satisfactory form of test is that generally known as the Hopkinson test. This test requires two machines similar in every respect, and having their shafts mechanically coupled. The machines are also electrically coupled as shown in Fig. 110, and the field of one of them is slightly weakened so as to disturb the balance of the e.m.f.'s in the local circuit of the armatures and cause the desired current to circulate between them. Both machines will be loaded, one as a generator, the other as a motor. It is evident that power must be supplied to the combination of the machines in order to keep them running, and that this power represents the sum of the losses in the machines. In the original Hopkinson test, the power was supplied mechanically, and its measurement presented a good deal of difficulty. The arrangement shown in Fig. 110 is a modification of the

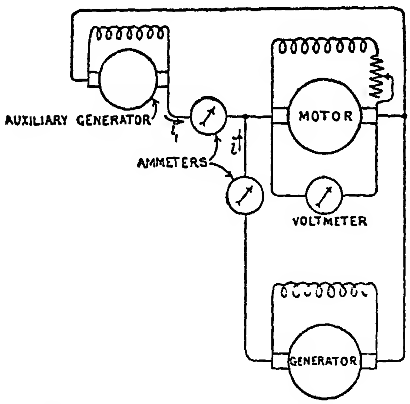


FIG. 110.—Hopkinson's efficiency test.

original Hopkinson test due to Kapp. The additional power required to keep the machines running is here supplied electrically by an auxiliary generator or secondary battery. If V is the p.d. across the terminals of the machines, i the generator current, and i_1 the current supplied to the motor by the external source, then V $(i + i_1)$ represents the total power absorbed by the motor. The two machines being nearly equally loaded, we may assume their efficiencies to be equal. If, then, x represents the efficiency of either machine, the brake-power of the motor is x V $(i + i_1)$. Since this also represents the mechanical power absorbed by the generator, the useful electrical power developed by the generator will be $x\,x\,V(i + i_1) = x^2\,V(i + i_1)$. But the useful power developed by the generator is also equal to V i. Hence

$$x^2\,V(i + i_1) = Vi,$$

$$\text{or } x = \sqrt{\frac{i}{i + i_1}} \quad . \quad . \quad . \quad . \quad (1).$$

The great advantage of the method is that a full-load test may be carried out with an expenditure of power which is only a small fraction of the power required to drive either machine separately. This is due to the fact that the mechanical power developed by the motor is not wasted, but is transmitted to the generator, and a large portion of it, after transformation into electrical power by the generator, is returned to the motor. There is thus a *circulation of power* between the two machines.

The Hopkinson test is particularly valuable in connection with very large machines, as the power available for the test is in such cases frequently quite insufficient to run either machine separately under full load. It has become the standard method of testing in all cases where two machines of the same size and type are available.

The formula (1) which we have obtained for the efficiency is based on the assumption that each machine has the same efficiency. This will be the case if (the machines being quite similar) each machine is running under the same conditions as regards load and excitation. Now a reference to Fig. 110 shows that the

In deducing this formula, we have supposed that the loss in the ammeter introduced into the armature circuit and in the connecting leads is negligible. If necessary, this loss may be taken into account.

motor is more heavily loaded than the generator, and that the motor field is slightly weaker than that of the generator. There being a difference in the load and excitation, the efficiencies will not be quite the same, and formula (1) will only give us the geometric mean of the two efficiencies. In the case of large machines, however, the difference of load will not be sufficient to affect the efficiency. But with small machines, whose efficiency is comparatively low, the percentage difference of load may be so great that the assumption regarding equality of the two efficiencies is no longer legitimate, and the method breaks down. It may, however, be modified so as to be applicable to small machines as well. For this purpose, the machines are both fully excited, and the balance of their e.m.f.'s is disturbed by introducing the necessary number of secondary cells into the armature circuit (in series, if necessary, with a low variable resistance for exact adjustment; the power lost in the resistance being taken into consideration in calculating the efficiency). The power required to keep the machines running is supplied *mechanically* (as in the original Hopkinson test), and in order to enable this power to be determined accurately, a small standardised motor—i.e., one whose efficiency for various loads has been previously determined by brake tests—may be used for driving the coupled machines under test. The motor may be either direct-coupled or belted to the machines.* Half the total power supplied to the coupled machines † represents the loss in each, and enables us to determine the efficiency.‡

§ 134. Hopkinson's Efficiency Test as Applied to Series-wound Machines.

In the account of the Hopkinson efficiency test given in § 133, we have supposed the machines to be provided with a shunt

* In order to find the loss in the belt approximately, the auxiliary motor is run light, and the power absorbed by it determined; similarly, the machines under test are run light as motors, and the power absorbed by them is measured; the sum *s* gives the power required to keep all three machines running light. The auxiliary motor is next belted to the two machines, and the power required to keep them running determined; the difference between this and *s* gives, approximately, the loss in the belt.

† Including the power supplied to the armatures by the cells; this covers the heating loss in the armature windings and brush contacts.

‡ For an account of the various modifications proposed from time to time of the original Kapp-Hopkinson test, the reader may be referred to an article by C. F.

winding. The same test may be applied to compound-wound machines, the series coils being cut out during the test, and only the shunt winding used. The loss which takes place in the series coils under normal conditions of running may be easily calculated from the resistance of the coils and the current, and added to the other losses as found by the Hopkinson method when using the shunt winding only. A simple modification of the connections allows of the use of the Hopkinson method in the case of two similar *series-wound* machines, such as two large railway motors. The arrangement of connections is shown in Fig. 111. The two machines are as before coupled mechanically (either directly, if the efficiency of the motors alone is required; or through their gearing, if the combined efficiency of motor and

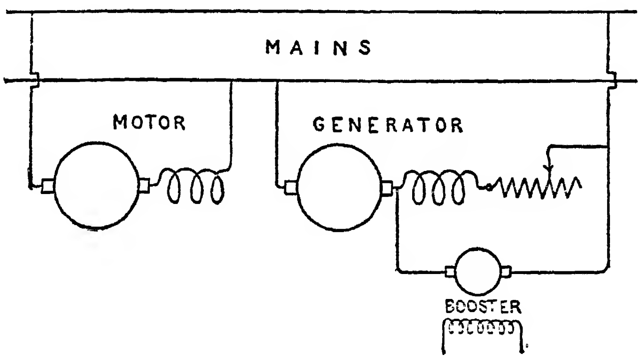


FIG. 111.—Hopkinson's efficiency test as applied to series wound machines.

gearing is in question). One of them—which runs as a motor—is connected directly across the supply mains. The other, which is to act as the generator, has its armature connected in series with the armature of a small auxiliary independently excited machine, termed a *booster*, which adds its e.m.f. to that of the generator and thereby enables it to supply current to the mains. The field of the generator is connected in series with an adjustable resistance and then across the brushes of the booster armature. The booster is driven by a small motor. The resistance

Guilbert in *La Lumière Électrique*, vol. 13. p. 72 (1911). A modification of the method applicable to a *single* machine is described by W. Lulofs in the *Journal of the Institution of Electrical Engineers*, vol. 43. p. 150 (1909).

Electric Journal, vol. 3., p. 525 (1906).

in the generator field and the booster excitation are so adjusted that when the motor is taking full-load current from the mains the generator field is also supplied with full-load current. It is evident that the hysteresis, eddy-current and frictional losses are then equal in the two machines. The generator *armature current* will necessarily be less than the current taken by the motor, as the motor torque must exceed the generator torque, and hence with equal fields the motor current must exceed the generator armature current. Let V=p. d. across motor terminals; i_m= motor current; V_g=p. d. across generator armature; i_g=current through generator armature. Then $Vi_m - V_g i_g$ gives the sum of the total losses in the motor and generator, with the exception of the loss in the generator field. The calculation of the efficiency proceeds on the lines already explained in connection with Field's efficiency test (§ 132), and it is advantageous to obtain i_g in the form $i_g = i_m - i_d$, i_d being measured by a differential method as in § 132.

§ 135. Swinburne's Efficiency Test.

Cases may arise where two machines are not available, and some other method of testing must then be resorted to, such as that originally suggested by Swinburne, which consists in measuring the losses, and calculating the efficiency from them and the known output of the machine. In this method, the machine is run light as a motor, with a p.d. across the brushes equal to the full-load e.m.f., the exciting current being adjusted to give the normal speed. The power w_o absorbed by the machine represents the hysteresis, eddy-current and frictional losses, and the heating loss in the shunt coils (the armature resistance loss is negligible; or, if not negligible, may be allowed for). It is then assumed that these losses remain constant at all loads, and that the only increase which occurs in the losses as the load is increased is that due to heating of the armature coils and brush contacts. This loss may be calculated (§ 124) from the known resistances of the armature winding and brush contacts and the current; let it be represented by a. If V is the p.d. and i the current delivered by the machine when running as a generator, the efficiency is

$$\frac{Vi}{Vi + w_o + a}$$

Convenient as this method is, it is, unfortunately, not quite reliable. For the hysteresis and eddy-current losses at full load are not the same as, but are in general greater than, those at no load, owing to the field distortion brought about by armature reaction (§ 127).

The following table represents the approximate full-load efficiencies which are obtainable with machines of modern design:—

Output in kilowatts...	·5	1·0	2·0	3	5	10	20	50	100	500	1,000	2,000
Percentage efficiency	60	65	70	73	76	81	85	88	90	93	94	95

As regards the distribution of the losses, it may be mentioned that the loss due to heating of the field and armature coils and brush contacts varies from about 15 per cent. for a 5 kw. machine to about 3 per cent. for machines of 1,000 kw. and over. The hysteresis and eddy-current losses vary from about 4 per cent. in the case of small machines down to about 2 per cent. in large ones. The frictional loss generally lies between 3 and 1 per cent.

§ 136. Determination of Temperature Rise. Heating Tests.

A very important part of the complete test of a machine is the determination of the temperature rise. Two methods of measuring the rise are available. One of these consists in using a thermometer whose bulb is applied to the surface (armature or field coil) whose temperature is required, the bulb being covered with some waste or cotton wool to prevent loss of heat by radiation. The second method is that in which the temperature rise is calculated from the increase of resistance of the winding. If r_0 is the resistance of the winding at 0°C., and if t_1, t_2, be the temperatures respectively of the room in which the test is being carried out and of the winding when the maximum temperature (after a sufficiently long run) has been reached, the corresponding resistances r_1 and r_2 of the winding are given by

$$r_1 = r_0 (1 + a\, t_1)$$
$$\text{and } r_2 = r_0 (1 + a\, t_2),$$

a being the temperature coefficient of resistance. Dividing the first equation by the second, we obtain

$$\frac{r_1}{r_2} = \frac{1 + a\, t_1}{1 + a\, t_2},$$

$$\text{which gives } t_2 = \frac{r_2 - r_1}{a\, r_1} + \frac{r_2}{r_1} t_1,$$

$$\text{so that } t_2 - t_1 = \left(\frac{r_2}{r_1} - 1\right)\left(\frac{1}{a} + t_1\right).$$

If the temperatures be measured on the Centigrade scale $a =$ ·004, and $\frac{1}{a} = 250$, so that the required temperature rise $t_2 - t_1$ is given by

$$t_2 - t_1 = \left(\frac{r_2}{r_1} - 1\right)(250 + t_1),$$

r_1 being the resistance at the commencement and r_2 that at the end of the test.

It must be carefully noticed that whereas the thermometer method only gives us the *surface* temperature rise of the winding, the increase of resistance method gives the *mean* temperature rise of the entire winding, which is necessarily *higher* than the surface rise. The increase of resistance method is far superior and capable of a much higher degree of accuracy than the thermometer method, and is being very generally adopted. Owing to the higher values given by this method, a greater temperature rise should be permitted when the method is specified, so that, instead of the 35° to 45° C. temperature rise as measured by the thermometer, a rise of 50° to 60° C. may be allowed in the case of the increase of resistance method.

In order to ascertain the ultimate temperature rise of the different parts of a machine of which no duplicate is available, and to which the Kapp-Hopkinson test is inapplicable, it is necessary to run the machine under full load until the temperatures are found to have reached steady values. Now in the case of large machines, such a procedure becomes very costly. Two methods have been suggested and used for coping with this difficulty. In one of these, due to R. Doczekal,* the machine is run a sufficiently long time to enable the temperatures to become steady, but under conditions which, although approximating to full load conditions, do not involve the expenditure of full-load power, and thus enable a large saving to be effected in the cost of the energy required for the test. In the second method, which has been studied in considerable detail by

* *Elektrotechnik und Maschinenbau*, vol. 28, p. 181 (1910).

W. R. Cooper, the machine is run under full load for a comparatively short time only, and the final temperature rise is calculated from the observed temperature changes during the short run, or while the machine is cooling down after having been stopped.

The principle of Doczekal's method, which effects an economy similar to that obtained by the use of the Kapp-Hopkinson method in the case of two similar machines, is as follows. If in a machine whose field is supplied with the normal full load exciting current from some independent source the brushes be gradually displaced forwards, the brush p.d. will steadily decrease until it finally vanishes when the displacement becomes equal to half a pole-pitch. Suppose now that the brushes, while near the position of zero e.m.f., are short-circuited through an ammeter, and that their position is adjusted so that full load current flows through the armature. Then the heating losses in the field and armature coils, brush contacts and commutator, are the same as those occurring at full load. But the armature core loss is different from that at full load, owing to the peculiar distribution of the flux under the conditions considered. The temperature rises of the field and armature coils may be taken as approximately correct. But in the case of the temperature rise of the armature core, a correction becomes necessary, and is, as shown by Doczekal, given approximately by the following simple formula:—

$$t_f = 2t_r - t_c,$$

where t_f = true temperature rise of core under full-load conditions; t_r = observed temperature rise of core in test; and t_c = temperature rise of armature winding.

It might be supposed that the reliability of this method is seriously impaired by heavy sparking at the brushes. According to Doczekal, however, such is not the case.

Examples.

1. The resistance of the armature winding of a 1,500-kilowatt, compound-wound, 600-volt generator is ·0042 ohm at the working temperature; the resistance of the shunt winding which is connected across the terminals is 53 ohms; and that of the series winding is ·00076 ohm. The brush contact drop is 2·5 volts. Find the electrical efficiency of the generator.

The Electrician, vol. 71, p. 972 (1913).

2. In a Soames brake, the distance of the point of suspension of the weight fromthe knife-edge is $15\frac{3}{4}$ inches. If the weight is 16 lbs., and the motor speed 800 revolutions per minute, what is the brake horse-power of the motor?

3. The resistance of the armature winding (exclusive of brush contacts) of a tramway motor is ·45 ohm, and that of its (series) field winding ·50 ohm, both resistances being measured at the working temperature. The brush contact drop amounts to 2·5 volts. Two such motors are connected up for a Field's efficiency test (§ 132). With a current of 50 amperes supplied to the motor at a p.d. of 490 volts across its terminals, the speed is found to be 420 revolutions per minute, and the generator armature is found to give a current of 40 amperes at a p.d. of 420 volts across the brushes. Find the combined efficiency of a motor and its gearing when the motor takes a current of 50 amperes at 520 volts, assuming as an approximation that the sum of the hysteresis, eddy current, and frictional losses is proportional to the speed.

4. Two machines are coupled as in the Hopkinson test (§ 133). With a p.d. of 240 volts across the terminals, the local current circulating between the armatures is adjusted to 100 amperes, and it is then found that the auxiliary generator supplies a current of 32·1 amperes. Find the efficiency of each machine.

5. The resistance of the armature winding (exclusive of brush contacts) of a 300-kilowatt, 500-volt, shunt-wound dynamo is ·0136 ohm, the resistance of the shunt winding being 250 ohms. The brush contact drop amounts to 2 volts. When the machine is run light as a motor at its normal speed and a p.d. across its brushes equal to the full-load e.m.f., the armature is found to take a current of 27·8 amperes. Find approximately the efficiency of the machine.

CHAPTER XIII.

§ 137. Rosenberg's constant-current dynamo and its practical applications—§ 138. Motor car lighting generators—§ 139. Special type of welding generator—§ 140. Homopolar generator.

§ 137. Rosenberg's Constant-current Dynamo and its Practical Applications.

For certain purposes—as, for example, for motor car and train lighting, running arc lamps in series, or supplying single arc lights—dynamos are required capable of maintaining an approximately *constant current* in spite of any variations in the resistance of the external circuit or in the speed at which the dynamo is driven. Some of the earliest forms of dynamos, intended for series systems of arc lighting, were of this type. In what follows we propose giving an account of a type of *constant current dynamo* invented by E. Rosenberg and extensively used for train lighting as well as for some other purposes.

The essential feature of this machine is the method by which the field flux is automatically varied in such a manner as to maintain the current in the external circuit approximately constant between certain limits of speed and load. The field magnet cores and yoke are of relatively small cross-section, and are "saturated" under normal conditions of working. The field magnets do not, however, as in ordinary machines, provide the useful flux which generates the working e.m.f. in the armature, but merely serve to produce an auxiliary flux by means of which the exciting current required for the main field is generated in the armature itself. The arrangement will be understood by reference to Fig. 112, which shows a four-pole machine of this type. The brushes which make contact with the conductors at a_1, a_2, a_3, &c., and which, in dynamos of ordinary construction,

Elektrotechnische Zeitschrift, vol. 26, p. 393 (1905), and vol. 27, p. 1035 (1906); see also *The Electrician*, vol. 55, p. 297 (1905), and vol. 58, p. 372 (1906).

are in connection with the external circuit, are in this machine *short-circuited*, all the positive and negative brushes being connected together. The short-circuit current so produced gives rise to a powerful cross-flux (§ 80), and it is this cross-flux that constitutes the *main flux* which generates the useful e.m.f. of the machine. The short-circuit current plays the part of the *exciting current*. In order to utilise the cross-flux, which (§ 80) is displaced relatively to the auxiliary flux by half a pole-pitch, additional sets of brushes, arranged so as to make contact with the conductors at m_1, m_2, &c., are provided, and these brushes form the main brushes of the machine, being in connection with the external circuit. In Fig. 112, the auxiliary flux provided by the field is indicated by the chain-dotted lines, while the useful or cross-flux is shown by the dotted lines.

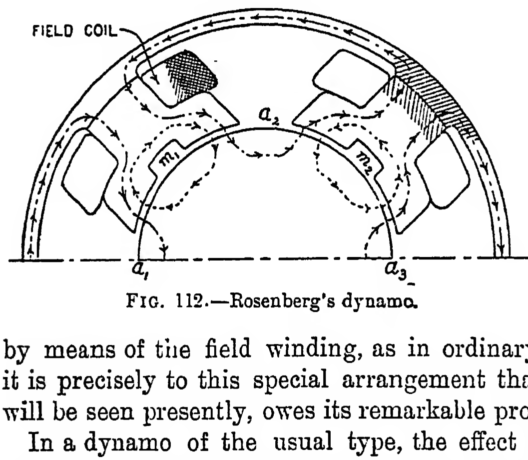


FIG. 112.—Rosenberg's dynamo.

At first sight, the arrangement described would seem to be unnecessarily complicated, since the main flux could be obtained directly by means of the field winding, as in ordinary machines. But it is precisely to this special arrangement that the machine, as will be seen presently, owes its remarkable properties.

In a dynamo of the usual type, the effect of short-circuiting the brushes when the full field excitation is maintained would be disastrous, as the enormous currents produced in the armature would in a very brief interval of time destroy the insulation by raising the conductors to a very high temperature, and finally even melting them. It is evident that this danger must be guarded against in the Rosenberg dynamo, and in cases where it is likely to occur fuses are employed for automatically opening the connection between the positive and negative auxiliary brushes (i.e. the brushes at a_1, a_2, &c., in Fig. 112). The auxiliary flux must be extremely weak in order to prevent the short-circuit current from rising above the limit of safety. On the other hand,

the cross-flux must be sufficiently powerful to generate the e.m.f. required for the main circuit. This accounts for the peculiar proportions of the machine shown in Fig. 112, which is an actual scale drawing.* It will be noticed that the yoke ring and field cores, which are only called upon to carry the auxiliary flux, are comparatively slender; while, on the other hand, the pole-pieces, which have to carry the cross-flux, are very massive.

It has already been mentioned that the Rosenberg dynamo is used for a variety of purposes, and the exact arrangement of the windings depends on the particular use to which the machine is to be put. One of its earliest applications was to train lighting, and we shall first consider the arrangements adopted in this

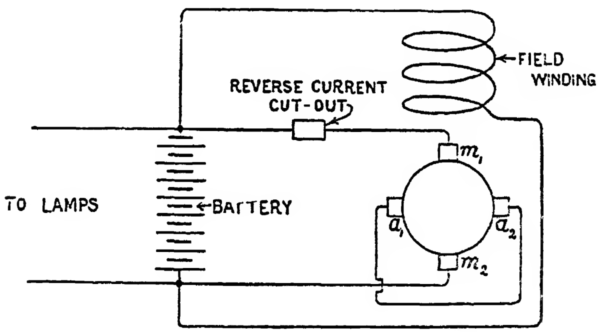


FIG. 113.—Connections of train lighting dynamo.

case. The dynamo is arranged to be driven by belt from a pulley mounted on one of the carriage axles. Since the supply of current must be maintained while the train is at rest, and since the dynamo is not then running, a battery of secondary cells is provided. A complete diagram of connections is shown in Fig. 113. The battery supplies current to the field winding, which provides the auxiliary flux. As soon as the dynamo is started the short-circuit current between the auxiliary brushes a_1, a_2, gives rise to a cross-flux, and an e.m.f. is generated by this cross-flux which may be utilised in the main circuit by means of the main brushes m_1, m_2.

One remarkable feature of such a machine will now become evident. Since the direction of the short-circuit current, and

* Except as regards the air-gap, which is shown exaggerated.

hence of the cross-flux, depends on the direction of rotation, a reversal of the rotation will be accompanied by a simultaneous reversal of the cross-flux, and the e.m.f. obtainable through the main brushes m_1, m_2, will always have the same direction, independently of the direction of rotation. Thus *the polarity of the main brushes remains unaltered if the direction of motion be reversed.*

With gradually increasing speed, the e.m.f. due to the cross-flux increases, rapidly at first, then more slowly, and when it rises above that of the battery, the dynamo begins to charge the battery and supply current to the lamps and to its own field coils. Above a certain speed the current will increase extremely slowly with increase of speed. This is due to the fact that the current flowing between the brushes m_1, m_2, exerts an extremely powerful *demagnetising effect* on the auxiliary flux, the position of the brushes m_1, m_2, being such that the demagnetising effect is as great as it is possible to make it (§ 80). The ampere-turns on the armature corresponding to the main current directly oppose the field ampere-turns, and since the former increase in direct proportion to the current, while the latter increase only very slowly (the voltage across the battery terminals varying between comparatively narrow limits), it follows that the auxiliary flux will steadily decrease with increasing speed and current. It is obvious that there must be a definite limit to the main current, no matter how high the speed; this limit would be reached when the demagnetising ampere-turns on the armature due to the main current became nearly equal to the field ampere-turns, since in that case the auxiliary field would become indefinitely small, and the speed would have to increase indefinitely in order to provide the necessary short-circuit current which supplies the cross-flux.

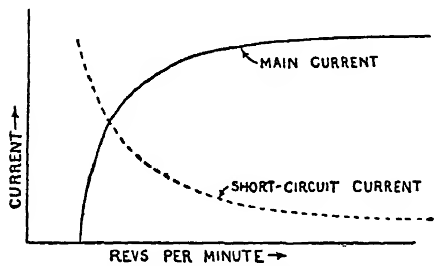


FIG. 114.—Relation connecting current with speed.

The behaviour of the main and short-circuit currents is represented graphically in Fig. 114. It will be seen that the short-circuit current steadily *decreases* with increasing speed; this

shows that, owing to the demagnetising effect of the main current, the auxiliary flux changes more rapidly than in proportion to the reciprocal of the speed. The auxiliary flux under normal working conditions is extremely weak, and does not exceed some 10 per cent. of the flux produced by the field winding when the machine is at a standstill.

The object of the deep slots in the middle of the pole-pieces, shown in Fig. 112, is to secure symmetry of the field with reference to both directions of rotation.

In order to prevent a short circuit of the battery by the armature of the machine when the train is at rest or moving at a very slow speed, a *reverse-current cut-out* is inserted. The special form of cut-out actually used for this purpose is a so-called *aluminium cell* or *rectifier* (sometimes also termed *electrolytic valve*). This consists of two metallic plates, one of which is of aluminium (or some aluminium alloy), and the other of some other metal (iron), immersed in a suitable electrolyte. So long as the aluminium is used as the cathode the current passes freely through the cell; a reversal of current, however, by coating the surface of the aluminium plate with a thin non-conducting layer of oxide, practically insulates it and stops the current. The best results have been obtained by using a solution of ammonium borate as electrolyte.†

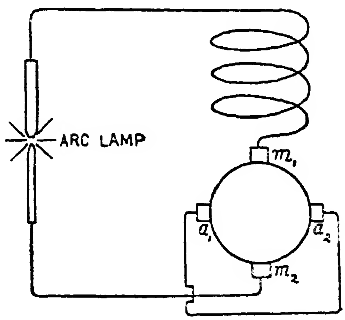


FIG. 115.—Arc lighting machine.

When a machine of this type is required for supplying a single large arc light (such as those used in search-light projectors), the arrangements are modified as shown in Fig. 115. The field now carries a series winding. The machine would in this case be driven at an approximately constant speed, and it possesses the

If the auxiliary flux varied inversely as the speed, the short-circuit current would obviously remain constant.

† Various other solutions, such as ammonium bicarbonate and ammonium phosphate, have been used as electrolytes. The ammonium borate solution is prepared by dissolving 500 grammes of crystallised boric acid in 10 litres of water, and adding 1 litre of ammonia, of density ·91.

characteristic of being able to maintain the current at an approximately constant value in spite of changes in the length of the arc. The exact shape of the characteristic curve of such a machine will depend on the ratio of the ampere-turns in the series field coil to the ampere-turns on the armature due to the main current. The former ampere-turns oppose the latter, the value of the auxiliary field depending on their difference. The larger the number of field-turns, the greater will be the auxiliary flux, and the larger the current for a given external resistance. The exact effect desired may be obtained by placing a variable shunt across the series coil, and adjusting it by trial to give the required result. In Fig. 116 are shown two characteristics, the full-line curve corresponding to the case in which the entire current is allowed to flow through the field coil, while the dotted curve shows the effect of diverting part of the current through a shunt connected across the coil. Among the great advantages connected with the use of such a machine are the possibilities of doing away with all steadying resistances (§ 166) and of bringing the carbons into contact, thereby short-circuiting the machine, without any risk of overloading it or the motor or engine employed to drive it. So far, indeed, from causing an overload, the striking of the arc absorbs less power than that required to maintain the arc when the carbons are at their normal distance apart.

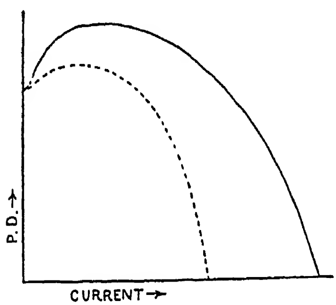


FIG. 116.—Characteristics of Rosenberg's dynamo.

§ 138. Motor-car Lighting Generators.

Numerous types of small generators for motor car lighting have been used. These are all arranged to work in conjunction with a secondary battery, the latter supplying current to the lamps when the car is at rest and the generator not running. Such generators are driven through suitable gearing by the car engine, and hence their speed varies between wide limits. The generators are required to supply an approximately constant current to the battery over a

certain range of speed. When the speed drops below a certain limit, the generator is automatically disconnected from the battery.

The main requirement in the case of such a generator is close regulation, efficiency being of secondary importance, and generally having a very low value (in the neighbourhood of 30 per cent.).

In Fig. 117 are shown diagrammatically two types of motor car lighting generators—the Brolt and the C. A. Vandervell.

In the Brolt machine, instead of a single pair of brushes at opposite ends of the neutral diameter, as in machines of ordinary construction, there are four brushes, arranged in two short-circuited pairs, the brushes of each short-circuited pair being equidistant from the neutral point. The result of this arrangement is to short-circuit the

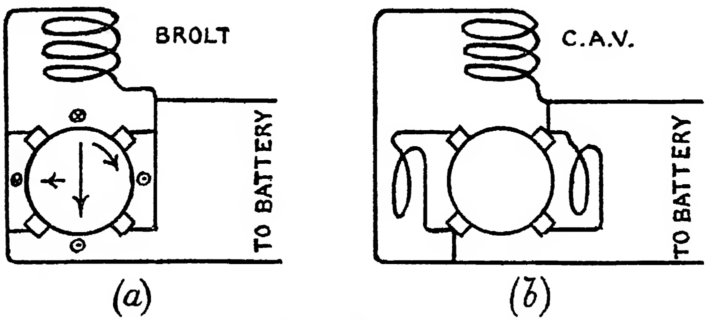


FIG. 117.—Motor car lighting generators.

armature coils in the interpolar space; a similar result could be obtained (though less conveniently from a mechanical point of view) by the use of a single pair of very thick brushes, if each brush were made to span an angular distance on the commutator surface equal to the angular distance between the brushes of each short-circuited pair. Let in Fig. 117 (*a*) the magnetic field due to the shunt winding of the generator (shown as a coiled line) have a downward direction, as indicated by the long vertical arrow, and let the armature rotate in the direction shown by the curved arrow. As soon as the armature is loaded, a cross-field will be produced having a direction from right to left, as shown by the short horizontal arrow; the armature current in the conductors under cover of the poles having the direction indicated by the small circles at the ends of the vertical diameter. In this cross-field are situated the short-circuited coils lying in the interpolar space, and the e.m.f.'s (and currents) in them have the

direction indicated by the small circles along the horizontal diameter. It will be seen that the ampere-turns arising from the short-circuited coils tend to wipe out the original field, and their de-magnetising effect increases with any increase of armature current. When the generator is giving its normal current, the demagnetising effect is such that a very slight increase of current would reduce the main magnetic flux so much as to prevent the current from increasing by more than an insignificant amount, and the machine will approximately regulate for constant current.

In the C. A. Vandervell machine, a similar arrangement of brushes is used, but the demagnetising action is intensified by providing an auxiliary set of poles (at right angles to the main poles) surrounded by a winding which short-circuits the brushes of each pair, the cross-field being thereby greatly strengthened, and with it also the demagnetising armature ampere-turns.

§ 139. Special Type of Welding Generator.

As another example of a generator of special construction, we shall consider an arc welding generator constructed by the General

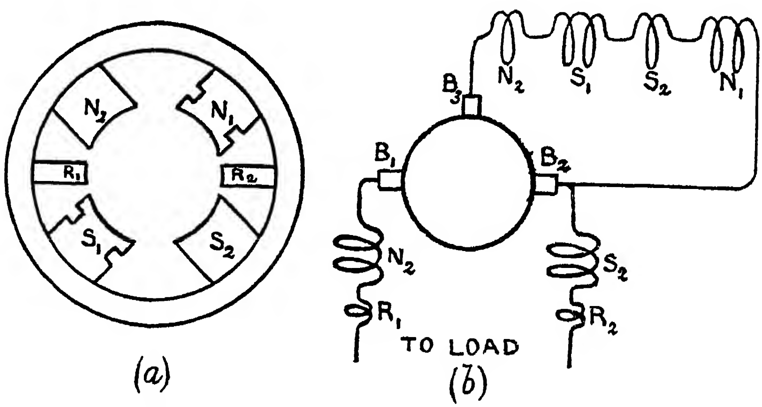


FIG. 118.—General arrangement of welding generator.

Electric Co. of the United States. A generator of this type is required to have a strongly drooping characteristic, and a machine with a pure shunt winding would not be suitable.

The machine is, magnetically considered, a two-pole one; but each pole is divided into two components, so that there are four main polar projections, as shown in Fig. 118 (*a*). Instead of a single

continuous north polar surface, we have two neighbouring north polar surfaces, provided by the poles N_2 and N_1; and similarly two neighbouring south polar surfaces, provided by S_2 and S_1. In addition to the main polar projections, there are the two reversing poles R_1 and R_2. The poles N_1 and S_1 are strongly constricted over a portion of their length, with a view to producing saturation of the constricted portion, so as to render the flux of these poles but slightly variable over a wide range of their exciting ampere-turns. The poles N_2 and S_2, on the other hand, contribute a flux which is highly variable with the ampere-turns.

The arrangement of connections is shown in Fig. 118 (*b*), where the

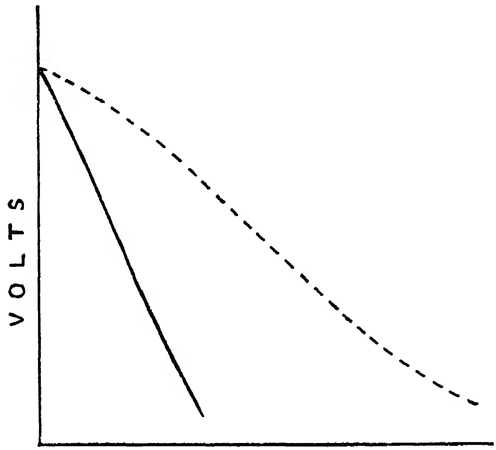


FIG. 119.—Characteristics of welding generator.

coiled lines represent the exciting coils, and are lettered so as to correspond to Fig. 118 (*a*). Besides the two main brushes B_1 and B_2, there is a third brush, B_3, half-way between B_1 and B_2. The shunt coils, indicated by the thin lines, are connected between B_2 and B_3. Only two main polar projections, N_2 and S_2, are provided with series coils (shown by the thick lines), and these are connected up so as to *oppose* the shunt coils N_2 and S_2, and when the series ampere-turns exceed a certain limit, they reverse the polarity of the poles N_2 and S_2.

The total e.m.f. developed by the armature may be regarded as made up of two components, one of which is contributed by the flux due to N_1 S_1, and the other by the flux due to N_2 S_2. Now,

owing to the strong saturation of N_1 S_1 over their constricted portions, the flux contributed by them will drop slowly with decrease in the shunt current, so that the e.m.f. component due to N_1 S_1 may be regarded as roughly constant over a wide range of p.d. On the other hand, the component due to N_2 S_2 will decrease rapidly with increasing load current, partly owing to the increase in the demagnetising series ampere-turns, partly to the decrease in the shunt ampere-turns. As already mentioned, if the series ampere-turns exceed a certain limit, the e.m.f. arising from the flux due to N_2 S_2 will change sign, becoming a counter e.m.f. The total armature e.m.f. will thus decrease rapidly with increase of load current.

Arrangements are provided for varying the number of series turns, and by this means the steepness of the drooping characteristic may be controlled. In Fig. 119 are shown two characteristics, the full line corresponding to the total number of series turns, while the dotted one represents the characteristic obtained by using only a fraction of the series turns.

§ 140. Homopolar Generator.

An interesting type of machine which has been used to a slight extent is the so-called *homopolar* or *unipolar* dynamo. The most striking feature of this type of generator is the absence of a commutator, the e.m.f. induced in the armature conductors always having the same direction. It is evident that this latter result can only be secured by the use of a magnetic field whose direction with respect to the plane containing the conductor and its velocity of motion remains unaltered. Such a field could be obtained by arranging a solid cylindrical core inside a hollow cylindrical core, a portion of the annular space between the two

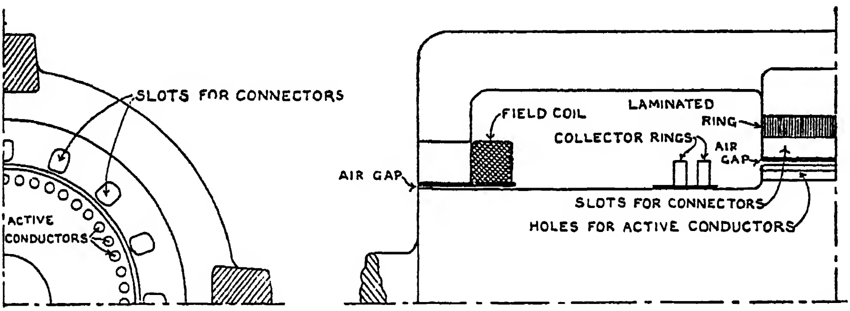


FIG. 120.—Homopolar generator.

cores being occupied by the exciting coil, and another portion forming the space in which the radial magnetic field is produced, across which the conductors are made to move. As in an ordinary dynamo, the conductors are most conveniently mounted on the inner core. A series connection of the conductors is obtained by the use of connection or collector rings or slip-rings, and connection between the brushes bearing on the slip-rings is effected by means of stationary connectors in which no e.m.f.'s are induced.

Fig. 120 illustrates the general arrangement of a dynamo of this type constructed in 1906 by the General Electric Co. of America, the machine being designed for an output of 2,000 kilowatts at 260 volts and 1,200 r.p.m. Both the field and the armature cores are of solid steel, there being no

owing to the strong saturation of N_1 S_1 over their constricted portions, the flux contributed by them will drop slowly with decrease in the shunt current, so that the e.m.f. component due to N_1 S_1 may be regarded as roughly constant over a wide range of p.d. On the other hand, the component due to N_2 S_2 will decrease rapidly with increasing load current, partly owing to the increase in the demagnetising series ampere-turns, partly to the decrease in the shunt ampere-turns. As already mentioned, if the series ampere-turns exceed a certain limit, the e.m.f. arising from the flux due to N_2 S_2 will change sign, becoming a counter e.m.f. The total armature e.m.f. will thus decrease rapidly with increase of load current.

Arrangements are provided for varying the number of series turns, and by this means the steepness of the drooping characteristic may be controlled. In Fig. 119 are shown two characteristics, the full line corresponding to the total number of series turns, while the dotted one represents the characteristic obtained by using only a fraction of the series turns.

CHAPTER XIV.

§ **141.** Secondary cells and their advantages—§ **142.** Chemical changes during charge and discharge—§ **142a.** Féry's theory of the reactions in a lead cell —§ **143.** Physical changes during charge and discharge—§ **144.** Capacity of a secondary cell—§ **145.** Defects of secondary cells. Types of plates — § **146.** Charging with constant current and at constant p.d.— § **147.** Testing of secondary cells; Determination of capacity, efficiency, and resistance—§ **148.** Rules for maintaining secondary battery in efficient working condition—§ **149.** Treatment of badly sulphated cells—§ **150.** Accumulator room. Containing boxes and stands for cells—§ **151.** Weight and cost of secondary cells—§ **152.** The Edison nickel-iron alkaline cell—§ **153.** Buffer battery— § **154.** Use of buffer battery with shunt-wound generators—§ **155.** Use of buffer battery with compound-wound generator. Reversible automatic booster —§ **156.** Pirani booster—§ **157.** Highfield booster—§ **158.** Lancashire booster— § **159.** Entz carbon controller for reversible booster—Examples.

§ 141. Secondary Cells and their Advantages.

In connection with almost every continuous current generating station, *accumulators* or *secondary cells* are used. Such cells provide a convenient reservoir of energy, which may be made to serve a variety of useful purposes, thereby offering a number of advantages, among which the more important are: (1) the possibility of entirely shutting down the plant, and so saving running expenses, during the hours when the demand for current is very small *; (2) the possibility of reducing the size and cost of the generators by allowing the accumulators to discharge in parallel with them during the period of heaviest load; (3) the possibility of maintaining the continuity of supply in spite of a temporary complete shut-down of the generating machinery caused by some sudden breakdown.

By a *secondary cell, storage cell,* or *accumulator,* is meant an appliance which enables us to effect the transformation of electrical into chemical energy, and the subsequent re-transformation of the chemical energy into electrical energy. Although there are endless combinations which may be made to perform this double transformation, commercially only two types of cell are of importance, viz., the lead secondary cell with an acid electrolyte, and the nickel-iron cell with an alkaline electrolyte. The *lead* secondary cell consists essentially of two sets of lead

* This is only possible in the case of small plants.

plates coated with "active material" and immersed in a solution of sulphuric acid in water. The active material which forms the coating of the plates undergoes cyclical chemical changes the nature of which is explained below.

The important part played by the lead secondary cell in modern continuous current engineering practice is due mainly to the following important characteristics possessed by this type of cell: (1) its extremely low resistance; (2) the remarkable constancy of its e.m.f. (about two volts) during the greater portion of the discharge; (3) reasonable efficiency (about 75 per cent. under average favourable conditions of working).

§ 142. Chemical Changes During Charge and Discharge.

In a secondary cell which has been discharged, the "active material" on both sets of plates consists largely of lead sulphate. This sulphate is not the ordinary white sulphate represented by the formula $PbSO_4$, a substance which is practically non-conducting and which cannot be readily split up by electrolytic means—but some more complicated compound containing a larger proportion of lead than corresponds to the above formula. Since, however, the exact composition of this compound is unknown, we shall use the simple formula $PbSO_4$ to denote it in representing by means of chemical equations the main chemical changes which take place during the process of charging.

The plates by which the charging current enters the cell, and which therefore form the anode during the charge, are termed the *positive plates;* the other plates, which during charge form the cathode, are termed the *negative plates.* The electrolyte consists of sulphuric acid, and is split up by the current into its ions, H_2 and SO_4, the latter appearing at the anode, and the former at the cathode. The following reactions then take place:—

(1) At the positive plates, $SO_4 + PbSO_4 + 2\,H_2O = PbO_2 + 2\,H_2SO_4$.

(2) At the negative plates, $H_2 + PbSO_4 = Pb + H_2SO_4$.

At the end of the charge, therefore, the positive plates are coated with a layer of *lead peroxide*, PbO_2, and the negative plates with a layer of metallic lead in a fine state of division, or *spongy lead.*

If the cell is allowed to discharge, the current flows through it

in the reverse direction, the positive plates acting as cathode, and the negative ones as anode. Hence the hydrogen resulting from the electrolysis of the sulphuric acid now appears at the positive plates, and the SO_4 at the negative plates. The following reactions take place during discharge:—

(1) At the positive plates, $H_2 + PbO_2 + H_2SO_4 = PbSO_4 + 2\,H_2O$.

(2) At the negative plates, $SO_4 + Pb = PbSO_4$.

The above equations must be regarded as merely conveying a very rough idea of the main reactions which take place in a secondary cell.

As the charge is nearing completion, the amount of $PbSO_4$ to be oxidised on the positive plates, and the amount of $PbSO_4$ to be reduced at the negative plates, become so small that they are insufficient to absorb all the ions liberated at the plates. Hydrogen begins accordingly to be given off at the negative plate and oxygen at the positive plate, bubbles of gas rising to the surface and then bursting, with the production of spray. This free evolution of hydrogen and oxygen is spoken of as the *gassing* of the cell, and is an indication of the completion of the charge. The production of the spray is frequently a troublesome matter, as the spray is strongly acid, and attacks any metal-work (other than lead) in its neighbourhood. Hence all connecting cables, coupling-bolts between consecutive cells (if not made of lead), &c., must be carefully protected against corrosion by the acid spray by coatings of anti-sulphuric enamel, vaseline or other suitable substances not attacked by sulphuric acid. The amount of spray produced may be reduced by placing thin glass plates in an inclined position on the top of the lead plates in the cell. These glass plates are partly immersed in the electrolyte, so that the bubbles rising through the liquid on reaching the under surface of the glass plate have to travel slowly upwards, coalescing and forming larger bubbles, and thereby reducing the amount of spray produced when the surface of the liquid is reached.

§ 142a. Féry's Theory of the Reactions in a Lead Cell.

Although the account just given of the chemical reactions taking place in a secondary cell is that generally accepted, its correctness has

R 2

been questioned by C. Féry,* whose investigations appear to throw doubt on the view that the material on a freshly charged positive plate consists of PbO_2. The following facts are regarded by Féry as affording definite evidence that the material on a charged positive plate is a higher oxide of lead than PbO_2: (1) the colour of the material is a very dark chocolate, almost black, and not red like that of PbO_2; (2) if some of the material is placed in a porous pot around a platinum electrode, it gives an e.m.f. of nearly 2·5 volts against a zinc electrode, whereas if chemically prepared PbO_2 is used in the same way, the e.m.f. is only ·7 volt; (3) chemical analysis also shows that the material contains a higher percentage of oxygen than PbO_2. Féry expresses the opinion that the composition of the active material on a charged positive plate is most probably represented by Pb_2O_5, and that during discharge this is reduced to PbO_2 (there being no formation of $PbSO_4$ on the *positive* plate during normal working). According to this view, the chemical changes at the positive plate would be represented by

$$2PbO_2 + SO_4 + H_2O = Pb_2O_5 + H_2SO_4$$

during charge, and by

$$Pb_2O_5 + H_2 = 2PbO_2 + H_2O$$

during discharge.

Later investigations by C. Féry and C. Chéneveau † indicate that besides the main reactions corresponding to the above equations, certain secondary actions occur. Féry's theory has been unfavourably criticised by L. Jumau, J. Grennell, and E. Denina. ‡

§ 143. Physical Changes during Charge and Discharge.

Corresponding to the chemical changes taking place in the cell we have certain characteristic physical changes going on

* *La Lumière Électrique*, vol. 34, p. 305 (1916); *Revue Générale de l'Électricité*, vol. 5, p. 627 (1919).

† *Comptes rendus*, vol. 181, p. 606 (1925).

‡ *Revue Générale de l'Électricité*, vol. 20, pp. 235–241 (1926).

during the process of charge and discharge. One of these consists in the periodic fluctuations in the density of the electrolyte. The reason of these fluctuations will be at once evident from the chemical equations given in § 142, from which it will be seen that during charge there is production of sulphuric acid at both sets of plates, the SO_4 groups which become detached from the plates entering the electrolyte as H_2SO_4. Thus the density of the acid rises during charge. Similarly, it will be seen that the density falls during discharge, some of the acid being withdrawn from the solution to form $PbSO_4$ on the plates It is evident that the upper and lower limits for the density will depend on the total amount of acid present. The volume of the acid relatively to the size and number of the plates varies to some extent in different types of cell, but 1·190 and 1·210 may be taken as representative values for the density at the end of a discharge and charge respectively.

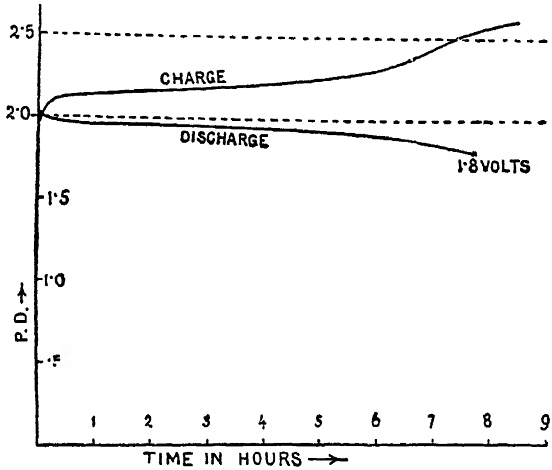


FIG. 122.—Charge and discharge curves.

Closely connected with changes of density are the changes taking place in the e.m.f. and p.d. of a cell. It has been shown by Gladstone and Hibbert* that the e.m.f. yielded by a given set of plates depends on the density of the acid, and that it increases with increase of density. From this it follows that the p.d. of a cell will rise and fall with the density of the acid. Hence there should be a progressive increase of p.d. during charge, and a progressive decrease during discharge. That such is actually the case will be seen by reference to the curves in Fig. 122,

* *Journal of the Institution of Electrical Engineers*, vol. xxi., p. 412 (1892).

which represent the typical behaviour of a cell during charge and discharge.

If the electrolyte were always maintained at a uniform density throughout, then since the mean density during charge is the same as that during discharge, we should expect the mean e.m.f. to be about the same during charge as during discharge, so that the mean p.d. during charge would exceed that during discharge by only the small amount corresponding to twice the resistance drop in the cell. Such, however, is not the case, and the average e.m.f. is found to be considerably higher during charge than during discharge—in spite of the fact that the average density of the electrolyte is the same in both cases. This result is easily explained by the fact that the density does not remain uniform, there being considerable local variations of density in the neighbourhood of the plates. During charge, acid is produced (§ 142) at both sets of plates, and hence the layer of acid in contact with the plates is of higher density than the bulk of the electrolyte—causing the e.m.f. to be higher than that corresponding to the mean density of the entire mass of electrolyte. Similarly, during discharge, acid is absorbed at the plates, and the layers in immediate contact with them become impoverished in acid and lower the e.m.f. These *local* changes in the density also account for other observed facts. Thus, during the initial stages of the charge the p.d. is found to rise comparatively rapidly (Fig. 122): this is due to the rapid formation of a layer of denser acid around the plates; when a certain density has been reached by this layer, however, convection and diffusion come into play, and prevent any further *rapid* rise in the density—the e.m.f. then rising slowly. Again, it is observed that when the circuit is broken on the completion of the charge, there is a very rapid drop in the e.m.f., even if the cell is allowed to remain an open circuit: this drop is accounted for by the rapid diffusion of the very dense acid covering the plates into the less dense surrounding portions of the electrolyte.

When a cell begins to gas, indicating the completion of the charge, its p.d. may be from 2·5 to 2·7 volts. A reference to the charge curve of Fig. 127 shows that shortly before the completion of the charge the p.d. begins to rise much more rapidly than it did during the greater portion of the charge. This increase in

the rate of rise is due to the change in the nature of the chemical reactions taking place in the cell—the oxidation and reduction of the active materials giving place to the liberation of oxygen and hydrogen.

The discharge of a cell must be regarded as having been completed when the p.d. has fallen to 1·8 *volts.* Any attempt to discharge the cell below this limit causes a very rapid further drop in the p.d., and results in permanent damage to the cell. This point is of extreme importance, and should be carefully borne in mind. Even when the p.d. has dropped to 1·8 volts, there is a large amount of peroxide remaining on the positive plates, and a large amount of spongy lead on the negative plates. But if the discharge be continued below this limit, a non-conducting layer of the white sulphate of lead is formed on the plates which is very difficult to get rid of. The formation of this non-conducting sulphate, which resists electrolytic action, is technically known as the "sulphating" of the cell. Sulphating also occurs if cells are allowed to stand inactive for any considerable period of time, and takes place very rapidly if the cells are left in a discharged condition.

Another noticeable physical change which takes place in the cell during charge and discharge is that in the *colour* of the plates. In a discharged cell, the positive or peroxide plates are of a reddish brown colour, and the negative of a whitish grey. As the charge proceeds, the colour of the positive plates deepens to a dark rich chocolate-brown, while the negative plates lose their whitish colour and assume a deeper grey having a metallic lustre. Although the colour is not always a reliable guide, yet as a rule the state of a cell as regards the amount of charge contained in it may to some extent be judged by the colour of its plates.

§ 144. Capacity of Secondary Cell.

By the *capacity* of a secondary cell is meant the number of ampere-hours which it is capable of supplying during a single discharge under given conditions. The capacity of a cell is a highly variable quantity, depending not only on the rate of discharge—*i.e.*, the discharging current—but also on the temperature and the previous history of the cell. The capacity

decreases somewhat rapidly with increasing discharge current, as will be seen from Fig. 123, which shows the relation connecting these two quantities. Temperature has a very marked effect on the capacity, which *increases rapidly* with rise of temperature.

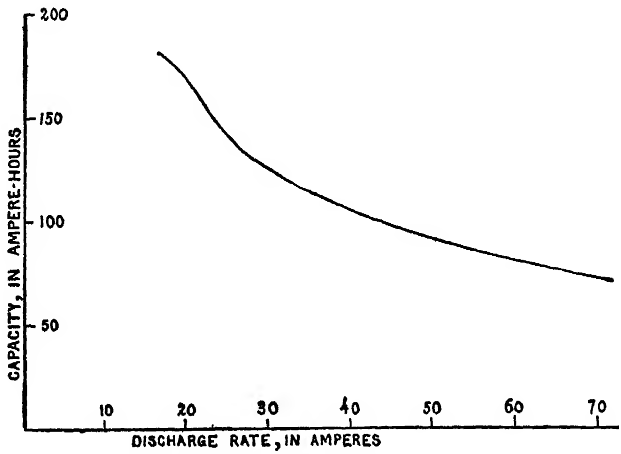


FIG. 123.—Relation connecting capacity with rate of discharge.

Thus, in a case investigated by Heim*, a rise of about 30° C. in temperature increased the capacity by 80 per cent.

The capacity of a cell steadily decreases in course of time. Rapidly decreasing capacity under normal working conditions is a sure indication that the battery is deteriorating, and that it requires careful examination.

§ 145. Defects of Secondary Cells. Types of Plates.

The great variety of constructions to be found in different types of lead secondary cells is due to attempts to deal with the various difficulties which have been experienced in the practical working of such cells. These difficulties may be briefly enumerated as follows: buckling of the plates; loss of active material by gradual disintegration of the plate, the material becoming detached from its metallic support and dropping to the bottom of the cell; and internal short-circuiting of the cell either by direct

* *Elektrotechnische Zeitschrift*, vol. xxii., p. 811 (1901); see also Hibbert, *Electrical Review*, vol. l., p. 883 (1902).

contact of the plates brought about by buckling, or by the formation of conducting bridges consisting of particles of active material which have become wedged between the plates.

It is usual to divide secondary cell plates into two great classes. To the first class belong plates of the *Planté* type. In this type of plate, the layer of active material is formed from the plate

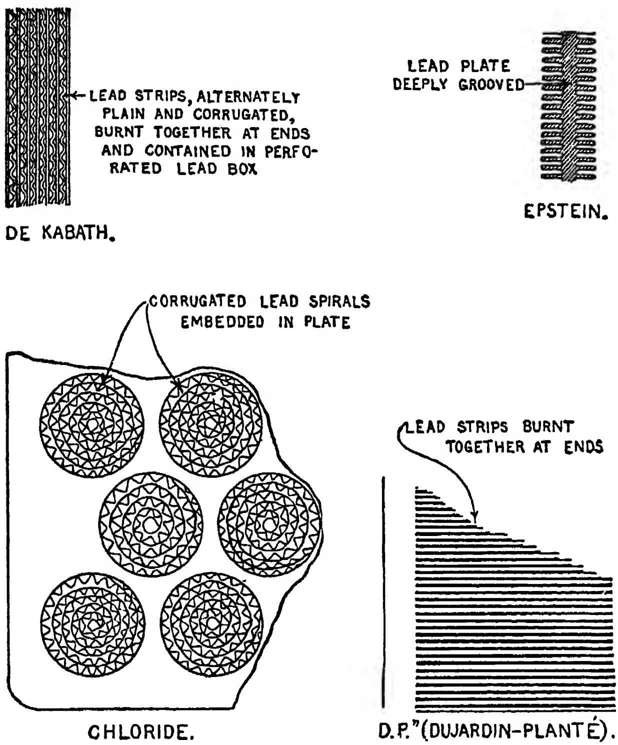


FIG. 124.—Types of Planté plates.

itself. In the original Planté cell two plain plates of lead were taken, and the necessary depth of active material was obtained by repeated charges and discharges in opposite directions with intervening periods of rest, it having been found that the depth

Gaston Planté was the inventor of the lead secondary cell; he carried out numerous researches with metals other than lead, and was finally led to adopt this latter metal as by far the most suitable for a secondary cell.

of the active layer was thereby gradually increased. This procedure, known as the *forming* of the plate, is costly, and is frequently modified nowadays. The initial production of the active layer on a Planté plate may be accomplished partly by chemical means,* and then completed electrolytically during the first charge, when the plate is said to be *formed*.† The actual depth of active material on a Planté plate is in all cases relatively small, and the desired capacity is obtained by increasing the effective *area* of the plate rather than the depth of the active layer. To this end, the surface of the plate is deeply grooved,

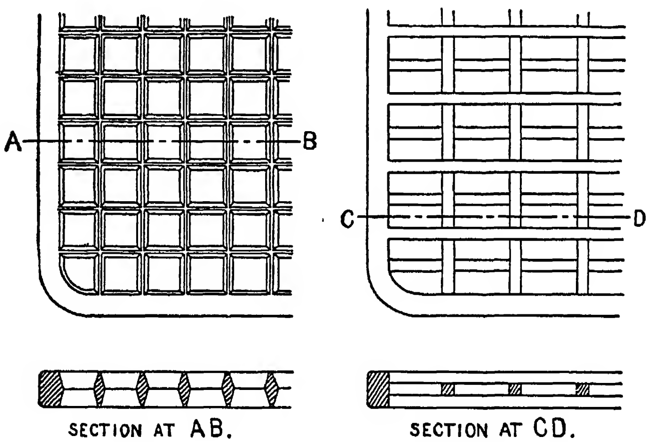


FIG. 125.—Types of grid for pasted plates.

or the plate is built up of thin strips or wires of lead, which give a large area of contact with the electrolyte as compared with the apparent area of the plate. Some of these modes of construction are illustrated in Fig. 124.

The second type of plate is the *pasted* or *Faure* plate. In this, the active material forms the bulk of the plate, the metal portion of which usually consists of a very light open framework, in the form of a more or less complicated *grid*, the openings of which

* The surface of the plate may be attacked by boiling it in dilute nitric acid and allowing it to dry.

† By the addition of sodium nitrate (or nitrite) to the electrolyte, a considerable depth of active material may be obtained in a short time without any preliminary chemical treatment of the plate. All traces of nitric acid must be carefully removed from the plate before it is used in the cell.

are completely filled with the active material. The latter is prepared by mixing litharge with dilute sulphuric acid to form a paste for the negative grids, red lead being similarly used in connection with the positive grids. The acid used in the preparation of the pastes has a density between 1·15 and 1·20. When the paste is spread over the grid, it sets hard after a time. During the forming process the red lead is oxidised to lead peroxide, and the litharge is reduced to spongy lead.

The active materials on the two plates do not behave in the same way, that on the positive plate tending to expand in course of time, while that on the negative plate tends to contract. For this reason, the positive grids are frequently made different from the negative ones. Two different forms of grid are illustrated in Fig. 125. An enormous number of different kinds of grids have been tried at different times, and almost every conceivable arrangement has been used. The principal aim has in all cases been to prevent the active material from coming away from its metallic support.

The number of negative plates in a cell is always greater by one than the number of positive plates. In the earlier types of cell, equal numbers of positive and negative plates were used, so that one of the outside plates was positive and the other negative. It was soon found, however, that owing to the unequal action on the two sides of the outside positive plate (the active material undergoing much greater expansion on the side facing the negative plate), this plate buckled very badly. By adding an extra negative plate, the action was equalised on both sides, and the trouble disappeared. The *total* number of plates (positive and negative) in a secondary cell is therefore always an *odd* number, the two outside plates being both negative.

§ 146. Charging with Constant Current and at Constant P.D.

Two methods are in use for charging secondary cells. In the one commonly used in Great Britain, the cells are charged with a *constant current*. Since, as we have seen, the e.m.f. of the cells steadily rises during charge, arrangements must be provided for gradually increasing the voltage of the charging circuit if the current is to be maintained constant. The charging is invariably

effected by means of a shunt-wound dynamo (§ 99), and if the dynamo is not supplying any other circuits at the same time, its voltage may be gradually raised as the charge proceeds by means of a field rheostat (the dynamo must, of course, be designed to allow of the required increase of e.m.f.). More commonly, however, the increase of voltage is obtained by the aid of a small auxiliary dynamo known as a *booster*. The armature of the booster is designed for the full charging current, and for a voltage corresponding to the rise from the normal e.m.f. of the cells during discharge (about two volts per cell) to the final p.d. towards the end of the charge (about 2·7 volts per cell). The connections of the main generator, battery and booster are shown in Fig. 126. It will be noticed that by the use of a booster the main generator may be run at a constant voltage, or that its voltage may be regulated quite independently of the p.d. required to maintain the charging current, the additional p.d. being always provided by the booster. Thus the main generator may be employed for supplying other circuits while charging the battery.

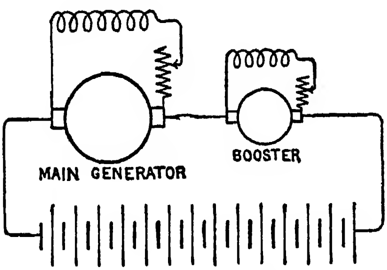


FIG. 126.—Connections of main generator and booster.

The second method of charging, which is practised a good deal on the Continent, and which came into use later than the constant current method, is that in which the charging p.d. is maintained constant at about 2·4 volts. With this method of charging there is an enormous initial rush of current, and the bulk of the charge is effected in a very short time. The current falls off rapidly, and the final stages of the charge take much longer than its initial stages, but the entire duration of the charge is much shorter (by about 50 per cent.) than with the constant current method. The method of charging at constant p.d. results in a larger capacity from a given size of cell, but leads to a lowering of the efficiency.*

* See a paper by Messrs. Cahen and Donaldson in *The Electrician*, vol. xli., p. 674 (1898).

§ 147. Testing of Secondary Cells. Determination of Capacity, Efficiency and Resistance.

In investigating the behaviour of a secondary cell, the more important quantities of which measurements have to be carried out are: (1) the capacity of the cell; (2) its efficiency; (3) its resistance. The *durability*, which determines the cost of maintenance, is an extremely important feature, but one regarding which a reliable judgment can only be formed from actual experience; the durability depends not only on the care bestowed on the battery, and on the nature of the load with which it has to deal, but also on the local climatic conditions of the place where it is installed.

For the purpose of determining the capacity, for a given rate of discharge (§ 144), and the efficiency of a secondary cell, it is not sufficient to take a single charge and discharge as a basis for this determination. This is due to the fact that the cell is never entirely exhausted—*i.e.*, deprived of *all* the energy stored up in it—but only discharged down to a p.d. of 1·8 volts. Even in a discharged cell there is still a certain amount of energy, and a considerable amount of active material on both plates which has not been converted into sulphate. Now the behaviour of a cell on charge or discharge depends on its previous history—*i.e.*, on the nature of the preceding charges and discharges—and we cannot assume that at the end of a given discharge the condition of the cell is precisely the same as it was at the beginning of the immediately preceding charge. In order to make sure of this point, it is necessary to take a number of successive charges and discharges, carried out under precisely similar conditions, and to compare the curves connecting voltage with time for the consecutive charges and discharges. If these curves are found to be exactly repeated each time, we are justified in concluding that the cell has reached a truly *cyclic state*, and that at the end of any discharge it is left in precisely the same state as it was at the beginning of the preceding charge. But if the curves show some modification during each succeeding charge and discharge, indicating that there is some sort of progressive change going on in the cell, the charges and discharges must be continued until this change practically disappears, and each cycle of charge and discharge becomes similar to the preceding one.

In testing a cell for capacity and efficiency, therefore, a sufficient number of charges and discharges must be carried out to reduce the cell to a strictly cyclic state, and only then can any legitimate conclusions be based on the results of the experiments. Such tests are therefore necessarily very tedious. The readings to be taken are those connecting time with p.d. and current (readings should also be taken of the density of the electrolyte). The current would always be maintained constant during a discharge, and the capacity corresponding to the given current is equal to the product of the current into the time of discharge. The capacity is generally expressed in ampere-hours.

It is usual to distinguish two kinds of efficiency, viz., the *ampere-hour efficiency* and the *watt-hour efficiency*. By the former is meant the ratio of the ampere-hours of discharge to the ampere-hours of charge, and by the latter the ratio of the watt-hours of discharge to the watt-hours of charge. When the cell is being tested under constant-potential conditions of charge (§ 146), the current is variable, and the ampere-hours of charge are given by the area of the curve connecting current with time. The ampere-hour efficiency of a cell is generally very high, exceeding 90 per cent. In order to determine the watt-hours of discharge, we multiply the current by the area of the discharge curve (*i.e.*, the curve connecting discharge p.d. with time, Fig. 122). If charging is carried out at constant current, the watt-hours of charge are determined in a precisely similar manner. But when charging at constant p.d., the watt-hours of charge are given by the product of the p.d. into the ampere-hours of charge (which are represented by the area of the curve connecting charging current with time). The watt-hour efficiency varies with different rates of discharge, but is generally of the order of 70 to 85 per cent.

The resistance of a secondary cell may be measured by determining the difference between the e.m.f. and the p.d., and dividing this difference by the current. Since this difference is very small for all ordinary rates of discharge, it could not be obtained with any high degree of accuracy by simply taking the readings of a voltmeter connected across the cell (1) on open circuit and (2) with the current passing through the cell, although approximate values might be obtained in this way. More accurate determinations may be made by using the arrangement

shown in Fig. 127. A compensating circuit, consisting of a couple of cells supplying current to a variable resistance and a resistance intermediate points of which are accessible, is arranged so that the p.d. across the terminals of the cell under test when traversed by a current may be balanced against the drop of potential over a section of the resistance in the compensating circuit. The balance may be adjusted by the aid of a milli-voltmeter (or calibrated dead-beat moving-coil galvanometer) whose resistance is very high in comparison with the resistance in the compensating circuit. Balance having been obtained, the circuit of the cell under test is suddenly opened, and the reading of the milli-voltmeter immediately on breaking the circuit is obtained. This reading divided by the current gives the resistance of the cell.

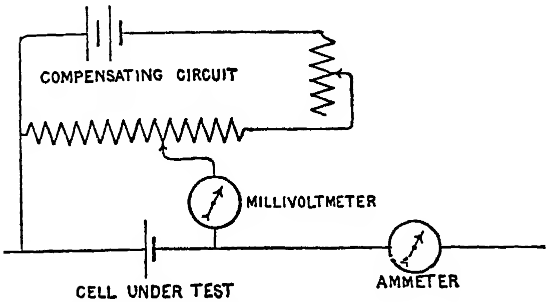


FIG. 127.—Measurement of resistance of secondary cell.

The resistance of a cell is found to increase greatly towards the end of a charge or discharge. With the usual types of cell, the resistance is of the order of 1·5 ohm per square inch of total area of positive plates.

§ 148. Rules for maintaining Secondary Battery in Efficient Working Condition.

The proper working and cost of maintenance of a secondary battery are so greatly dependent on the care with which it is handled that it is a matter of extreme importance to adhere closely to the following rules, the strict observance of which practical experience has shown to be essential to the satisfactory working of the battery.

The battery should never be discharged below the 1·8 volt limit of p.d.* In connection with this rule, it is important to

* This limit applies to the normal discharge rate. In the case of cells intended for very high rates of discharge, the limit is (owing to the larger resistance drop in the cell) lower, and its value for various discharge rates is generally given by the makers.

observe that it is the p.d. (when the battery is still discharging), and not the e.m.f., which is referred to.

The battery should be charged as soon as possible after each discharge. If a battery is left standing without re-charge after having been discharged the non-conducting white sulphate is gradually formed on the plates; as already mentioned (§ 142), this sulphate is extremely difficult to get rid off by electrolytic action.

Should it be necessary to leave the battery standing idle for any considerable length of time, the last charge preceding the period of rest should be continued until all the cells gas freely. If possible, a short charge (until gassing occurs) should be given to the battery about once a fortnight.

Should the period during which the battery is to remain inactive extend over many months, it is best to empty the cells and take out the plates, proceeding as follows:—The battery having been fully charged, the electrolyte is drawn off. The cells are then immediately filled with pure water, and the plates allowed to soak for about 12 hours. They may then be removed, dried, and stored away.

The level of the electrolyte should be kept not less than half an inch above the tops of the plates. Owing to the evaporation of water which takes place, it is necessary to add *water* to the electrolyte from time to time in order to maintain the required level. *Distilled water* should be used for this purpose, and not tap-water. Neither rain-water nor condensed water from boilers is suitable.

The maximum normal discharge rate as stated by the makers should not be exceeded. Frequent heavy overloads not only greatly reduce the capacity of the battery, but result in permanent damage to it and shorten its life. One very serious effect of excessive discharges is a loosening of the contact between the active material and its support, the material dropping away from the plate and collecting at the bottom of the cell.

It is generally found that the cells forming a battery do not all behave in the same way, some being better than others. For this reason, each individual cell should be examined about once a week, its p.d. while discharging being measured by a suitable low-reading voltmeter, and the density of the electrolyte being determined by a hydrometer. If it is found that any cell is

much weaker than the others, it should be treated with special care until it is brought up to their level. Two methods of treatment are available. Either the weak cell is cut out of circuit during the period of discharge, so that, while receiving the same amount of charge as the others, it is not allowed to discharge, and so gradually gains on the others; or, while allowed to discharge with the others, it receives an additional amount of charge. This additional charge may be given to the weak cell either by means of a couple of auxiliary cells specially kept for the purpose, and known as *milking* or *hospital cells*, or by means of a small low-voltage "milking" booster driven by a motor. The milking cells (or booster) are provided with a couple of flexible leads ending in special terminals for connection to the weak cell, and of sufficient length to reach across to any cell of the battery.

The weak condition of a cell is sometimes due to *internal short-circuiting*. If on account of excessive discharges or for some other reason particles of active material become detached from the plates, and if there are no barriers between the plates, such particles, if large enough, may lodge between two neighbouring plates instead of dropping to the bottom of the cell. They then form a more or less perfect solid conducting bridge across the plates, which has the same effect as if the circuit of the cell were closed through an external resistance. A cell so short-circuited will therefore go on discharging even if the external circuit be open, and will gradually fall behind the others. In examining the cells, such internal short-circuits should carefully be looked for, and if any particles are seen bridging across the plates, they should be pushed down by means of a clean rod or strip of glass.

In order to keep a check on the excess of the ampere-hours put into the cells during the consecutive charges over those taken out during the discharges, it is useful to provide two ampere-hour meters, one in the charging and the other in the discharging circuit. The ratio of the readings on the discharge and charge instruments gives the average ampere-hour efficiency; this may be as high as 95 per cent. if the cells are charged and discharged

Thin diaphragms or separators of specially prepared wood supported between the plates for the purpose of preventing internal short-circuiting have been used by some makers.

daily, but with longer intervals between the consecutive charges it will drop to 90 per cent. or even less.

§ 149. Treatment of Badly Sulphated Cells.

It not infrequently happens that owing to neglect the plates of secondary cells get coated, more or less completely, with a layer of the white badly-conducting sulphate of lead. One of the most serious results of such sulphating is a large drop in the capacity of the cell. If the sulphating is not excessive, the cells may be restored to their normal capacity by a prolonged overcharge at about half the normal charging current. If this treatment should fail, the following more laborious but very effective method may be employed.

The acid having been withdrawn, the cell is filled with a solution of *pure* sodium sulphate (the commercial sulphate generally contains sodium chloride, which is extremely harmful, as the plates would be rapidly destroyed by the chlorine liberated during electrolysis). The density of the solution is not of much importance, but about 200 grammes of the hydrated sulphate (Na_2SO_4, 10 H_2O), or about 90 grammes of the anhydrous salt (Na_2SO_4) may be used per litre (equivalent to about 2 lbs. and ·88 lb. respectively per gallon). The cell is then charged at the normal rate until the sulphate is reduced. This takes not more than 60 hours, and the removal of the sulphate may be judged by the appearance of the plates. The sodium sulphate solution is then drawn off, and distilled water is run in in order to wash away the remainder of the sodium sulphate. It is not important that the washing should be thorough, as any traces of sodium sulphate remaining in the plates are beneficial rather than otherwise. Finally, the water is withdrawn and the original acid put back into the cell.

§ 150. Accumulator Room. Containing Boxes and Stands for Cells.

Owing to the extreme importance of keeping the electrolyte as free from impurities as possible, the cells should be placed in a position where there is but little danger of contamination of the

The Electrician, vol. 67, p. 830 (1911).

acid. Ammonia has been found to be a very dangerous impurity. Hence the site of the accumulator room should never be chosen in the neighbourhood of stables, etc. The purer the atmosphere, in fact, the better for the cells.

The accumulator room should be large enough to allow of easy access to every cell. It should have a concrete floor, covered with a layer of asphalte, and the floor should be given a sufficient slope towards a drain, so that any acid or water on the floor will immediately drain off. If a tiled floor is used, the tiles should be set in pitch, not cement (the latter absorbing acid). Good ventilation should be provided by means of a blower forcing fresh air into the accumulator room, and driving out the acid spray through suitable outlets.

The containing boxes for secondary cells are made of glass, antimonious lead, or wood lined with lead. Glass boxes are used for smaller, and lead-lined wood boxes for larger, cells. Portable and motor-car ignition cells are generally contained in boxes of celluloid or ebonite.

The stands supporting the cells are of pitch-pine, and are constructed without the use of nails or other metal fastenings, which would be attacked by the acid spray. In cases where the battery voltage is high, the stands should be insulated from the floor by porcelain insulators. Acid-resisting paint or varnish is used to protect the stands from the acid.

Owing to the fact that the surfaces of the containing-boxes are always more or less moist, it becomes necessary to support each cell on suitable insulators. A common form of battery insulator is shown in Fig. 128. The insulating surface here consists of an oil surface, the oil being contained in the annular trough formed in the lower part of the insulator.

In the case of glass containing boxes, the bottom of the box is too uneven to be conveniently supported on the insulators, as the concentration of the entire weight of the cell on the comparatively small surface of contact between the boxes and the insulators would unduly strain the former, and might cause fracture. For this reason, the boxes are generally supported on a layer of

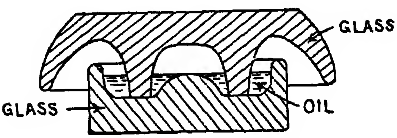


Fig. 128.—Oil insulator.

Transactions of the Faraday Society, vol. 1, p. 271 (1905).

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sawdust contained in a varnished tray of wood which rests on the insulators.

Although the accumulator room should be well lighted in order to facilitate inspection of the cells, it should be shielded from the full glare of the sun, as this is liable to crack glass boxes and to cause excessive evaporation.

§ 151. Weight and Cost of Secondary Cells.

The weight of a secondary cell for a given capacity depends very largely on the type of cell, determined by the particular purpose for which the cell is to be used. As a general rule, reduction of weight can only be obtained at the cost of durability. A strong plate is also a heavy plate. In the case of stationary batteries, weight is not a very important consideration, and a heavier and more durable type of plate is used than in the case of batteries for traction purposes. Stationary batteries have a capacity of from two to three ampere-hours per lb. of total weight (acid included). Traction batteries may yield up to six ampere hours per lb. of total weight. Assuming the mean p.d. per cell during discharge to be 1·9 volts, the above results correspond to about 3·8 to 5·7 watt-hours per lb. of total weight for stationary batteries, and up to about 11·4 watt-hours per lb. of total weight for traction batteries. The weight of the electrolyte alone is generally from a quarter to one-third of the total weight of the complete cell.

The price of secondary cells is about £1 15*s*. per 100 ampere-hours, or, roughly, £8 15*s*. per kilowatt-hour.

If the cells are all arranged on the floor level, the amount of floor space (including gangways between rows of cells) may be taken at about three square feet per kilowatt-hour.

The above figures are based on the assumption of a nine hours' discharge rate. As the rate of discharge is increased, the capacity and kilowatt-hours decrease (§ 144).

§ 152. The Edison Nickel-Iron Alkaline Cell.

In this cell, the frames supporting the active material are made of nickel-steel, in order to secure the greatest possible mechanical strength and rigidity. The active material on the positive plate consists of alternate layers of nickel oxide and

flakes of metallic nickel (the latter being intended to increase the conductivity of the material). This active material is contained in a series of tubes (about 4 inches long and $\frac{1}{4}$ inch in diameter) formed of perforated nickel-steel, and bound round with eight steel bands for greater strength. Each positive plate contains thirty such tubes, which are mounted vertically in two rows arranged above one another, each row consisting of fifteen tubes. The active material on the negative plate consists of iron oxide; this is contained in rectangular boxes or pockets of perforated nickel-steel sheet; each negative frame holds twenty-four such pockets arranged in three rows, one above the other.

The electrolyte is a 20 per cent. solution of caustic potash, with a slight amount of lithium hydrate added.

The chemical reactions in such a cell may be expressed by the following equations:—

(1) Charge:

$Fe_3O_4 + 8K + 4H_2O = 3Fe + 8KOH$ (negative plate).

$2Ni_3O_4 + 8(OH) = 6NiO_2 + 4H_2O$ (positive plate).

(2) Discharge:

$3\,Fe + 8(OH) = Fe_3O_4 + 4H_2O$ (negative plate).

$6NiO_2 + 8K + 4H_2O = 2Ni_3O_4 + 8KOH$ (positive plate).

No change takes place in the density of the electrolyte during charge or discharge.

The containing case of the Edison cell is made of nickel-steel, and is fitted with an air-tight cover which is welded on to the case. The terminal rods pass through insulating stuffing-boxes in the cover. The cover is fitted with a valve cap so arranged that while it allows the cell to be filled with the electrolyte and allows any gases which may be liberated to escape, it prevents the access of air; were the latter admitted, the carbon dioxide which it contains would be absorbed by the caustic potash and would weaken the electrolyte.

The average value of the p.d. during discharge is about 1·2 volts, while the charging voltage is from 1·7 to 1·8 volts. The energy efficiency corresponding to a 7-hour charge rate is about

The initial discharge p.d. is about 1·5 volts, but within the first fifth of the discharge this drops to about 1·3 volts, and then continues dropping more slowly down to about 1 or ·9 volt, when the discharge is regarded as completed. The rate of voltage drop during discharge is much higher than in a lead cell—this is one of the disadvantages of the Edison cell.

58 per cent., and the ampere-hour efficiency about 82 per cent. The efficiency is improved by increasing the rate of charging.

The watt-hours per lb. of complete cell depend on the size of the cell, and vary from about 9 in the case of small (40 ampere-hour) cells to about 13·5 in large (450 ampere-hour) cells.

The cost of Edison cells varies from £30 to £16 per kilowatt-hour.

In view of the much higher cost and lower efficiency of the Edison cell as compared with the lead cell, it is obvious that the Edison cell must possess certain important advantages over the lead cell in order to enable it to compete with it. The following are among such advantages:—

The Edison cell is capable of giving enormously high rates of discharge, and may even be short circuited, without damage. It may be discharged until the voltage disappears, although the discharge is ordinarily stopped when the p.d. has dropped to 1 or ·9 volt. It is not damaged by overcharging, and may be left standing idle for an indefinite period even if discharged. It is mechanically very strong, and is capable of withstanding very rough handling, severe vibration and shocks. It gives off no corrosive spray. Its life is very long, so that it will outlast several lead cells. It has a slight advantage over the lead cell in point of weight.

On account of the above advantages, the Edison cell is particularly well adapted for traction work, and for use in all cases where skilled attendance is not available, and where the cell is liable to be neglected.

§ 153. Buffer Battery.

In the case of generating stations supplying a traction load, the load undergoes violent fluctuations. Secondary batteries are frequently used in such stations for the purpose of equalising the load on the generators. The battery is connected in parallel with the generator or generators, and the arrangements are such that the battery e.m.f. just balances the 'bus bar p.d. when the load corresponds to the normal load for the generators. If the load falls below this limit, the generator begins to charge the battery; if it rises above it, the battery begins to discharge, and

prevents the generator from becoming overloaded. A battery so used is termed a *buffer battery.* It is evident that a buffer battery allows of a reduction in the generating plant, the peaks of the load being taken by the battery. The advantages of a buffer battery may be briefly stated as follows: (1) the engines and generators may be kept running continuously under full load, irrespective of the external load on the system; this results in high efficiency, *i.e.*, low cost of energy; (2) the wear and tear on the machinery is reduced, owing to the absence of the severe stresses which arise with a fluctuating load; (3) the battery may be used during the hours of light load, thereby shortening the hours of running for the generators. The fact that a buffer battery allows of a reduction in the size of the generating plant may be regarded as an additional advantage, since although part of the money saved on the boilers, engines and generators must be spent on the battery, the cost of the latter, for a given output reckoned at one hour's discharge rate, is less than that of an equivalent amount of steam generating plant.

§ 154. Use of Buffer Battery with Shunt-wound Generators.

On the Continent, traction generators are generally provided with a simple shunt winding, so that the p.d. drops with increase of load. The buffer battery being connected directly across the 'bus bars, and containing a number of cells such that the battery e.m.f. just balances the generator p.d. at normal load, an increase of load above the normal results in a drop of p.d., and causes the battery to discharge in parallel with the generator; on the other hand, a decrease of load results in an increase of p.d., and causes the generator to charge the battery. It is obvious, however, that the buffer action of the battery—*i.e.*, its regulating effect on the generator load—will be more or less imperfect, since a variation of the generator load is absolutely essential in order to bring the battery into action; further, the extent of this buffer action will depend on the characteristic curve of the generator and the resistance of the battery, the buffer action being the more energetic (*i.e.*, the generator load being kept the more nearly constant), the more strongly drooping the generator characteristic. This simple arrangement of having a buffer battery directly across

the generator terminals is extensively used in Germany. It has been found that the buffer action of a battery depends very largely on the state of the battery itself.

§ 155. Use of Buffer Battery with Compound-wound Generator. Reversible Automatic Booster.

In England and the United States, traction generators are generally compound-wound, and have a rising characteristic, so that with increasing load the 'bus bar p.d. rises, and the greater drop along the *feeders*† is compensated for. If the generator were compounded for constant voltage at all loads, it is evident that a battery across the generator terminals would have absolutely no effect in regulating the load; while if—as is usually the case—the generator were over-compounded, a battery across its terminals, so far from exerting any buffer action, would intensify the fluctuations of load. It is therefore evident that in either of these two cases some auxiliary arrangement must be provided if the battery is to act as a buffer between the generator and the external load. Such an arrangement is furnished by a *reversible automatic booster*. Although an absolute necessity with compound-wound generators, a reversible booster may also be used with shunt-wound machines, in which case it greatly improves the buffer action of the battery.

By a reversible automatic booster is meant an auxiliary generator whose armature is connected in series with the buffer battery, and whose field windings are so arranged that an increase of load above the normal load of the generator produces an e.m.f. in the booster armature which is added to that of the battery, and which increases in proportion to the excess of load, thereby

According to H. Wille (*Elektrische Bahnen und Betriebe*, vol. 3, p. 9 (1905)), a battery whose buffer action has become feeble may be restored to a more sensitive condition by the following method of treatment: The battery is charged until it begins to gas freely. It is then given about two hours' rest; charging is next resumed until the battery again begins to gas freely. This is followed by another period of two hours' rest, and a further charge until gassing commences, and so on, the treatment being continued until the battery starts gassing as soon as charging commences. The battery will now be found to exert a much more powerful buffer action.

† By *feeders* are meant cables which run out from the generating station and *feed* the current into the overhead wire, whence it is supplied to the car motors. Each feeder supplies about half a mile of overhead wire.

enabling the battery to discharge and so deal with the overload; the opposite effect is produced if the load decreases below the normal, the booster e.m.f. now opposing that of the cells, and enabling the generator to charge them. A reversal of the booster e.m.f.—*i.e.*, a reversal of the field flux—must clearly take place as the external load passes through the value corresponding to the normal load on the generator; at this point, the booster field must be wiped out. The booster is driven by a shunt-wound motor.

§ 156. Pirani Booster.

A large number of different types of reversible boosters have been devised. One of the earliest is that due to Pirani. The

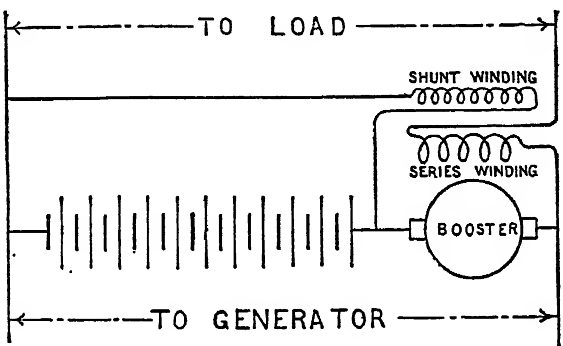

FIG. 129.—Pirani booster.

connections of the booster in their simplest form are shown in Fig. 129. The booster field is provided with two windings, a shunt winding connected across the battery, and a series winding traversed by the load current. These two windings are connected up differentially—*i.e.*, so as to oppose each other—and are adjusted to balance one another when the load current has the normal value. An increase of load causes the series winding to

* See L. Schröder [*Elektrotechnische Zeitschrift*, vol. 17, p. 805 (1896), and vol. 27, p. 252 (1906)]; Stirm [*Ibid.*, vol. 30, p. 297 (1909)]; J. S. Highfield [*Journal of the Institution of Electrical Engineers*, vol. 30, p. 1040 (1901)]; G. A. Grindle [*Ibid.*, vol. 30, p. 1098 (1901)]; C. Turnbull [*Ibid.*, vol. 36, p. 591 (1906)]; M. J. E. Tilney [*Ibid.*, vol. 36, p. 605 (1906)]; W. A. Ker [*Ibid.*, vol. 44, p. 486 (1910)]; R. Rankin [*Ibid.*, vol. 48, p. 283 (1912)]; F. Sarrat [*Bulletin de l'Association des Ingénieurs-Electriciens, Liège*, vol. 3, p. 439 (1903), and vol. 4, p. 180 (1904)]. See also *The Electrician*, vol. 53, p. 303 (1904), vol. 54, pp. 322, 438 (1904); and *Western Electrician*, vol. 37, p. 210 (1905).

preponderate, and gives rise to an e.m.f. in the booster armature which is added to that of the battery; a decrease of load producing the opposite effect. This form of booster is sometimes known as the *simple differential booster.*

A serious disadvantage of the Pirani booster in the simple form just described is the very large amount of copper in the field windings, the effective ampere-turns being represented by the difference of the ampere-turns contributed by the two windings. In order to cheapen the construction of the booster, the arrangement adopted in practice is that represented by Fig. 130. The booster field has only a single winding, which is supplied with current by a special small exciter, the differential windings being arranged on the field of this latter machine. Although there is still an abnormally large amount of copper on the field of the *exciter*, yet owing to the smallness of this machine, the total amount of copper in the two machines is less than it would be on a single machine; further, the loss in the series winding is materially reduced. The arrangement also possesses the additional advantage that, the differential windings on the exciter having been adjusted once for all to exact balance at normal load, the amount of boosting is easily controlled by the field rheostat in the booster field; whereas with a single machine both windings would have to be adjusted if it were desired to control the boosting effect without upsetting the balance at normal load. Instead of allowing the entire load current to flow through the series winding, only a fraction of it may be utilised, the remainder flowing through a suitable shunt. The booster, exciter and motor armatures are all mounted on a common shaft.

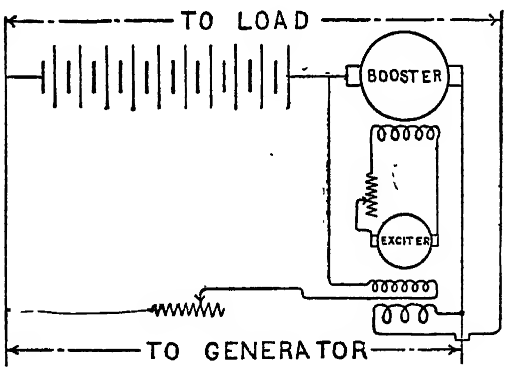

FIG. 130.—Pirani booster with separate exciter.

See end of § 159 for necessity of *laminating* booster field.

§ 157. Highfield Booster.

One of the best known and most successful forms of reversible booster is Highfield's booster. The connections of this machine are shown in Fig. 131. The booster field is provided with two windings, one of which is a series winding traversed by a suitable fraction of the load current, while the other, which is a fine-wire winding, is connected in series with an exciter and then across the battery terminals. The booster and exciter are mechanically coupled to each other and to a shunt-wound motor which drives them. The exciter, whose e.m.f. opposes that of the battery around the local circuit formed by the battery, exciter and fine-wire booster field coil, is, in addition to its shunt winding, provided with a series field coil (not shown in Fig. 131), and maintains a practically constant p.d. across its terminals. At normal load the booster e.m.f. is such that when added to the battery e.m.f. it exactly balances the generator p.d. An increase of load by increasing the current through the booster series coil raises the booster e.m.f., causing the battery to discharge. The opposite effect takes place when the load decreases.

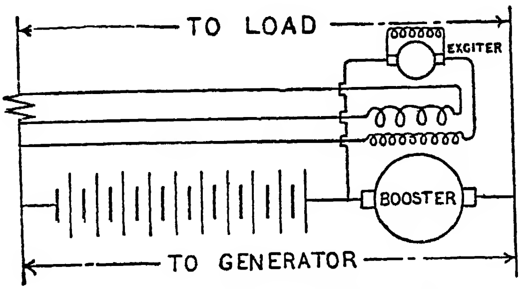

FIG. 131.—Highfield reversible booster.

The object of the special exciter is to render the battery current practically *independent of the battery e.m.f.* Let us, for instance, suppose that the battery is fully charged, and that the load is at its normal value. Then the battery e.m.f. will exactly balance the exciter e.m.f., and the fine-wire booster coil will receive no current. But if when the load is normal the battery happens not to be fully charged, its e.m.f. will be less than that of the exciter, and the latter will send a current through the fine-wire booster coil which will strengthen the effect due to the series coil, the booster e.m.f. being thereby increased by exactly

the amount corresponding to the drop in the battery e.m.f., and hence the battery will still receive no current, in spite of its lower e.m.f. A similar regulating effect occurs when the battery is either discharging or being charged, the fine-wire booster coil being so designed that any difference between the exciter and battery e.m.f.'s calls into play an exciting current which changes the booster e.m.f. by an amount equal to this difference, a drop in the battery e.m.f. resulting in an equal rise in the booster e.m.f., and *vice versâ*. It will be noticed that the exciter generally runs as a generator when the battery is discharging, and as a motor when it is being charged (the exciter e.m.f. being generally greater in the first case, and less in the second, than that of the battery).

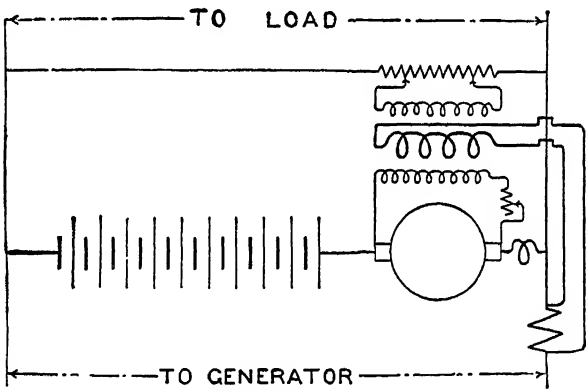

FIG. 132.—Lancashire Booster.

§ 158. Lancashire Booster.

The Lancashire booster, shown in Fig. 132, is provided with four field coils. One of these is connected across the mains, and is provided with a rheostat of the potentiometer type. The second coil is connected across a diverter or shunt *in the generator circuit*—an important feature which distinguishes the Lancashire from other types of booster. The third coil is an ordinary shunt coil across the brushes of the armature. The fourth coil is a series coil in the battery and booster circuit, and its object is to compensate for resistance drop in the armature (and in the coil itself) and armature reaction.

The action of the booster is as follows. When the generator current is at its normal value, the coil across the mains and that across the diverter in the generator circuit balance each other, and there is no e.m.f. in the booster armature. There is no current in the self-exciting coil across the brushes, and no current in the series coil in the battery circuit, since the e.m.f. of the battery just balances the p.d. of the mains, and there is no current in the battery circuit. Now suppose the load to increase suddenly, so that the generator current increases. This will cause the current in the diverter coil to increase, and will produce an e.m.f. in the booster armature which is added to the battery e.m.f. and causes the battery to discharge. As soon, however, as

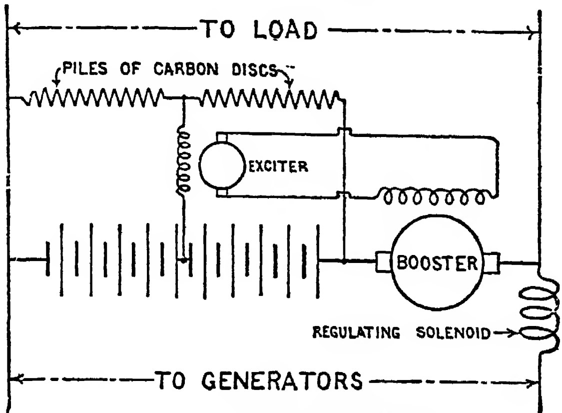

Fig. 133.—Entz reversible booster.

there is a current through the booster armature a p.d. appears across the brushes, and the self-exciting coil comes into play, strengthening the booster field and causing a heavier discharge from the battery. This action does not go on indefinitely, however, because as the battery current increases the generator current decreases and tends to weaken the booster e.m.f. A new state of equilibrium will be reached when the generator current is only slightly in excess of its original value. The changes take place in the reverse direction when there is a decrease of load. Thus in this type of booster the generator current is allowed to change between certain limits, and these changes in the current control the action of the booster in such a manner as to bring the generator current back to nearly its original value.

§ 159. Entz Carbon Controller for Reversible Booster.

A method of booster control (invented by Entz) which has come into favour is that shown diagrammatically in Fig. 133. The simple field winding of the booster is connected to the armature of an exciter, whose field coil is across the middle point of the battery and the middle point of a carbon regulating resistance, the latter being across the battery terminals. The carbon regulator consists of piles of carbon discs, the resistance of one set of discs increasing while that of the other set decreases, or *vice versâ*, according as the rocking lever shown in Fig. 134 moves one way or the other. It is evident from Fig. 133 that the exciter field will receive no current if the two carbon disc resistances on either side of the point of connection of the exciter field are equal, since this point will then be at the same potential as the middle point of the battery. But an increase in one of the resistances, accompanied by a decrease in the other, will cause a current to flow through the exciter field one way or the other, depending on the direction in which the change of resistance takes place. The booster e.m.f. is thus either opposed or added to that of the battery, enabling the latter to receive a charge or to discharge. The rocking lever which controls the relative values of the carbon disc resistances has attached to it at one end an adjustable spiral spring, and at the other a soft iron core which is suspended inside a solenoid traversed by the generator current. Slight variations in the latter are sufficient to call into play the buffer action of the battery. In order to protect the battery and booster against heavy overloads, the play of the rocking lever is limited by two adjustable stops.

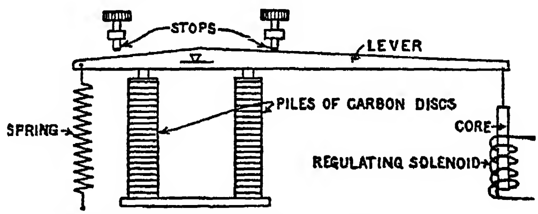

FIG. 134.—Carbon controller of Entz booster.

Since a booster is required to respond as promptly as possible to fluctuations of load, its field system should be *laminated*

throughout. The same remark applies, of course, to the exciter. If the field were made solid, the changes in its magnetism would be greatly retarded by the heavy eddy-currents induced in the field cores.

Examples.

1. After a number of charges and discharges at a constant current of 21 amperes, the following series of readings of the p.d. was found to be reproduced each time in the case of a certain secondary cell :—

Time, in hours	0	·2	·3	·5	1·0	2·0	3·0	4·0	5·0	6·0	6·5	7·0	7·5	7·75	8	8·5
Charge p.d.	1·9	2·14	2·16	2·16	2·16	2·17	2·19	2·21	2·24	2·30	2·36	2·45	2·55	2·58	2·60	2·62
Discharge p.d.	2·12	1·98	1·98	1·98	1·98	1·97	1·96	1·95	1·925	1·90	1·88	1·86	1·82	1·80	—	—

Find from the above (*a*) the capacity, in ampere-hours ; (*b*) the quantity efficiency ; and (*c*) the energy efficiency of the cell.

2. After a steady state had been reached in the case of a cell which was being charged at a constant p.d. of 2·51 volts, and discharged at a constant current of 36 amperes, the readings were as follows :—

Time, in minutes	0	2	10	20	30	40	50	60	70	80	87	100	120	140	142
Charging current	172	168	154	124	85	49	26	16	12	10·5	10	—	—	—	—
Discharge p.d.	2·05	1·94	1·93	1·92	1·92	1·92	1·91	1·905	1·90	1·89	1·88	1·875	1·845	1·815	1·81

Find (*a*) the capacity ; (*b*) the quantity efficiency ; and (*c*) the energy efficiency.

3. An electric launch is to be fitted with a battery of secondary cells capable of maintaining an output of $3\frac{1}{4}$ horse-power for ten hours. A type of battery is available giving 12 watt-hours per lb. weight of battery complete, and costing at the rate of £11 8*s*. per kilowatt-hour. Find (*a*) the weight and (*b*) the cost of the battery.

4. In a central station, a battery is required capable of maintaining a current of 400 amperes at 500 volts during two hours. Find approximately (*a*) the weight ; (*b*) the cost of the battery ; and (*c*) the floor-space required, assuming all the cells to be on the same level.

CHAPTER XV.

§ 160. The electric arc—§ 161. The arc between carbon electrodes—§ 162. Nature of carbon arc—§ 163. Distribution of potential in arc. Relation connecting p.d. and current—§ 164. Temperature of arc—§ 165. Arrangement and sizes of carbons in arc lamps—§ 166. Steadying resistance. Arc lamp mechanisms—§ 167. Körting and Mathiesen clockwork lamp—§ 168. Crompton-Pochin brake-wheel lamp—§ 169. Flame arcs—§ 170. Angold magazine flame arc lamp—§ 171. Mercury vapour lamps—§ 172. Quartz tube mercury vapour lamps—§ 173. Carbon filament incandescent lamp—§ 174. Metallic filament incandescent lamps.

§ 160. The Electric Arc.

The occurrence of *sparks* across a break in a circuit conveying a current at the instant of interrupting the current was known from very early times, and was soon followed by the discovery of the electric *arc*. If we suppose that by means of two conductors arranged to be movable with respect to each other, a circuit containing a sufficiently high e.m.f. (not less that 60 volts, say) may be closed or opened, and if when the circuit is closed the current has a sufficiently high value (about 10 amperes, say), then on separating the conductors by a *short* distance (about $\frac{1}{8}$ inch), we find that no interruption of the current takes place, but that a flame is formed between the conductor ends and that the current continues to flow across this flame. The ends of the conductors or electrodes between which the flame is formed are raised to a very high temperature, and may be melted if the current is sufficiently large. Such a conducting flame, consisting of an incandescent column of gases, is known as an *electric arc*. If the distance between the conductors be increased beyond a certain limit, which depends on the e.m.f. of the circuit, the arc goes out, and the current is interrupted. The starting of an arc between two conductors is spoken of as the *striking* of the arc. An arc may be formed not only between solid conductors, but also between liquid conductors, or between a solid and a liquid conductor.

§ 161. The Arc between Carbon Electrodes.

The arc formed between two rods of carbon is particularly striking, on account of the dazzling light to which it gives rise. It is also of very special importance on account of the numerous practical applications which it has received in electric lighting, electric welding, and electric furnace work.

The general appearance of the carbon arc (in air) and of the ends of the vertical carbon rods or *carbons* between which it is formed is shown in Fig. 135. It will be noticed that the flame or arc itself consists of a central portion or *core* of a purple colour, and of an outer envelope which is greenish. The shapes of the ends of the two carbons are entirely different. The positive carbon, or that by which the current enters the arc, becomes hollowed out, forming a *crater*, while the negative carbon, by which the current leaves the arc, becomes more or less pointed.

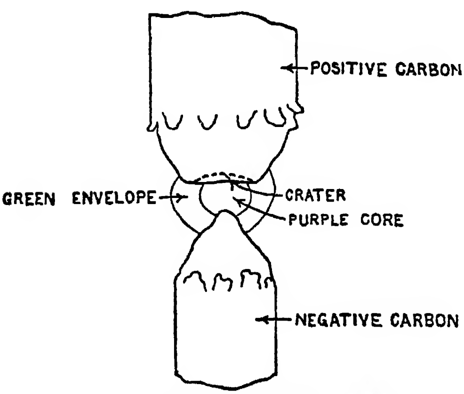

FIG. 135.—General appearance of arc.

Besides the difference of shape, there is another very important difference between the two carbons which is readily detected by a mere cursory examination. The bulk of the intense light emitted by the arc is easily seen to come from the surface of the crater formed at the end of the positive carbon; a much smaller amount of light comes from the tip of the negative carbon. Very little light is obtained from the flame or arc itself.

The length of arc, or distance apart of the carbons, varies in practice from $\frac{1}{8}$ to $\frac{3}{32}$ inch in those types of arc lamp in which the arc burns in air.

§ 162. Nature of Carbon Arc.

In spite of the enormous amount of experimental investigation devoted to the subject, the nature of the arc, and of the processes going on in it, is still a matter of uncertainty. According to

Mrs. Ayrton,* who made a very exhaustive study of the behaviour of the arc under different conditions, the purple core of the arc consists of carbon *mist* (or condensed carbon vapour), i.e., minute particles of carbon held in suspension in the column of incandescent gases; while the outer green sheath consists of burning gases. Further, there is some indication of the existence of a very thin layer of true carbon *vapour* (as distinguished from the condensed vapour or mist) in immediate contact with the surface of the crater. The resistivity of the vapour being much higher than that of the mist, the resistance of the thin layer of vapour next the crater is much greater than that of the long column of mist. There will thus be a relatively large drop of potential across the thin layer of vapour, while the potential drop along the column of mist will be relatively small. The bulk of the power supplied to the arc will be absorbed by the layer of vapour, and this power is employed in producing further volatilisation of carbon from the surface of the crater. The area of the crater depends on the current; as the current increases, this area expands, and with it also the cross-section of the column of mist.

The temperature of the arc is maintained partly by the power supplied to it electrically, and partly by the slow combustion of the carbons. The power arising from this latter cause is about 25 per cent. of the total power.†

§ 163. Distribution of Potential in Arc. Relation connecting P.D. and Current.

The distribution of potential in the arc has been experimentally investigated by means of thin exploring electrodes of carbon which could be introduced into the arc, and it is on the results of this investigation that Mrs. Ayrton has founded her explanation of the nature of the arc given in the preceding paragraph. The p.d. between the positive carbon and an exploring carbon pencil placed in the arc as close as possible to the crater surface (see Fig. 136) was found to vary from about 32 to about 39 volts, decreasing with increase of current

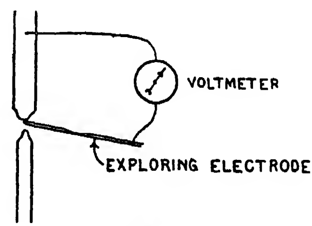

FIG. 136.—Method of using exploring electrode.

* *The Electric Arc*, p. 391.

† E. Rasch, *Das Elektrische Bogenlicht*, p. 42.

and increasing with length of arc. The p.d. between the negative carbon and an exploring electrode in the arc placed close to the surface of the negative carbon was found to be about 8 volts. The relation connecting the total p.d. V across the arc (i.e., the p.d. between the carbons) with the current i and length of arc l was found by Mrs. Ayrton to be represented very accurately by the formula—

$$V = a + b\,l + \frac{c + d\,l}{i},$$

where a, b, c and d are constants whose values depend on the quality and size of the carbons used.

From the above equation it is evident that under no circumstances can the p.d. across the arc be reduced below the limit represented by the constant a in the equation. The arc therefore behaves as if it contained a counter-e.m.f., and for a long time this apparent counter-e.m.f. was assumed to have a real existence. This view has now been generally abandoned,* as it has been found that there is no trace of any e.m.f. in the arc immediately after the current has been switched off, and before any material change has had time to take place in the condition of the arc. The arc, it may be mentioned, retains its physical condition and conducting power for an appreciable interval of time after switching off the current, so that if the carbons remain separated and the switch is again closed, the arc will be re-established, provided the interval of time between the opening and closing of the circuit is not too long.

§ 164. Temperature of Arc.

The temperature of the arc is the highest which may be produced by artificial means, all known substances except carbon readily fusing in the arc. This temperature approaches 4,000° C. The high temperature of the arc is usefully applied in electric welding and in arc furnaces (such as those used for preparing calcium carbide). The temperature of the crater is found to be independent of the current, an increase of current simply causing the crater area to expand, but not increasing its brightness. This constant temperature of the crater is supposed to correspond to the boiling-point of carbon.

* See, however, W. Duddell, *Phil. Trans.*, vol. cciii., p. 305 (1904).

§ 165. Arrangement and Sizes of Carbons in Arc Lamps.

The bulk of the light coming from the crater surface, it is usual to make the negative carbon somewhat smaller in diameter than the positive one, in order that as little light as possible may be intercepted by the former. The positive carbon in lamps of ordinary construction is placed above the negative one, so that the light may be thrown downwards, where it is wanted.

The carbon ends being at a high temperature, and their side surfaces in free contact with air, slow combustion of the carbons takes place. The surface of the crater itself is protected from direct contact with the air by the thin layer of carbon vapour with which it is covered. But loss of carbon takes place here by volatilisation, the volatilised carbon subsequently condensing and burning in the green flame which surrounds the purple core of the arc.

The luminous intensity of the arc, for a given amount of power supplied to it, is found to increase very considerably as the size of the carbons is reduced.* So far as the efficiency of the arc is concerned, therefore, there would be a considerable advantage in using carbons of the smallest practicable size. Unfortunately, a reduction in the size of the carbons involves an increased rate of their consumption, and more frequent re-carboning or "trimming" of the lamps. The sizes of carbons generally in use are such that the positive and negative carbons are consumed at the same rate of about $\frac{1}{2}$ inch per hour, the total rate of consumption being 1 inch per hour.† The currents in use vary from 5 to 20 amperes, those most commonly in use being 10 or 12 amperes. The following Table gives the usual sizes of positive and negative carbons for various currents :—

Current	5	8	10	15	20
Diameter of positive carbon in mm.	13	15	18	20	22
" negative " "	9	11	12	13	15

The lamps are arranged to take either 12-inch or 9-inch carbons. Allowing 2 inches for the minimum length of carbon which must

* See Blondel, *L'Éclairage Électrique*, vol. x., p. 497 (1897); T. Hesketh, *The Electrician*, vol. xxxix., p. 707 (1897); also G. N. Eastman, *Electrical World and Engineer*, vol. xlv., p. 772 (1905).

† If the diameters of the carbons were equal, the positive carbon would be consumed much more rapidly than the negative one.

be allowed to remain in the holders in order to prevent the latter from becoming over-heated and fused, we see that a lamp fitted with 12-inch carbons will burn $\frac{12 - 2}{\frac{1}{2}} = 20$ hours, and one fitted with 9-inch carbons $\frac{9 - 2}{\frac{1}{2}} = 14$ hours.

The positive carbons are provided with a *core* of much softer carbon. This core is consumed more readily than the surrounding harder carbon, and its effect is to keep the arc central and steady; with both carbons solid, the arc exhibits a tendency to wander about the end surface of the positive carbon, the position of the crater constantly changing, and the light flickering in consequence.*

§ 166. Steadying Resistance. Arc Lamp Mechanisms.

It is found that in order to obtain steady working, a certain minimum resistance must be connected in series with the arc. This resistance is represented partly by the series coil of the regulating arc lamp mechanism, partly by an external resistance. The *steadying resistance* outside the arc also prevents an excessive rush of current at the instant when the carbons are in contact with each other, and before the arc has been formed or *struck* by separation of the carbons. The steadying resistance required in series with an arc absorbs about 10 or 20 per cent. of the p.d. across the arc.

Owing to the gradual consumption of the carbons, some automatic arrangement is obviously necessary in order to keep the carbons at the proper distance apart, by moving or *feeding* them forward as they are consumed. This forward feed is effected by the arc lamp mechanism. The mechanism is generally controlled by two coils or solenoids, one of which is in series with the arc and carries the main current, while the other is connected as a shunt across the arc. The two solenoids are arranged to produce opposite effects so far as the length of arc is concerned, i.e., they act *differentially;* an increase of current through the series coil lengthening the arc, and an increase of

* A type of lamp in which a *tungsten* arc is used, and which is known under the trade name of "Pointolite," has come into use for special purposes (kinematograph projection and small searchlights). The arc burns in a sealed glass bulb filled with nitrogen. It forms the nearest practical approach to a point-source of light, burns steadily and is much safer as regards fire risk than a carbon arc burning in air. See *Journal of the Institution of Electrical Engineers*, vol. 54, p. 15 (1916).

current through the shunt coil (which results from an increase of p.d. across the arc) shortening it.

Numerous forms of arc lamp mechanisms have been devised, but those which have been most commonly in use may be divided into (1) clockwork and (2) clutch-wheel or brake-wheel types.

§ 167. Körting and Mathiesen Clockwork Lamp.

The Körting and Mathiesen arc lamp, whose mechanism is shown in Fig. 138, may be taken as a typical example of the clockwork type of lamp. The upper carbon-holder is heavily weighted, and is suspended from one end of a chain which passes over a chain-wheel and carries the lower carbon-holder at its other end—(the two carbon-holders are, of course, insulated from each other), as shown diagrammatically in Fig. 137. A rotation of the chain-wheel in a clockwise direction causes the upper carbon to descend and the lower one to ascend, thereby shortening the arc. Assuming the two carbons to be consumed at the same rate, it follows that the position of the arc in space will remain constant; a lamp capable of maintaining the arc in a constant position is sometimes spoken of as a *focussing lamp*. Contact is maintained with the lower carbon-holder through the chain, while the insulated upper holder is connected to a flexible copper conductor insulated by having a number of beads strung on it. The chain-wheel is, as shown in Fig. 138, mounted on an axle carrying a toothed wheel which forms the first of a train of wheels ending in a four-armed star-wheel. The train of wheels is contained between two plates, forming a frame, which is pivoted at the point P, and is thus free to rock about an axis passing through P. It will be noticed that a motion of the clockwork frame about P in a counter-clockwise direction causes the thin metal plate which acts as a detent to engage the star-wheel. thereby locking the train; while a motion in a clockwise

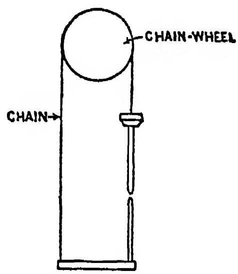

FIG. 137.—Arrangement of carbons and chain.

direction releases the star-wheel, allowing the upper carbon-holder to move down and the lower one to move up. Attached to the upper end of the frame is a bar of soft-iron which forms an armature, acted on by two U-shaped electromagnets, arranged as shown in the figure, the horizontal magnet being wound with coils connected in series with the arc, and the vertical one with coils forming a shunt across the arc. In order to secure a

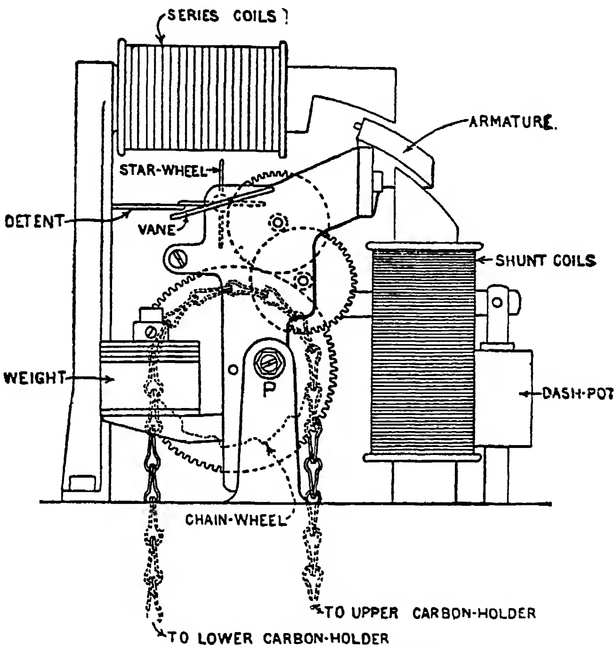

FIG. 138.—Mechanism of Körting and Mathiesen arc lamp.

smooth feed, a metal arm projecting from the clockwork frame carries a brass cylinder which fits very accurately over a piston attached to the base-plate, the arrangement forming an air dash-pot which prevents any jerky or irregular motion of the frame.

When the lamp is not in use, the weight of the upper carbon holder is sufficient to turn the clockwork frame into its extreme clockwise position, the star-wheel being free and allowing the carbons to move until they come into contact. Switching on the current causes the series magnet to pull the frame in a counter clockwise direction, separating the carbons, thus striking

the arc and at the same time locking the train of wheels. As soon as the arc is struck, a current passes round the shunt magnet and prevents the arc from being lengthened beyond the normal limit. As the arc lengthens the current decreases, and the p.d. across the arc increases; hence the pull due to the series magnet is weakened and that due to the shunt magnet strengthened. The armature slowly moves towards the poles of the shunt magnet, and at a certain stage the star-wheel is released, allowing the carbons to feed forward; as the arc shortens the pull due to the series magnet increases, and that due to the shunt magnet decreases, until the star-wheel is once more locked and the feed arrested. The motion of the train is retarded by a vane mounted on the axle carrying the star-wheel.

168. Crompton-Pochin Brake-wheel Lamp.

In Fig. 139 is shown the mechanism of a brake-wheel type of lamp—the Crompton-Pochin "S" type lamp. In lamps of this class the feed of the carbons is effected by the alternate release and clamping of a brake-wheel mounted on an axle which also carries the pulleys supporting the flexible bands by which the carbon-holders are suspended. The lamp illustrated in Fig. 139 is a focussing one, both carbons being movable. Each carbon-holder is suspended by a flexible copper conductor, which also serves to convey the current to the holder. The two flexible conductors are coiled in opposite directions around two brass pulleys mounted on a common axle, but insulated from each other by a disc of ebonite. By the side of one of these pulleys is mounted the flanged brake-wheel. The brake consists of a flexible copper band coiled once around the brake-wheel and having the cylinder of an air dash-pot attached to its lower end. The upper end of the brake-band is attached to a point in a rocking lever, or "see-saw" lever, whose position is controlled by two coil-and-plunger mechanisms, one of the coils carrying the main current and the other being connected as a shunt across the arc. Contact is maintained with the two halves of the axle supporting the pulleys by means of wire brushes rubbing against the ends of the axle, as shown in the figure. The right-hand drawing in the figure shows an end view of the pulleys and brake-wheel. When the lamp is not in use, the weight attached

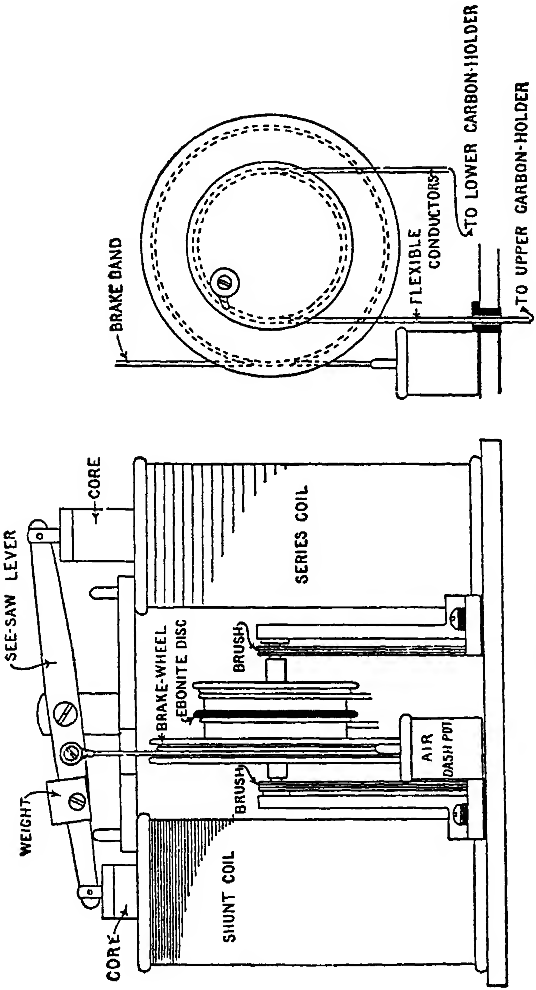

FIG. 139.—Crompton-Pochin brake-wheel lamp.

to the left-hand side of the lever causes that side to drop against a stop, the brake being thereby left entirely slack, and allowing the carbons to come into contact. As soon as the current is switched on the series coil sucks in its core, pulling down the right-hand side of the lever, tightening the brake around the brakewheel, and then causing a rotation of the brake-wheel which separates the carbons and strikes the arc. The current which appears in the shunt coil as soon as the arc has been struck prevents any lengthening of it beyond the normal limit, releasing the brake-wheel when this limit has been exceeded.

§ 169. Flame Arcs.

The ordinary types of arc lamps described above, in which the electrodes consist of pure carbon, have now become obsolete, having been, for indoor use, displaced by gas-filled tungsten lamps, and for outdoor use, by the much more efficient *flame arc lamps*. In these the arc is established between a positive electrode containing a considerable percentage of mineral matter and a negative electrode, which is either of pure carbon or which may also contain mineral matter. Although attempts at increasing the efficiency of the arc light by the use of mineralised carbons date back to the early days of electric lighting, it was only at a much later date that renewed attempts in this direction were crowned with success. One of the main difficulties encountered bv the early experimenters was the formation of a slag at the electrode ends which rendered the arc extremely unsteady. The success of the modern type of flame arc lamp is in no small measure due to H. Bremer, who must be regarded as the pioneer in this field, and who was the first to design a serviceable type of flame arc lamp.

The *flame arc* lamp derives its name from the fact that the bulk of the light comes from the intensely luminous arc *flame*, and not from the positive crater as in the ordinary arc, whose flame is relatively only feebly luminous. The high luminosity of the flame is due to the presence in it of the vapours of the metallic salts which are incorporated with the carbons. Besides the luminosity of the arc itself, other important differences between the flame and the ordinary arc are :—(1) The flame arc is very much longer (10 to 15 mm. as against 2 to 3 mm. in the ordinary

arc). (2) The drop of potential between the surface of the crater and the flame arc is relatively small (about 17 volts). (3) The rate of consumption of the mineralised flame carbons is much more rapid (about 1·2 inch per hour for *each* carbon). (4) There is a much more copious production of poisonous and corrosive fumes than in an ordinary arc.

In flame arc lamps the carbons are arranged not vertically above one another, but *side by side and inclined to each other at a small angle* (see Fig. 141). Immediately above the carbon tips is placed a dome-shaped casting lined with fire-clay and known as an *economiser*, which performs several useful functions. In the first place, it limits the supply of oxygen to the carbon ends, and reduces their rate of consumption; next, since with equal rates of consumption the carbon which burns more slowly will tend to project to a greater distance below the economiser, and so be in an atmosphere richer in oxygen, its rate of consumption will thereby be accelerated, so that the rates of consumption will be equalised. Again, the economiser will act as a reflector and give a better distribution of the light. Finally, the loss of heat by convection will be reduced.

A device employed in many flame arc lamps is the *blow magnet* for spreading out the flame of the arc. It consists of a small electromagnet whose field tends to blow the arc downwards, the flame itself having a tendency to rise upwards. The result is a flattening or spreading out of the arc which tends to steady it and improve its efficiency.

The *colour* of a flame arc is determined by the particular kind of chemical which is incorporated with the carbon electrodes. The chemicals used for this purpose are the *fluorides* of calcium, strontium, cerium and titanium. Calcium fluoride gives a yellow arc; strontium fluoride, a red one; while cerium or titanium fluoride yields a white arc.

The positive electrode in flame arc lamps generally consists of a flame-cored carbon, while the negative one is either a cored carbon of the ordinary type, or is provided with a small flame core. The outer shell of pure carbon is prepared by mixing lamp-black (70 to 80 per cent.) with finely-ground gas retort carbon (30 to 20 per cent.), and this mixture is then made into a paste with tar, moulded under considerable pressure, and then carbonised in a furnace. The flame core is prepared by mixing about equal amounts of the fluoride to be used and finely ground

carbon with a solution of potassium silicate, and forcing the mixture into the opening of the carbon tube or shell. The ratio of the flame core diameter to the external diameter of the carbon is about 1 : 2 in the case of a positive carbon, and 1 : 3 in the case of a negative one.*

Owing to the comparatively high resistivity of the flame core, some arrangement must be made to prevent an excessive drop of potential along the flame carbon itself. Several means have been employed for attaining this object. The carbons may be coppered; or a metal wire (copper or zinc) may be embedded in the carbon (outside the flame core, so as not to affect the character of the arc); or a sliding contact may be arranged at a short distance from the arc, so that the drop of potential takes place over a short length of carbon only.

The difficulty arising from the rapid consumption of flame carbons has been overcome by the use of a *magazine* lamp, provided with magazines which hold a number of carbons, the lamp mechanism being so arranged that when a pair of carbons has been used up, another pair automatically comes into action.

§ 170. Angold Magazine Flame Arc Lamp.

Instead of the simple carbon-holders used in ordinary flame arc lamps, magazine lamps are provided with magazine holders capable of holding a number of carbons. The magazine of the Angold lamp is shown in Fig. 140. It is capable of holding nine carbons. It consists essentially of a stout aluminium plate provided with a movable arm controlled by a spiral spring which causes the arm to press the carbons towards the grooved bar or guide down which the carbon in use is slowly fed by a catch or finger pressing against its upper end. The carbons are kept in the same plane by means of cover plates at the top and bottom of the magazine which form open boxes of width equal to the diameter of the carbons. The carbon in use is slowly fed downwards, and when its upper end has passed below the bottom

* See a paper on "Yellow Flame Arcs," by M. Solomon, in the *Journal of the Institution of Electrical Engineers*, vol. 49, p. 737 (1912).

cover plate, the spring-controlled arm pushes the carbons towards the guide-bar, one of the carbons dropping into the groove of this bar and so being placed in readiness for use when the remaining fragment of the previous carbon has been ejected.

The *arc striking and regulating mechanism* is shown in Fig. 141. One of the magazines is fixed, while the other is pivoted at its upper end, and has its lower end connected to a bell-crank lever actuated by the usual differential coil-and-plunger mechanism. The cores of the differential electro-magnet are so placed relatively to the coils that the latter tend to suck them up (as indicated by the arrows in Fig. 141). When the lamp is not in use the carbon ends are in contact. If now the current is switched on, the series coil sucks up its core, causing the see-saw lever to move so as to strike the arc. The shunt coil then comes into play, and the arc is kept burning at its normal length. As the carbons are consumed, however, their distance apart increases, and finally the arc length becomes too great even when the distance apart of the magazines has reached its minimum value. The feeding mechanism then comes into play.

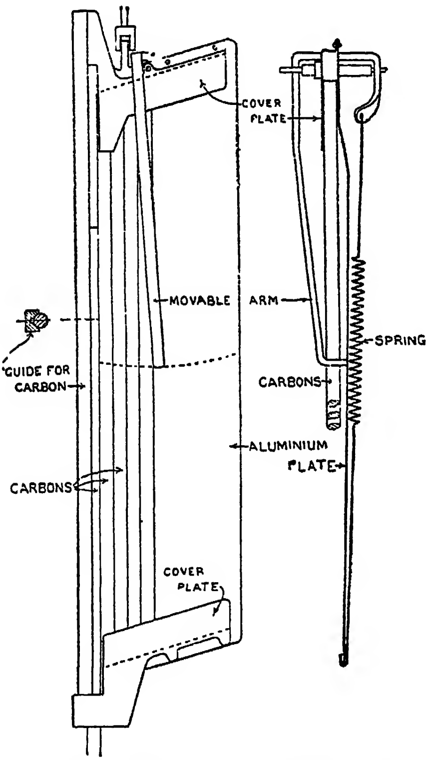

FIG. 140.—Magazine of Angold flame arc lamp.

The action of the *feeding mechanism* will be understood by reference to Fig. 142, which shows the general arrangement of the working parts; the details of construction are complicated. Surrounding each carbon guide-bar (see Fig. 140) is a slider

through which passes freely a push-bar. Each slider is fitted with a pivoted finger (not shown) bearing on the top of the carbon, so that as the sliders are pushed down, they carry the carbons with them. The two push-bars are rigidly attached to but insulated from a carriage which travels along two phosphor-bronze rack rods. Inside this carriage are pivoted two pawls which engage the rack rods; the pawls are mounted so that they

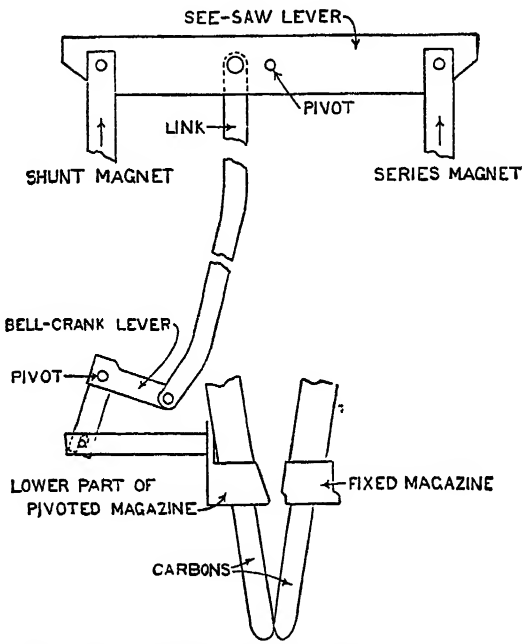

FIG. 141.—Arc striking and regulating mechanism.

can occupy one or other of the two positions shown in the lower part of Fig. 142, and they are coupled so that they both occupy the same position, but the coupling is loose, allowing of a certain amount of play between the pawls. The pawls are controlled by a spring in such a manner that it is impossible for them to remain in any intermediate position. One of the rack rods is fixed, while the other is capable of a reciprocating motion through a short distance. Suppose now that the pawls occupy the full-line position, and that the movable rack rod is displaced

upwards. This causes the pawl engaging it to dig into the rack and become locked to it. The other pawl, which engages the fixed rack, is free to slip *upwards* over the teeth of the rack (there is, as mentioned above, sufficient play between the two pawls to allow of this). The result then is, that the carriage is carried *upwards* bodily with the movable rack rod, the pawl engaging the fixed rack slipping over its teeth during the motion. When the movable rack rod begins its *downward* stroke, the pawl engaging

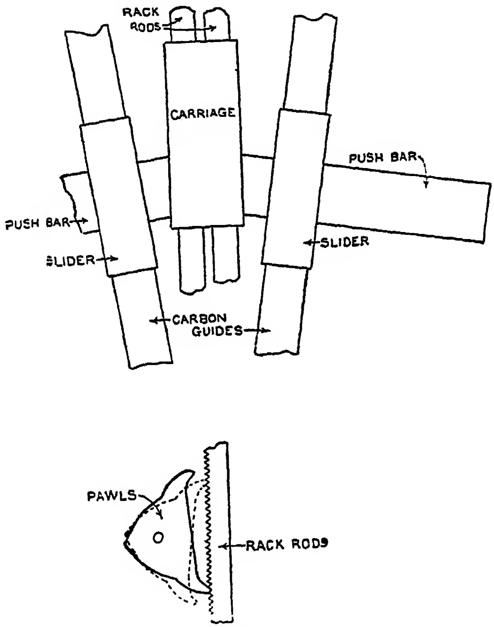

FIG. 142.—General arrangement of feeding mechanism.

the fixed rack digs into it and becomes locked. No displacement of the carriage downwards can therefore take place, while as the movable rack slips through the carriage the pawl engaging the rack slips over its teeth. We thus see that, with the pawls in the full-line position, a reciprocating motion of the rack results in a step-by-step *upward* travel of the carriage. Similarly, with the pawls in the dotted position, a reciprocating motion of the rack produces a step-by-step *downward* travel of the carriage.

The reciprocating motion of the movable rack rod is obtained by means of a special feeding magnet whose circuit is

automatically broken when it has sucked up its core, and again automatically closed after the core has returned to its lowermost position. It is evident that for feeding the carbons a downward travel of the carriage is required, and hence the pawls will occupy the dotted position shown in the lower part of Fig. 142. The feed of the carbons is not continuous, but intermittent, taking place about every two minutes. The feed is controlled by the see-saw lever of the arc regulating mechanism, which carries a catch that prevents the carbon contacts which complete the circuit of the feeding magnet from closing when the see-saw lever passes beyond a certain position. When the carriage to which the push-bars are attached has descended sufficiently far, a lever which controls the position of the pawls comes into contact with an abutment, and is snapped over so as to bring the pawls into the full-line position of Fig. 142. No further feed is now possible, and the carriage begins to travel *upwards* by a step-by-step motion, such travel continuing without intermission until in the uppermost position of the carriage the lever controlling the pawls comes into contact with another abutment, and snaps the pawls over into the dotted position, causing the carriage to travel downwards until the catches or fingers on the sliders engage the tops of the new pair of carbons, and the feed continues as before. The upward travel of the carriage takes less than

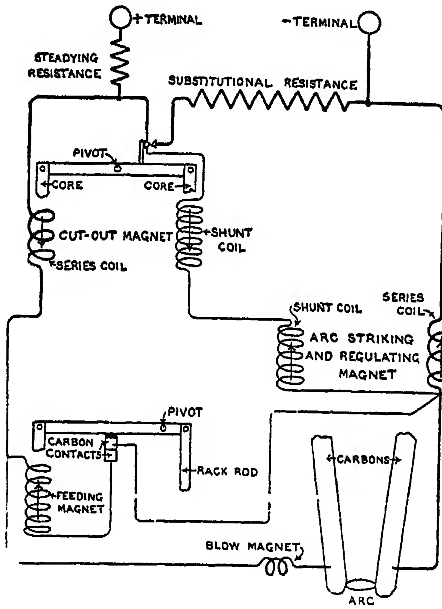

FIG. 143.—Circuits of Angold magazine flame arc lamp.

During the upward travel of the carriage the catches glide along the sides of the carbons.

one minute, and the arc continues to burn all the while. When the push-bars have engaged the new pair of carbons, these latter continue to push the remaining ends of the old pair downwards when a feed is taking place, until these ends drop out of the guide-bars into a suitable receptacle at the bottom of the lamp globe. The arc is then temporarily extinguished (for about seven seconds), while a special cut-out magnet (see Fig. 143) introduces a substitutional resistance which maintains continuity of the circuit and prevents other series lamps in the same circuit from going out. The downward feed of the new carbons now continues until the arc is again struck.

The arrangement of the lamp circuits is shown in Fig. 143.

The voltage across the terminals of this lamp is 42. The burning hours are from 60 to 70. The carbons are 12 inches long.

§ 171. Mercury Vapour Lamps.

A form of lamp which has found a limited application in practice is the mercury vapour lamp. In this the arc is established through a long column of mercury vapour. The light comes from the glowing vapour, and not from either electrode. This lamp is remarkably efficient, but owing to the preponderance of rays in the green and violet regions of the

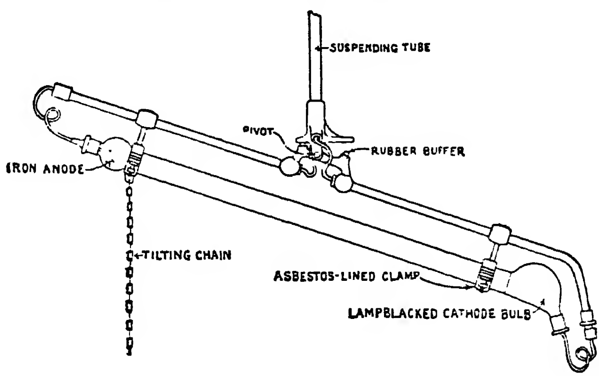

FIG. 144.—Cooper-Hewitt mercury vapour lamp.

spectrum it is unsuitable for purposes of general illumination, the effect produced being too ghastly. For photographic work, drawing offices, &c., it is, however, a very suitable and cheap form of illuminant.

One form of mercury vapour lamp—the Cooper Hewitt lamp—is shown in Fig. 144. It consists of a long glass tube (about five feet long) with expansions at the ends which contain the electrodes. The anode is a cup of iron, and the cathode a pool of mercury. The tube is about one inch in diameter. In the course of manufacture the tube is carefully exhausted before being sealed up. It is attached to a supporting metal tube through which pass the connecting wires, and which is pivoted at its middle as shown in the figure. Normally, the tube occupies the inclined position shown. In order to start the lamp, the tube is tilted by means of a chain, so as to cause some of the mercury contained in the cathode chamber to run down the tube in a fine stream. As soon as the mercury reaches the anode, momentary metallic connection is established between the electrodes, allowing a current to pass. Immediately afterwards the mercury thread breaks, starting the arc, which soon fills the entire tube. As soon as the arc has been started the tube is released, and the mercury flows back into the cathode chamber. A steadying resistance is connected in series with the vapour tube.

§ 172. Quartz Tube Mercury Vapour Lamps.

By enclosing the mercury vapour in a tube of quartz instead of glass, the temperature of the vapour may be raised considerably, and the efficiency of the lamp greatly improved. Such quartz tube lamps are now in use. Owing to the much larger amount of light given out per unit of length of such tubes as compared with glass tubes working at a much lower temperature, the total length of the tube may be made much shorter, and the tube is generally mounted in a fitting similar to that used for an arc lamp. The tube itself consists of a straight horizontal portion which is filled with mercury vapour when the lamp is at work, and bent end portions containing mercury and the leading-in wires. Close to the end

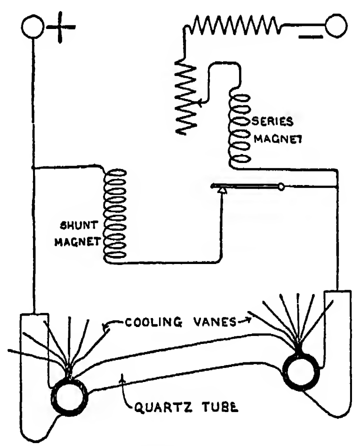

FIG. 145.—Quartz tube mercury vapour lamp.

portions of the tube are two short cross-tubes; embracing these are a number of thin sheets of copper or aluminium, the ends of which are spread out so as to form a set of cooling vanes and prevent the electrodes from becoming too hot (see Fig. 145). The general arrangement of the lamp circuit is shown in Fig. 145. When the lamp is not in use, the mercury pools in the end portions of the tube are insulated from each other. The closing of the switch causes the current to pass through the shunt or starting magnet, the series magnet, and the resistances in series with the lamp. The shunt magnet attracts its armature and tilts the tube so as to cause the mercury to run along the tube and form a conducting bridge between the two mercury pools. As soon as the pools are bridged, however, there is a large increase of current; the series magnet is now excited strongly enough to attract its armature and break the circuit of the shunt magnet, which drops its armature and thereby allows the tube to fall back into its normal position. The breaking of the mercury thread connecting the two pools starts the arc, and the tube becomes filled with incandescent mercury vapour.

At first, while the tube is cold, the p.d. across it is small, the current large, and the light emitted comparatively feeble. As the tube gets heated up, however, and the temperature of the vapour rises, the p.d. across the tube increases until it reaches a value about six times the initial value, while at the same time the current drops to about one-third of its initial value, and the light emitted greatly increases. The lamp takes about 10 minutes to reach a steady state, when the p.d. across the tube forms about 80 per cent. of the p.d. across the terminals of the lamp.

The life of quartz tube mercury vapour lamps is from 5,000 to 6,000 hours. After that period, the tubes become more or less porous, owing to the very high temperature to which they are exposed, and require renewal.

Quartz tube mercury vapour lamps are made in various sizes, from about 800 to about 3,000 candle-power, for voltages from 100 to 240. The currents taken vary from 1·5 to 6 amperes. Owing to their high candle-power, and to the large proportion of green rays in their light, they are unsuitable for ordinary indoor illumination. But for the lighting of large, open spaces, such as goods sheds, &c., they provide a very efficient form of illumination.

In order to improve the colour of the light, ruby globes, which absorb the green rays, are sometimes used.

An interesting application of the quartz tube mercury vapour lamp may be mentioned. At the high temperature reached in such a lamp, the mercury vapour emits powerful radiation in the ultra-violet region of the spectrum. Quartz, unlike glass, is highly transparent to such short wave-length radiation, so that the radiation passes freely through the tube. Now one well-known and remarkable property of such radiation is its destructive effect on living organisms. Hence quartz tube lamps are being successfully used for the purpose of sterilising water and other liquids.

§ 173. Carbon Filament Incandescent Lamp.

The oldest form of incandescent lamp is the carbon filament lamp. This consists of a thin filament of carbon enclosed in a vacuous bulb of glass. The vacuum must be very perfect to prevent oxidation and consequent destruction of the filament when raised to a high temperature. By applying a sufficient voltage across the terminals of the lamp the filament is raised to a white heat and emits light.

The process of manufacture of a carbon incandescent lamp is briefly as follows:—Pure cotton wool (which consists mainly of cellulose, $C_6 H_{10} O_5$) is dissolved in a solution of zinc chloride to form a highly viscous liquid. This liquid is then squirted through a tube with a sufficiently fine opening into a vessel containing alcohol. The issuing cellulose thread is allowed to remain in the alcohol for three or four days, during which time it becomes thoroughly hardened. It is then washed, wound on a large cylinder, and allowed to dry. The next process consists in cutting the thread into suitable lengths, and winding these on carbon frames or formers, which give the threads the shape of the finished filament. The formers are then packed in plumbago crucibles fitted with lids so as to exclude air, and are carbonised in a furnace which is raised to a very high temperature. The hydrogen and oxygen of the cellulose are thereby driven away, and a thread of pure carbon of great hardness is left. The

filaments) are now attached to the leading-in wires*: the ends of the latter are first formed into a tiny tube, into which the filament end is inserted, the tube being then tightened with a pair of tweezers: the joint is rendered mechanically and electrically perfect by depositing carbon on it—an operation performed by completely immersing it in benzene, and raising it to a white heat by sending an electric current through it. The filaments are next rendered uniform by the *flashing* process, which consists in placing them in an atmosphere of benzene vapour and sending a current through them sufficient to cause incandescence; the thinner portions of the filament having a higher resistance, and hence being at a higher temperature than the thicker ones, have carbon deposited on them (by the decomposition of the benzene vapour by heat) more rapidly, so that finally a perfectly uniform filament is obtained. This filament is sealed into a glass bulb, which is thoroughly exhausted. The ends of the leading-in wires projecting outside the bulb are soldered to tinned copper wires, and the latter are soldered to the brass contact-plates by means of which contact is established with the contact-pins in the lampholder. The copper wires and the brass contact-plates are embedded in some form of insulating cement which is surrounded by a brass ring provided with two diametrically opposite pins to fit the bayonet-joint in the lampholder. The material used in forming the "cap" of the lamp is in some cases plaster of Paris, but as this is highly hygroscopic and hence a poor insulator, and loses its hardness at high temperatures, it is not always satisfactory, and various other substances are used in its place (especially if the lamps are to be capable of withstanding moisture), such as a mixture of litharge and glycerine, or some vitreous substance.

In order to prevent the possibility of the formation of an arc across the lampholder contact-pins, these pins should be separated from each other by an insulating barrier. The construction of a typical lampholder will be understood by reference to Fig. 146. The contact-pins are, it will be noticed, separated by an S-shaped plate of porcelain which effectively prevents any possibility of arcing.

* These were formerly made of platinum, but owing to the great rise in the price of this metal, special alloys (iron-nickel) are now used.

§ 174. Metallic Filament Incandescent Lamps.

First introduced in 1879, the carbon filament lamp held the field against all competitors for nearly a quarter of a century. It is true that even in the early days of incandescent lighting attempts were made to construct lamps with filaments of some suitable metal (platinum, iridium), but all such attempts resulted in failure. The first serious competitor of the carbon filament lamp was the Nernst lamp, which made its appearance in 1897,

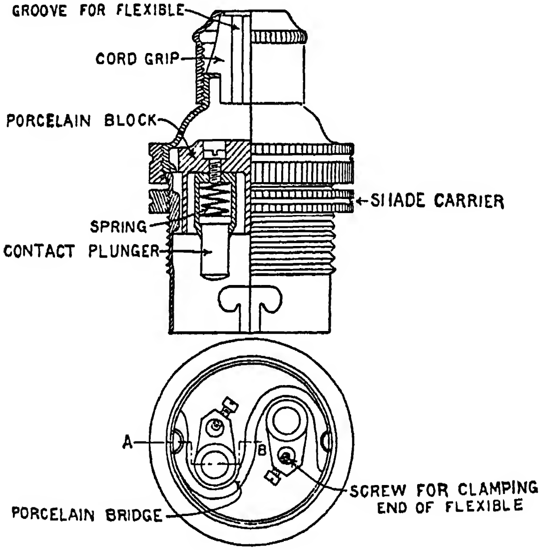

FIG. 146.—" S " type lampholder.

and whose " burner " consisted mainly of a mixture of the oxides of thorium and zirconium; the "burner" or "glower" being incombustible was freely exposed to the air. Since such a burner is non-conducting when cold, special arrangements had to be adopted for heating it up in order to render it conducting. This made the construction of the lamp more expensive than it would otherwise have been. The brittle nature of the glower was another difficulty. In spite of these disadvantages, the Nernst lamp made considerable headway for a time, owing to its high efficiency as compared with the carbon filament lamp. With the appearance of the modern metal filament lamp,

however, the Nernst lamp began to decline in importance, and is now obsolete.

In order to understand the reason why the metallic filament lamp enables us to produce light so much more economically than the carbon filament lamp, it becomes necessary to consider briefly the laws which govern the conversion of electrical into luminous energy in such cases. When light is obtained from a body by rendering the body incandescent, not only does the total radiation increase with the temperature, but also the ratio of the luminous or visible radiation to the total radiation. Hence with increasing temperature there is an increase of efficiency in the production of luminous radiation. If we neglect the effect of selective emission (*i.e.*, of the difference between the spectra of two bodies at the same temperature), then the efficiency of light production by this means becomes a matter of *temperature*, and of two bodies raised to incandescence the higher efficiency will be obtained with the body maintained at the higher temperature.

Thus in order to secure the highest efficiency it becomes necessary to maintain the incandescent body at the highest possible temperature, so that only the most refractory solids need be considered in this connection. Now since carbon is the most refractory substance known, it might at first sight be supposed that it would form the best material for the filaments of incandescent lamps, enabling us to work at the highest temperature.

But if we try raising the temperature of a carbon filament beyond a certain point difficulties arise. It is found that the filament begins to shoot off small particles which become deposited on the walls of the bulb, causing blackening of the lamp. The filament itself becomes rapidly disintegrated, and breaks after a comparatively short period of use. All this occurs at a temperature far below the melting point of carbon. It is found, in fact, that carbon filaments cannot be profitably run at a higher temperature than about 1800° C.

After carbon, the most refractory substances among conductors are certain metals, the most important of which is *tungsten* (or wolfram), the melting point of which is in the neighbourhood of 3000° C. It is found that a tungsten filament may be run at a temperature of about 2300° C.—*i.e.*, some 500° C. higher than a carbon filament—without excessive blackening of

the bulb or rapid disintegration of the filament. It is owing to its higher working temperature that the tungsten filament is so much more efficient. Among other metals possessing a similar property are tantalum and osmium.

The *osmium* lamp made its appearance in 1900. Owing to the extreme brittleness of its filament, it never achieved any commercial success.

The *tantalum* lamp was introduced in 1904, and enjoyed a considerable degree of popularity prior to the introduction of the drawn-filament tungsten lamp.

The first *tungsten* lamps were constructed in 1906. Originally, considerable difficulty was experienced in connection with this lamp owing to the delicate nature of the filament.* The early or *pressed* filaments were prepared by making a paste of finely divided metallic tungsten with an organic binding material, and squirting the paste into a filament, which was dried and carbonised by heating it *in vacuo*. It was then raised to a white heat in an atmosphere of hydrogen and nitrogen, with the object of driving off any carbon present in the form of cyanogen and hydrocarbons, and increasing the mechanical strength of the filament. Such pressed filaments were very delicate, and the lamps had to be handled with considerable care.

In the modern tungsten lamp, *drawn wire* filaments are used. Metallic tungsten is not ordinarily ductile, but may be rendered so by suitable methods of treatment. After the metal has been freed from all impurities, it is hammered at a very high temperature (1200° to 1300° C.) in a current of hydrogen (to prevent oxidation). This renders it ductile at ordinary temperatures. The drawing of the wire is facilitated by heating the diamond die as well as the wire. Such drawn wire filaments possess great mechanical strength, and have

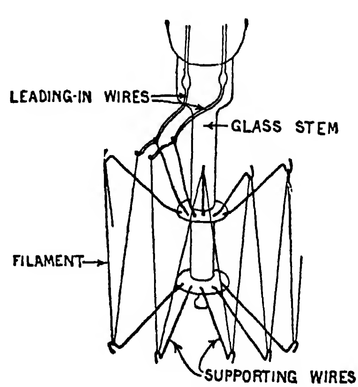

FIG. 147.—Method of supporting filament.

* For an excellent account of the tungsten lamp, see a Paper by O. Ruff in *Zeitschrift Vereines Deutscher Ingenieure*, vol. lvii., p. 1615 (1913).

enabled the tungsten lamp to supersede the carbon filament lamp.

Owing to the much lower resistivity of the metallic as against the carbon filament, the length of the former is, for a given voltage, much greater than that of the latter. In order to get the great length of filament into a bulb of reasonable size, the filament is arranged zig-zag fashion around a central glass stem projecting into the bulb, as shown in Fig. 147. The upper supports for the filament consist of springy molybdenum wire, the lower ones of nickel or nickel-steel wire.

A carbon filament has a negative temperature coefficient of resistance, and its resistance when cold is about 1·75 times its resistance at the working temperature. A tungsten filament, on the other hand, has a positive temperature coefficient, and its resistance at the working temperature is about eleven times its resistance when cold. Hence the initial value of the current when a lamp is switched on will be less than its final value for a carbon filament, but several times the final value for a metal filament.

The positive temperature coefficient of a tungsten filament is useful in protecting the lamp against the effects of abnormally high voltages, and in rendering it less sensitive than the carbon filament lamp to voltage fluctuations.

The efficiency of the tungsten lamp might be still further increased by running the lamp at a higher temperature. But, as already explained, an economical limit is reached beyond which it would not pay to go, owing to the blackening of the bulb and the shortening of the useful life of the lamp. Now the blackening of the bulb and rapid disintegration of the filament may be largely prevented by placing the filament not *in vacuo*, but in a bulb filled with some inert gas, such as nitrogen. When this is done, however, with a filament of the ordinary type, the loss of heat by convection and conduction becomes so great as to wipe out any advantage gained by running the filament at a higher temperature. In order to reduce this loss, the cooling of the filament by convection and conduction must be reduced. In the latest type of tungsten lamp—the gas-filled lamp—this object is achieved by winding the tungsten wire into a fine spiral which is supported at a number of points and hangs in a series of festoons between the supports. The nitrogen has a pressure of about $\frac{2}{3}$ of

an atmosphere when the lamp is cold, and the pressure rises to about 1 atmosphere when the lamp is in use. The resistance of a gas-filled tungsten filament lamp when hot is about fourteen times its resistance when cold. The working temperature of the filament is about 2800° C.

The filament of a carbon lamp becomes (in a well-lighted room) appreciably luminous when the terminal voltage is about a quarter of the normal working voltage; while a tungsten lamp shows luminosity at about one-twelfth of its normal voltage.

CHAPTER XVI.

§ 175. Visible and invisible radiation—§ 176. Luminous intensity. Spherical reduction factor—§ 177. Photometric units—§ 178. Practical standards of luminous intensity—§ 179. Law of inverse square. Brightness and Lambert's law—§ 180. Units of luminous flux and illumination—§ 181. Photometric measurements. Photometers—§ 182. Lummer-Brodhun photometer—§ 183. Flicker photometer—§ 184. Ulbricht's globe photometer—§ 185. Trotter's illumination photometer—§ 186. Efficiency of luminous sources—§ 187. Distribution of intensity—§ 188. Rousseau's construction—§ 189. Mean hemispherical intensity—§ 190. Rating of lamps. Reduction factor—§ 191. Useful life of lamps.

§ 175. Visible and Invisible Radiation.

THE energy sent out into space by any luminous source consists partly of luminous or visible radiant energy, partly of invisible radiation. The difference between the various kinds of radiant energy, and the different colours of the visible part of it, is a difference of wave-length,* the nature of the wave-motion which constitutes radiation being the same in each case. *Colour* in light is analogous to *pitch* in sound; and just as there are notes whose pitch is either too high or too low to render them audible, so there are light-waves whose wave-length is either too great or too small to affect the human eye.

In photometry we are concerned not with the total rate at which energy is being radiated by a given source, but with the rate at which the source radiates *luminous* or *visible* energy, and we restrict our attention entirely to the particular range of wave-length which corresponds to visibility. This range of wave-length is spoken of as the *visible spectrum.* In most cases the visible part of the radiation constitutes but a small fraction (not exceeding 8 per cent.) of the total radiation. From this it will be understood that our present methods of artificial illumination are

* The waves corresponding to *red* have a wave-length of about 76×10^{-6} cm. As the colour changes through orange, yellow, green, blue and indigo to violet, the wave-length decreases to about 41×10^{-6} cm., beyond which limit the waves cease to affect the eye.

extremely wasteful, since the production of the luminous radiation is accompanied by that of a much more powerful non-luminous radiation, this latter being entirely useless so far as purposes of illumination are concerned.

§ 176. Luminous Intensity. Spherical Reduction Factor.

The *luminous intensity* of a given source is the rate at which the source is sending out luminous energy—i.e., radiant energy comprised within the limits of wave-length corresponding to visibility. At first sight it might appear (and theoretically such is the case) that the simplest unit of luminous intensity would be the unit represented by a source which radiates a unit (say, one erg) of luminous energy per unit (say, a second) of time. But such a unit would be awkward to use in practical photometric measurements, owing to the difficulty of embodying it in a suitable practical standard. Hence the units of luminous intensity employed in practice are represented by various arbitrary standards, selected mainly with reference to their reliability from the point of view of constancy. If we imagine a spherical surface described around any given source as centre, and consider the amount of luminous energy which passes through a unit area of the surface per second, then it will be found that this depends on the position of the unit area relatively to the source. Along some directions the luminous energy incident per sec. on the unit area will be much greater than along others. The source is in such cases said to be non-uniform, or its intensity is said to vary along different directions. All sources used in practice are of this type. If we next imagine an ideal uniform source (i.e., one which would radiate the same amount of luminous energy per sec. to a unit of area, wherever this area might be placed on the surface of a sphere described around the source as centre), and if the actual and ideal sources be so related that each radiates the same *total* amount of luminous energy per sec., then the intensity of the uniform source represents what is known as the *mean spherical intensity* of the actual source.

Owing to the fact that no actual source is uniform, it is evident that when using any given source as a standard, it is necessary to specify the direction along which the radiation is

supposed to correspond to standard intensity. This direction is generally taken to lie in a horizontal plane passing through the source, and if the source is symmetrical about a vertical axis, no further specification is necessary; but in cases where there is no such symmetry, the exact direction with reference to the source must be specified.

By the *spherical reduction factor* of a source with respect to any particular direction is meant the ratio of the mean spherical intensity of the source to its intensity along that direction.

§ 177. Photometric Units.

The *total* luminous energy radiated by any source per second, or the total *rate* at which luminous energy is being sent out by the source, is conveniently spoken of as the total *luminous flux* emitted by the source, and from what has been said above it is evident that the total luminous flux is directly proportional to the mean spherical intensity of the source. The total luminous flux is frequently, but erroneously, termed the total "quantity of light" emitted by the source. This latter term should be restricted to denote the *product of luminous flux into time* (which corresponds to the luminous energy sent out by the source during the time considered).

The luminous energy received per second by any surface is termed the *luminous flux* incident on that surface. The luminous flux per unit of area of a surface at any given point is defined to be the *illumination* of the surface at that point.

The *brightness** of a luminous surface is its luminous intensity per unit of area in a direction normal to the surface.

It is important to distinguish brightness from illumination. A dead-black surface may be very strongly illuminated—i.e., it may be receiving a very large amount of flux per unit of area—and yet its brightness may be quite negligible, since the flux is almost entirely absorbed.

The most important units of luminous intensity in use at the present time are the *International candle*, the German unit known as the *Hefner unit*, and the French units known as the *bougie-décimale* and the *Carcel unit*. The relations connecting these units are as follows:—

* Sometimes termed *intrinsic brilliancy*, or *specific intensity*. Instead of intensity per unit area, we might consider flux emitted per unit area ($= 4\pi \times$ intensity per unit area); this has by some writers been termed *specific radiation* or *specific flux*.

1 Hefner unit = ·90 international candle.
1 International candle = 1·11 Hefner units.
= 1 bougie-décimale.
= ·104 Carcel unit.

§ 178. Practical Standards of Luminous Intensity.

Most of the practical standards of luminous intensity are *flame* standards. Among these may be mentioned the (now discarded) British candle, the pentane lamp, the Carcel lamp, and the Hefner lamp. The combustibles used in these standards are spermaceti wax, pentane, rape-seed oil and amyl acetate respectively. Standard spermaceti candles have been found to be so variable as to be practically useless in all accurate photometric measurements. They are therefore no longer employed.

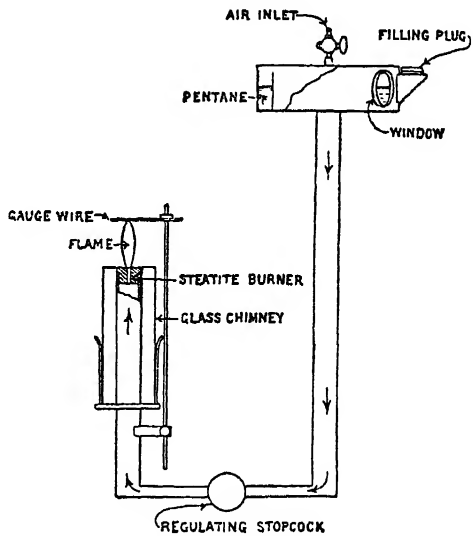

FIG. 148.—Simmance-Abady pentane standard.

Among proposed standards other than flame standards may be mentioned a unit area of incandescent platinum maintained at a definite temperature, and a unit area of the crater of the electric arc.

All flame standards suffer from a serious disadvantage: their intensity depends not only on the pressure of the atmosphere, but also on its humidity, and on the percentage of carbon dioxide present in it. Hence in accurate work account has to be taken of all these factors. In certain localities the normal variations of humidity throughout the year are sufficient to produce a variation of as much as 6 per cent. in the intensity of a flame standard.

For ordinary photometric measurements, by far the most convenient standard is a standardised incandescent lamp of the special construction described below.

In Fig. 148 is shown a form of pentane standard, devised by Simmance and Abady, whose intensity in a horizontal direction represents two candles. The combustible is pentane (C_5H_{12}) vapour. Liquid pentane is poured into the annular space of the cylindrical reservoir shown in the figure, through a side opening which is then closed tightly with a plug. Pentane evaporates very readily, and as its vapour is heavier than air, it falls down the vertical tube attached to the reservoir, at the same time sucking in air through the top air inlet. The mixture of air and pentane vapour descends to the horizontal tube, which is fitted with a regulating stop-cock, and then ascends to the steatite burner*; the latter is of very simple form, consisting of a cylindrical plug perforated by a single central hole. The correct height of flame is indicated by a platinum wire, and the flame is, by means of the regulating stop-cock, adjusted to just touch the wire.

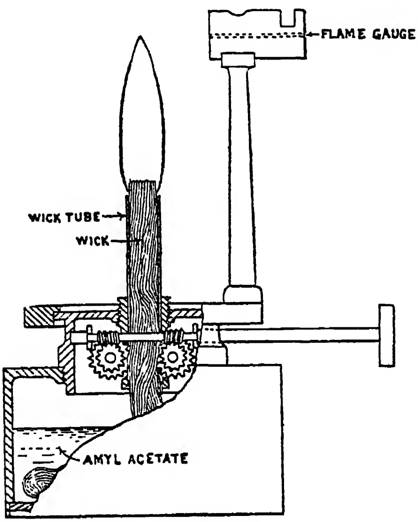

FIG. 149.—Hefner lamp.

The Hefner unit is represented by the intensity, in a horizontal direction, of a standard form of lamp, known as the Hefner lamp, which burns amyl acetate ($C_7H_{14}O_2$).† This lamp is shown in Fig. 149. The combustible is contained in a cylindrical bronze vessel whose inner surface is tinned. The

* Steatite, French chalk, or soapstone, is a silicate of magnesium having the composition $Mg_2Si_3O_8, 2H_2O$.

† Amyl acetate (pear essence) is prepared by distilling fousel oil (obtained in the distillation of spirits from fermented grain or potatoes) with acetic and sulphuric acids. It is characterised by its fruity, pear-like odour.

wick consists of a number of cotton strands laid together quite loosely, and passes through a German silver tube. It may be raised or lowered by means of toothed wheels which pass through the sides of the tube and bite into the wick. The wheels are rotated simultaneously in opposite directions by the double worm-wheel mechanism clearly shown in the figure. A gauge is provided for adjusting the height of the flame. There is hardly any consumption of the wick if the lamp is in proper working order.

As compared with the pentane standard, the Hefner lamp has the disadvantage of yielding a much redder light, which renders it less convenient for use when measuring the intensity of sources giving a white light.

For many purposes, a standard whose intensity is only one or two candles is somewhat too feeble when the source of unknown intensity to be compared with the standard is very powerful. Hence in practical photometry various sources of greater intensity are frequently employed. One such source is the Vernon Harcourt 10-candle pentane lamp. The general arrangement of this is similar to the 2-candle pentane standard already described, but the lamp is of more elaborate construction. Another standard, which, however, is only a secondary or auxiliary one, and which requires re-standardisation at intervals by comparison with a primary standard, is an incandescent lamp supplied at a definite accurately measured p.d. Dr. Fleming has introduced a form of this standard in which the carbon filament is contained in a much larger bulb (six or eight inches in diameter) than that ordinarily used. The filament is first mounted in an ordinary bulb, and run for about 100 hours at 5 per cent. above its normal p.d. It is then transferred to the larger bulb; this treatment renders the intensity of the lamp constant (for a given p.d.) over considerable periods. As the intensity, in a horizontal plane, of an incandescent lamp varies in different directions, it is necessary, in employing this standard, to use the intensity along a perfectly definite direction in a horizontal plane through the lamp (the axis of the lamp being, as usual, vertical).

* J. A. Fleming, *Handbook for the Electrical Laboratory and Testing Room*, vol. iii. p. 260; also *Journal of the Institution of Electrical Engineers*. vol. (1907).

§ 179. Law of Inverse Square. Brightness and Lambert's Law.

Let in Fig. 159 S represent a given source of light, and let a cone of small angle be described whose vertex is at the source. It is clear that owing to the rectilinear propagation of luminous flux in a homogeneous medium, any flux sent into the cone in the neighbourhood of its apex will remain within the cone, and if the medium is non-absorbent, then the same amount of flux must cross each section of the cone, wherever this section may be situated. Hence, considering two parallel plane sections, a and A, normal to the axis of the cone, we see that if F denote the flux contained within the cone, the illumination of the two sections will be $\frac{F}{a}$ and $\frac{F}{A}$, since (§ 182) illumination is flux per unit of area. But since the areas are directly as the squares of their distances from the vertex of the cone, it follows that the illuminations of a and A are to one another as $\frac{1}{l^2} : \frac{1}{L^2} = L^2 : l^2$, or *the illumination of any area normal to the flux varies* (along a given direction) *inversely as the square of its distance from the source.* This *law of the inverse square* forms the foundation of all photometric measurements.

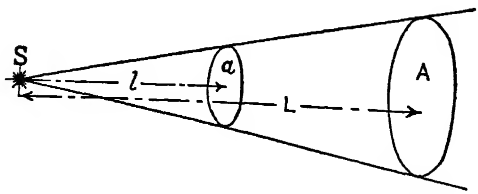

FIG. 150.—To illustrate law of inverse squares.

Imagine a small plane surface of area a_1 (Fig. 151) which receives a certain luminous flux, f, the surface being normal to the direction of the flux. The illumination of the surface is f/a_1. Imagine next that the surface is withdrawn, and for it substituted a plane surface of area a_2, which is such that the whole of the flux f formerly incident on a_1 is now received by a_2. The illumination of the surface a_2 is $f/a_2 = \frac{f \cos \theta}{a_1}$, where θ is the angle between the two surfaces, or the angle between the direction of the flux and the normal to the surface a_2. From this we see that *the illumination* of a surface whose inclination to the

direction of the flux is varied is *proportional to the cosine of the angle between the normal to the surface and the direction of the flux.*

The brightness (intrinsic brilliancy) or luminous intensity per unit of area of a surface which is either self-luminous or which emits light by diffuse reflection, depends on the direction along which the intensity is considered, and is found to vary directly as the cosine made by the given direction with the normal to the surface. This law (which, however, is only approximately true) is known as *Lambert's law* or the *cosine law.*

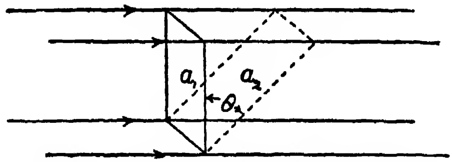

FIG. 151.—Effect of obliquity on illumination.

The following examples of the numerical values of brightness may be of interest:

Source.	*Brightness in c.p. per sq. inch.*
Candle flame	4
Oil lamp flame	8
Cooper-Hewitt mercury vapour lamp	15
Carbon glow-lamp	400
Tantalum "	600
Tungsten "	1,000
Flaming arc	5,000
Crater of pure carbon arc	200,000

§ 180. Units of Luminous Flux and Illumination.

If we imagine a perfectly uniform source, i.e., one having the same intensity along every direction, and if we suppose a sphere of unit radius described around this source as centre, then, assuming that the intensity of the source is unity, the luminous flux incident per unit area of the sphere is taken to represent the unit of luminous flux. If the intensity of the uniform source is equal to one (international) candle, the unit of luminous flux is termed a *lumen.* Since the total area of a spherical surface of unit radius is 4π, it follows that a uniform source of unit intensity emits 4π units of luminous flux. The illumination of the spherical surface of unit radius under the above conditions is, in accordance

with the definition of illumination, clearly unity, since the total flux through the surface is 4π, and the area of the surface is also 4π.

In dealing with British units, it has been customary to take the candle as the unit of intensity, and the foot as the unit of length. The unit of illumination is then known as a *foot-candle* or *candle-foot*. We might define a foot-candle as the illumination of a surface, every point of which is at a distance of one foot from a source of intensity equal to one candle, the surface being everywhere normal to the rays.*

In France, the *bougie-metre* and *carcel-metre* have been used as units of illumination. The former unit is ·104 of the latter.† Since the *bougie-décimale* is equivalent to the candle, it follows that the foot-candle is equivalent to about 10·8 bougie-metres, or to about 1·12 carcel-metres. The bougie-metre has been termed a *lux*.

The following are a few examples of the values of illumination in various cases. For reading, 1 candle-foot may be taken to be a minimum, and to secure comfort, about 3 candle-feet are desirable. For drawing, about 4 candle-feet should be provided. The ordinary illumination of shop interiors is of the order of 5 candle-feet, while for show windows as much as 20 candle-feet may be used.

§ 181. Photometric Measurements. Photometers.

The problems which most commonly arise in photometry are the following: (1) The comparison of the luminous intensity, along any given direction, of some unknown source with the intensity of a given standard; (2) the determination of the total luminous flux, or the mean spherical intensity ($= \frac{1}{4\pi} \times$ luminous flux, § 180) of a source; (3) the determination of the

* The reader may be helped towards a clearer understanding of the relationship of the various photometric units by considering their analogy to the corresponding magnetic units. Luminous intensity corresponds to pole-strength. A unit pole gives rise to a magnetic flux of 4π lines; a unit (uniform) source produces 4π units of luminous flux. Magnetic flux density or induction corresponds to luminous flux density or illumination. The unit of luminous flux might be defined as the flux emitted by a unit source per unit solid angle.

† The Carcel unit is represented by an oil lamp—the Carcel lamp—provided with an Argand burner, and burning well-purified rape-seed oil. 1 carcel = 9·6 *bougies-décimales*.

illumination at a given point or points of a surface. The first measurement is effected by means of some form of apparatus known as an *intensity photometer*, or photometer simply; the second involves the use of an *integrating photometer* or *lumen-meter;* while the instrument employed in connection with the third measurement is known as an *illumination photometer*.

Photometers for the comparison of the intensities of two sources are based on the law of the inverse square (§ 179). Numerous forms of this type of instrument have been devised, but the general arrangement in most cases is as shown in Fig. 152. The two sources under comparison, S_0 and S, are arranged at the ends of a table provided with a graduated scale along which may be slid a screen whose plane is normal to the line joining S_0 and S. Such a table or bench is known as a *photometric bench;* the screen, which in some cases is entirely opaque and in others translucent, is enclosed in a box known as the *photometric box*, which is provided with suitable apertures for admitting the fluxes from the two sources and enabling the observer to see the appearance of the two sides of the screen. The position of the screen is varied until its two surfaces appear equally illuminated. When this is the case, we must, by the law of the inverse square, have $\frac{S}{l^2} = \frac{S_0}{l^2_0}$,

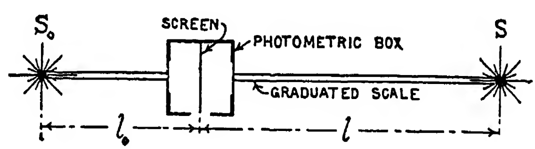

FIG. 152.—General arrangement of photometer.

where S and S_c are taken to represent the intensities of the unknown and the standard source respectively.

Hence $S = \frac{l^2}{l^2_0} S_0$.

The differences between the various kinds of intensity photometers consist in the different methods adopted to enable the observer to judge of the relative illumination of the two sides of the screen.

§ 182. Lummer-Brodhun Photometer.

One of the most accurate forms of intensity photometer is that devised by Lummer and Brodhun, and illustrated in Fig. 153.

The screen is of an entirely opaque white material (plaster of Paris or magnesia). Some of the light diffused by each side of the screen is received by a plane mirror (or totally reflecting prism), and after regular reflection passes into the combination of two prisms shown in the figure. One of these is an ordinary prism, while the other (*P*) has portions of its largest surface roughened or "frosted" by a sand-blast, the remaining portions being in optical contact with the surface of the other prism. Consider first the light coming from the left-hand side of the screen. On reaching the surface of contact of the two prisms, the light will freely pass through those portions of *P*'s surface which are in optical contact with the right-hand prism, and so will enter the small telescope and reach the eye of the observer. But over the frosted portions of *P*'s surface the light will be irregularly reflected or diffused, and will be prevented from passing into the right-hand prism. Thus the only light reaching the observer from the left-hand side of the screen is that passing through the clear or optically worked portions of *P*'s surface. Considering next the light coming from the right-hand side of the screen, we see that on reaching the common surface of the two prisms the light will be freely transmitted through the portions of the surface corresponding to optical contact, and will not reach the eye of the observer; while over those portions of the surface which are opposite, but not in actual contact with, the frosted parts of *P*, total reflection will occur, and the light will be received by the observer.

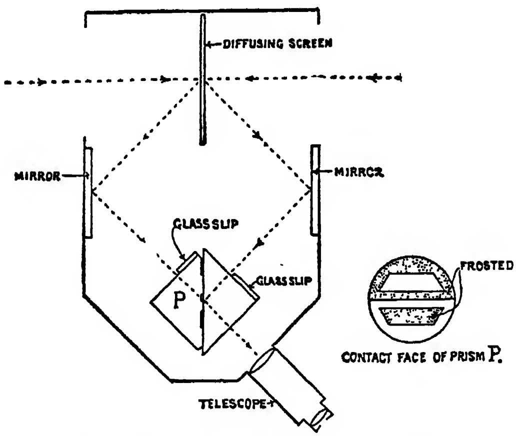

FIG. 153.—Lummer-Brodhun photometer.

To the observer, therefore, the field of view of the telescope appears divided into four regions, two trapezoidal and two peripheral regions. Of these, one trapezoidal region and the

peripheral region not immediately adjacent to it are illuminated by light from the left-hand side of the screen, while the remaining two regions are illuminated from the right-hand side of the screen. If the instrument were used in this simple form, then, corresponding to equality of illumination of the two sides of the screen, the four regions of the field would merge into a single uniformly illuminated field. The accuracy of the adjustment may, however, be increased by using, in addition to the principle of *equality of illumination*, another principle: that of *equality of contrast*. This is done by placing two glass slips against the sides of the prisms as shown, so that the light corresponding to either trapezoidal region suffers greater absorption than that coming from the peripheral regions. The appearance presented in the position of balance is now a uniformly illuminated field with two darker trapezoidal patches standing out against it, and the observer is guided partly by the more or less complete fusion of the peripheral regions, partly by equality of contrast between each trapezoidal patch and the peripheral region immediately surrounding it. It will be noticed that there is no *physical* line of demarcation between the different regions—a fact which materially contributes to the accuracy of the adjustment.

§ 183. Flicker Photometer.

Another type of photometer which has come into favour is the *flicker photometer*. The particular form to which Fig. 154 refers has been devised by Simmance and Abady. The name of the instrument is due to the fact that balance is obtained by adjusting the position of the photometric box so as to cause the flickering appearance of the screen to disappear. The essential part of the instrument consists of a rotating opaque white screen of special shape, driven by clockwork or a small electric motor. The two intersecting curved surfaces forming the edge of the rotating disc are the surfaces of two cones whose axes are parallel to the axis of

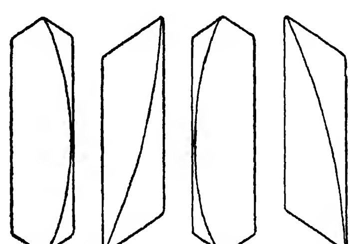

FIG. 154.—Rotating disc of flicker photometer.

revolution of the disc, but are at equal distances from, and on opposite sides of, it. Fig. 154 shows four positions of the disc as seen by the observer, the consecutive positions representing

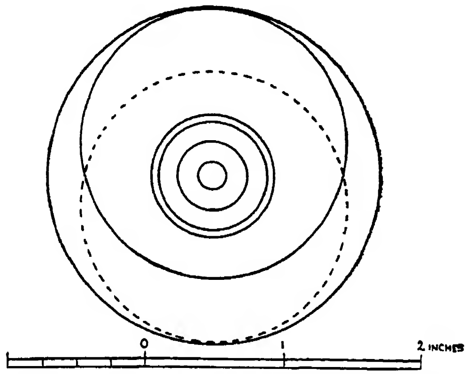

FIG. 155.—End view of flicker photometer disc.

angular displacements of one-quarter revolution. Fig. 155 shows an end view of the disc. The curved surface of each cone

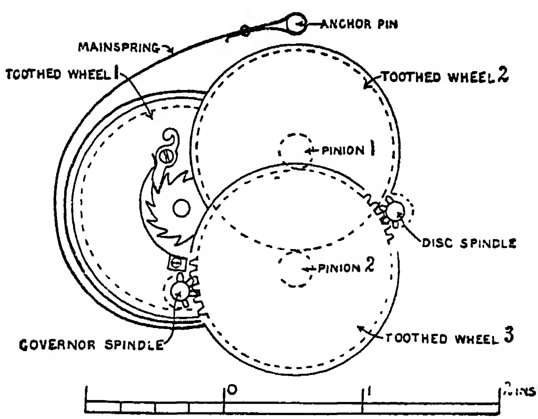

FIG. 156.—Wheel train of spring motor.

is illuminated by the light coming from one only of the sources under comparison. If the illuminations are unequal, then since the line of intersection of the two surfaces during the rotation of the disc appears to travel to and fro, a flickering appearance

will be presented by the edge of the disc. The position of the photometer box is adjusted until the flicker as nearly as possible disappears.

Figs. 156 and 157 show the working parts of the spring motor which drives the disc. Referring to Fig. 156, the torque of the mainspring is transmitted, by means of a toothed wheel fixed to the spring barrel, to the pinion 1, whose axle carries the toothed wheel 2. This latter drives the pinion 2, on the same axle with which is mounted the toothed wheel 3. This wheel drives two pinions, one of which is mounted on the same axle

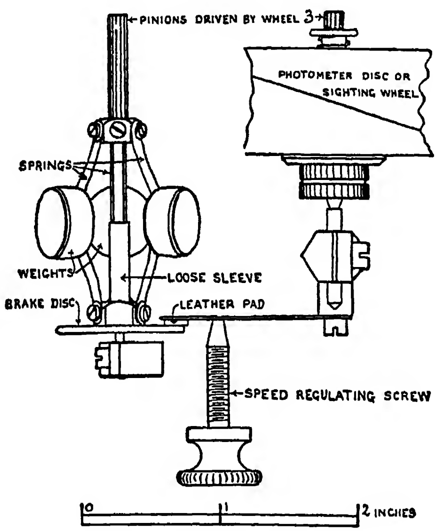

FIG. 157.—Details of spring motor.

as the disc, while the other is on the governor spindle. Fig. 157 shows details of the centrifugal governor and the speed-regulating screw.

A serious difficulty encountered in most photometric measurements is the *difference of colour* of the two sources under comparison. If the colour difference is very great, it is difficult to estimate equality of illumination of the two surfaces with any degree of certainty. One of the main advantages of the flicker photometer is that the colour difficulty is to a large extent eliminated.

§ 184. Ulbricht's Globe Photometer.

For the measurement of the mean spherical intensity, various photometers have been devised. One of the best known is Ulbricht's *Globe Photometer*, shown in Fig. 158.* The source of light is hung at the top of a large globe (about 6 feet in diameter) constructed of plaster of Paris supported by a wire and wire gauze framework. In some cases, sheet-metal hemispheres

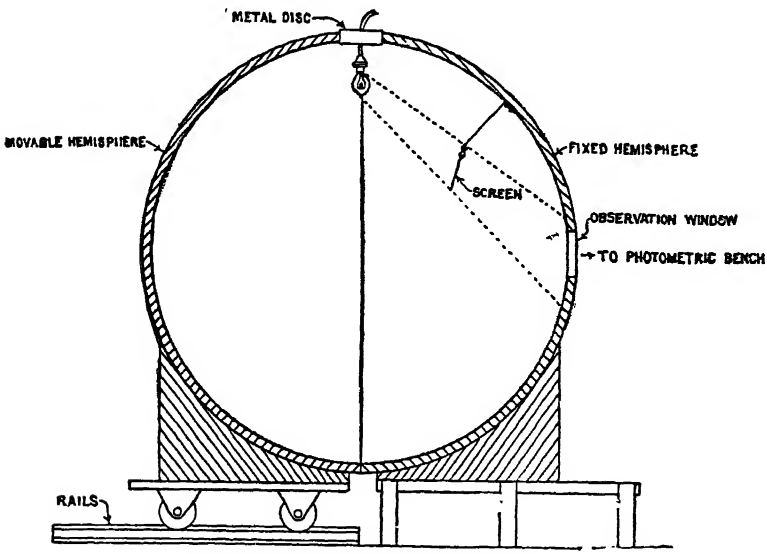

FIG. 158.—Ulbricht's globe photometer.

painted white with barium sulphate on their inner surface have been used.† The globe is divided into two hemispheres by a diametral plane, which is most conveniently arranged to be vertical, as shown in the figure (sometimes a horizontal dividing

‡ *Electrical World*, vol. liv., p. 727 (1909) ; *Elektrotechnik und Maschinenbau*, vol. xxviii., p. 659 (1910).

* The form of construction illustrated in Fig. 163 is due to M. Corsepius. See the *Elektrotechnishe Zeitschrift*, vol. xxvii., p. 468 (1906).

† According to E. Winkler (*Schweizerische Elektrotechnische Zeitschrift*, vol. iv., p. 110 (1907)), the best result is obtained by first coating the inner surface of each hemisphere with white enamel, and then giving it a double coating of zinc white mixed with unboiled milk.

plane has been used: this, however, is not so convenient). One of the hemispheres is fixed at the end of the photometric bench, and is provided with a circular observation window of milk-glass through which light passes out of the globe along the bench, while the other hemisphere is mounted on a truck running on a couple of rails.

By means of two handles attached to the movable hemisphere, this may be wheeled away from the fixed hemisphere so as to give ready access to the metal disc from which the lamp is supported. The observation window is screened from the direct light of the source by a screen which is large enough to give a safe margin of screening all round the window.

The theory of this photometer is, briefly, as follows: Referring to Fig. 159, let S denote the source, which may be situated anywhere inside the globe, and let r be the radius of the globe or sphere. Let a small area a, situated anywhere on the surface of the sphere, receive a luminous flux f from the source. Owing to the whiteness of the matt surface a large proportion of the incident flux will be diffusely or irregularly reflected, and the area a will itself act as a secondary source, sending out luminous flux to all portions of the spherical surface. The luminous intensity of the area a in a normal direction may be written $k\,f$, where k is a constant whose value depends on the reflecting power of the surface. Consider now the illumination at any point P of the sphere due to the flux sent out by a. Since the intensity of a along the direction towards P is, by the cosine law (§ 179), $k\,f\cos\theta$, the illumination at P due to a is $\frac{kf\cos\theta}{x^2}\times\cos\theta=\frac{k}{x^2}\cdot f\cos^2\theta$. Now since $\cos\theta=\frac{x}{2r}$, we have for the illumination at P due to a the value $\frac{k}{x^2}\times f\times\frac{x^2}{4r^2}=\frac{k}{4r^2}\cdot f$, and since $\frac{k}{4r^2}$ is constant, we arrive at the remarkable result

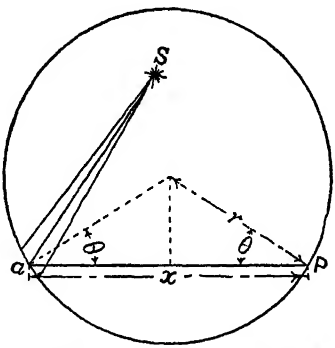

FIG. 159.—To illustrate theory of globe photometer.

that the illumination due to the flux reflected from a is *uniform over the entire sphere,* and is proportional to the total flux incident on a. A small area at P will in its turn act as a tertiary source, and, by the above reasoning, will again produce uniform illumination over the entire surface of the sphere. The final result, then, is that the flux received by the area a produces a certain uniform illumination of the entire sphere, this illumination being proportional to the flux incident on a. Similar reasoning applies to every portion of the spherical surface which receives flux from the source, and hence we see that the total illumination of the sphere *due to reflected flux* is proportional to the total flux sent out by the source, and hence to the mean spherical luminous intensity of the source.

It is to be particularly noted that it is not the *total* illumination at any point of the spherical surface that is proportional to the intensity of the source, but only that part of the illumination which is due to flux *reflected from the remaining portions of the spherical surface.* The *total illumination* is by no means uniform, as it includes, in addition to the illumination due to repeated reflections—which, as we have seen, is uniform—the illumination due to the source, which is clearly a variable quantity, depending not only on the way in which the intensity of the source varies along different directions, but also on the position of the source inside the globe. Hence it is that in determining the intensity of the source it becomes necessary to screen the milk-glass observation window from the direct light of the source by means of a screen, as shown in Fig. 158. The luminous flux coming through the observation window will then be proportional to the mean spherical intensity of the source, and it only remains to explain how the absolute value of the mean spherical intensity may be determined. This is most conveniently done by first using a source whose mean spherical intensity has previously been determined (by finding its intensity along a sufficient number of different directions) and then the source of unknown intensity, balance being in each case obtained against any suitable standard source. The intensities are inversely as the squares of the distances of the photometric box from the standard in the two cases.

Since the construction of a globe photometer presents considerable difficulties, Dr. W. E. Sumpner suggested, in 1910, the substitu-

tion of a rectangular box for the sphere,* and a cubical box (of 2-metre edge) has been used at the National Physical Laboratory. Such a cube does not, however, obey the law of Ulbricht's globe photometer, and simple direct comparisons of m.s.c.p. like those permissible with the sphere are only possible in the case of sources having similar polar curves of intensity (§ 187).

In carrying out photometric measurements, it is extremely important to exclude all extraneous light. Hence the walls of the photometric room, its floor and ceiling, and, in fact, every surface in it (such as that of the photometric bench itself) should be painted dead black.

§ 185. Trotter's Illumination Photometer.

A very convenient form of illumination photometer, originally devised by A. P. Trotter, and subsequently improved by him in

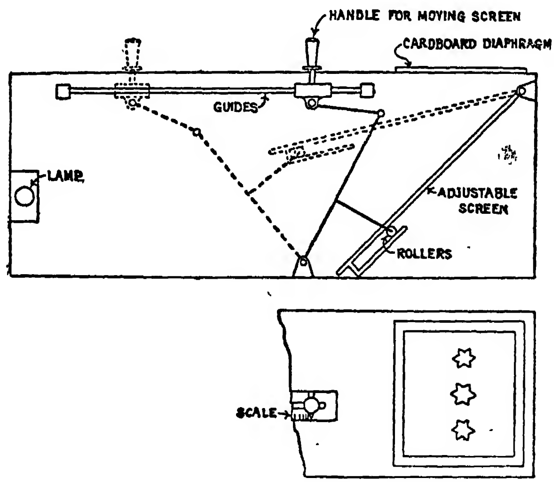

FIG. 160.—Trotter's illumination photometer.

conjunction with W. H. Preece, is shown in Fig. 160. Inside a rectangular box, which may be supported on a suitable tripod stand, are arranged a low-voltage incandescent lamp (supplied by a few secondary cells) and a movable hinged screen. The screen is illuminated by the lamp, and its illumination may be varied by altering its inclination (§ 179); this is done by means

Illuminating Engineer (London), vol. 3, p. 323 (1910); see also *Journal of the Institution of Electrical Engineers*, vol. 59, p. 143 (1921).

of the linkwork shown in the figure. Fixed at one end of the top of the box is a white cardboard diaphragm pierced with three star-shaped holes, through which the adjustable screen may be viewed. This diaphragm is illuminated by the external source or sources whose illumination in a horizontal plane at the point corresponding to the position of the photometer is required. By suitably altering the inclination of the screen while viewing it through the holes in the diaphragm, the two illuminations (of screen and diaphragm) may be made equal, in which case the star-shaped openings would become more or less indistinguishable. As, however, the illumination of the screen is not uniform (since different points of it are at different distances from the lamp), it is not possible to cause all three stars to disappear simultaneously, and the position of balance is taken to be that corresponding to the disappearance of the middle star.

The instrument is graduated by producing known illuminations of the diaphragm (by placing a source of known intensity at a known distance from it), and finding the corresponding positions of the handle which give balance.

By placing the instrument on its side, so that the diaphragm occupies a vertical position, the illumination in any vertical plane may be determined.

§ 186. Efficiency of Luminous Sources.

The most rational way of expressing the efficiency of a luminous source is to state the luminous flux obtainable per unit of power, as, *e.g.*, the lumens (§ 180) per watt; or, since the total flux is proportional to the mean spherical candle-power, as the m.s.c.p. (mean spherical candle-power) per watt. Unfortunately, in the past it has been customary to use the reciprocal of this ratio the watts per candle—and to speak of this as the "efficiency" of the source. If watts per candle are considered, it is more correct to speak of them as the "inefficiency" in order to prevent confusion.

In the following Table are given the approximate luminous inefficiencies of some of the more important types of electric lamps:—

The linkwork is designed so as to give an open scale. In the most recent form of this instrument, the mechanism for altering the inclination of the screen is somewhat different from that shown in Fig. 160.

Type of lamp.	*Watts per mean spherical candle-power.*
Ordinary arc lamp (clear globe)	1·3
Flame arc lamp	·35
Cooper-Hewitt lamp	·5
Quartz tube mercury vapour lamp	·25
Carbon filament incandescent lamp	3 to 4·5 (according to size and voltage)
Ordinary tungsten lamp	1·6
Nitrogen-filled tungsten lamp	·7

The candle-power of a carbon filament incandescent lamp varies approximately as the cube of the watts, and as the 6th power of the volts; and that of a tungsten lamp, as the 2·4th power of the watts, and the 3·8th power of the volts. It will be noticed that the variation of candle-power with voltage is much less in the case of the tungsten filament than in that of the carbon filament lamps.

Of the total power supplied to an incandescent lamp, only the small fraction (2 to 8 per cent.) radiated in the form of *luminous energy* produces a useful effect. The bulk of the power wasted goes towards producing non-luminous heat radiation. The remainder of the waste power is represented by the heating loss in the leading-in wires and joints (this does not exceed ·2 per cent.), thermal conduction along the leading-in and anchoring wires (this is of the order of 4 to 6 per cent., but varies between the wide limits of 2 and 14 per cent. in lamps of different types and voltages), and gas conduction and convection loss. The latter is inappreciable in vacuum bulb lamps of good make, but is considerable (over 20 per cent.) in lamps with nitrogen-filled bulbs.

§ 187. Distribution of Intensity.

The intensity of a given source varies along different directions, and the distribution of the intensity is a matter of great practical importance. In some cases—as, e.g., that of an ordinary arc lamp—the intensity is distributed symmetrically about an axis, and the law of distribution will be completely known if we determine the distribution of the intensity in any plane passing through the axis of symmetry. Graphically the distribution of intensity in any plane may be conveniently represented by drawing a series

* For an analysis of the minor losses in incandescent lamps, see a paper by E. P. Hyde, F. E. Cady and A. G. Worthing in *Illuminating Engineering*, vol. 4, p. 389 (1911).

of lines radiating from the point corresponding to the position of the source, and laying off along each line a length representing, to a convenient scale, the intensity along that direction. By joining the extremities of the lines we obtain a curve known as the *polar curve of luminous intensity.** The typical form of polar curve for a naked arc is shown in Fig. 161, where it is represented by the full-line curve O R S. It will be noticed that the bulk of the luminous flux is directed downwards (the greater part coming from the crater).

§ 188. Rousseau's Construction.

If the polar curve of intensity for a source symmetrical about an axis is known, the mean spherical intensity is easily determined by a graphical method due to Rousseau. Rousseau's construction is shown in Fig. 161. With O as centre and any convenient radius, describe a semi-circle. The revolution of this about its vertical diameter would generate a spherical surface. Let the diameter be divided into any convenient number of equal parts, and through each point of division let a plane be drawn which is perpendicular to the diameter. The spherical surface will thereby be sliced up into a number of parts of equal area, and if the sub-division has been carried sufficiently far, the mean illumination of each slice of the spherical surface surrounding the source may be regarded as equal to the illumination along the circle bisecting the slice. If, for instance, P be the middle point of A B, where A B represents one of the equal parts into which the diameter has been divided, and if a plane perpendicular to the diameter be drawn through P, then the mean illumination of the slice of the spherical surface included between the planes through A and B may be taken to be equal to the illumination at Q, or the illumination along the circle of intersection

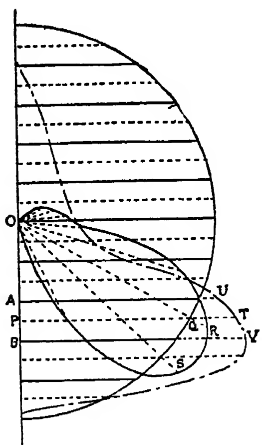

FIG. 161.—Rousseau's construction.

* This curve is sometimes also termed the *characteristic* of the symmetrical source.

with the spherical surface of the plane through P. Since, however, all points of the spherical surface are at the same distance from the source, the illumination at any point will be simply proportional to the intensity of the source along the line joining the given point to the source. Thus the illumination at Q will be proportional to O R, R being the point of intersection of O Q (or O Q produced) with the polar curve of intensity. Along P Q, or P Q produced, lay off P T = O R; by repeating the construction for the various slices, and joining all the points T so found, we obtain the chain-dotted curve shown in the figure. Now the total luminous flux passing through any slice, such as that corresponding to A B, is equal to the product of the area of the slice into the mean illumination; and since the area is proportional to A B, and the mean illumination to P T, the flux passing through the slice is proportional to A B × P T, or to the area of the strip A B V U. The same reasoning applies to every slice, and hence the total luminous flux is proportional to the area included between the vertical diameter and the chain-dotted curve. A similar construction carried out for a *uniform* source would give for the chain-dotted curve a straight line parallel to the vertical diameter, and the total flux from the source would be proportional to the area of the corresponding rectangle. If the intensity of the uniform source be so chosen that the area of this rectangle is equal to the area of the chain-dotted curve corresponding to the actual source, then the total flux from each source is the same, and the intensity of the uniform source is, by definition, the m.s.c.p. of the actual source. From this it at once follows that the m.s.c.p. of the given source is equal to the mean ordinate* of the chain-dotted curve, or to the height of a rectangle constructed on the diameter as base and equal in area to the chain-dotted curve.

If the source is not symmetrical about a vertical axis, Rousseau's construction may still be used, provided each radius of the polar curve corresponds to the mean intensity of the source along the surface of a cone whose semi-vertical angle corresponds to the angle made with the vertical by the given radius of the polar

Measured horizontally, the diameter being regarded as the axis of abscissae. It must be clearly understood, in estimating the mean ordinate of the chain-dotted curve, that the base of this curve corresponds to the *entire* diameter and not merely to that part of it included between the points at which the abscissae first vanish.

curve. Thus, in Fig. 161, O R would in such a case represent the mean intensity of the source along different positions of the generating line of a cone whose vertex is at O and whose semi-vertical angle is B O R.

Rousseau's construction is at present not of much practical interest in connection with rapid determinations of the m.s.c.p., as such determinations are much more conveniently carried out by means of special photometers, such as Ulbricht's Globe photometer (§ 184). Its importance, however, lies in the fact that it enables us to determine the constant of a photometer of this type.

§ 189. Mean Hemispherical Intensity.

The more powerful sources of light, such as flame arc lamps and large nitrogen-filled tungsten lamps, are frequently used for the illumination of large open spaces, in which case it is desired to obtain as large a flux as possible in a *downward* direction, any flux thrown upwards being in many cases practically useless. For this reason, it has become customary to consider what is known as the *mean hemispherical candle-power* (m.h.s.c.p.) of a source. Let a sphere be described around the source as centre, and let the sphere be bisected by a horizontal plane; then the mean hemispherical intensity of the given source is the intensity of a uniform source which is capable of producing a luminous flux through the lower hemisphere equal to that of the actual source; or the m.h.s. intensity is the intensity of a uniform source whose total flux is equal to twice the flux through the lower hemisphere due to the actual source.

§ 190. Rating of Lamps. Reduction Factor.

Although the most rational method of rating a given lamp is to state its m.s.c.p., manufacturers some time ago decided to list their lamps according to the *watts* taken by them, without any direct reference to candle-power. Formerly the nominal rating used by manufacturers was based on the mean c.p. in a horizontal

plane, or the mean horizontal candle-power (m.h.c.p.). The variations in the intensity in a horizontal plane are considerable and somewhat irregular.* A method frequently employed for determining the m.h.c.p. is to mount the lamp in a special holder which may be rotated about a vertical axis, at about 120 revolutions per minute, and to find the c.p. while the lamp is being rotated.

For a given type of lamp, the ratio $\frac{\text{m.s.c.p.}}{\text{m.h.c.p.}}$ is found to be approximately constant. This ratio is termed the *spherical reduction factor* for the given type of lamp, and in the usual types it varies from about ·78 to about ·90. For metal filament lamps of the type shown in Fig. 156, the spherical reduction factor has in most cases the value ·8. According to A. Russell,† the spherical reduction factor for carbon filament lamps may be determined from two measurements, that of the m.h.c.p. I_h and that of the candle-power I_v, in a vertical direction, by means of the formula—

$$\text{spherical reduction factor} = {\cdot}785 + {\cdot}11\,\frac{I_v}{I_h}.$$

§ 191. Useful Life of Lamps.

An important item in the total cost of lighting when using a given type of lamp is the cost of lamp renewals. This depends on the cost of the lamp (or of the renewable part of it) and on the useful life of the lamp. The candle-power of most lamps decreases with age (in some cases there is a rise of c.p. during the initial stages), and their inefficiency increases. It is nowadays customary to regard a lamp as having reached the end of its useful life when its c.p. decreases by 20 per cent. from its initial value. The useful life of a lamp depends very largely on its inefficiency; the higher the latter, the longer is the life of the lamp. The gradual deterioration which takes place in incandescent

* A. Russell, *Journal of the Institution of Electrical Engineers*, vol. xxxii., p. 631 (1903); G. B. Dyke, *Proceedings of the Physical Society of London*, vol. xix. p. 399 (1904).

† A. Russell, *Journal of the Institution of Electrical Engineers*, vol. xxxviii. p. 315 (1907).

lamps is due to two causes: the blackening of the bulb by the deposition of particles shot off from the filament, and a change in the nature of the filament surface.

Examples.

1. If the brightness (intrinsic brilliancy) of the crater of an arc is 150 candle-power per square mm. of its surface, and if the total crater area amounts to 16 square mms., what is the luminous intensity of the crater in a direction making an angle of 45° with the axis of the carbons?

2. In a certain photometer, the distance apart of the two sources is 60 inches, the standard source being represented by a Hefner lamp. If balance is obtained with an unknown source when the photometric box is at a distance of 12 inches from the Hefner lamp, what is the intensity of the source?

3. Two arc lamp posts are placed 120 feet apart. One lamp is 40 feet, and the other 35 feet above the level of the ground. Find the illumination, in candle-feet, at a point on the ground 35 feet from the lower lamp, on the line joining the two posts, if the intensities of the lower and higher lamps in the direction of the point are 750 and 420 candles respectively.

CHAPTER XVII.

§ **192.** Types of switches—§ **193.** Principles of switch design—§ **194.** Mechanical construction of switches—§ **195.** Conditions determining resistivity of switch contact—§ **196.** Regulating switches for secondary batteries—§ **197.** Field breaking switches—§ **198.** Methods of suppressing arcing at switch contacts —§ **199.** Automatic switches. Maximum current circuit-breakers—§ **200.** Retarding or time-limit devices. Thermal release circuit-breaker—§ **201.** Minimum and reverse-current circuit-breakers—§ **202.** Fuses.

§ 192. Types of Switches.

By a *switch* in the most general sense of the term is meant an arrangement of movable and fixed contacts by means of which a given change in the connections of a circuit may be effected. The simplest form of switch is that which is intended merely for closing or opening a circuit. If the "make" or "break" takes place at a single point only, the switch is spoken of as a *single-pole* switch. If, on the other hand, simultaneous makes or breaks are arranged to take place at two points, we have a *double-pole* switch. The movable contact of a switch may be arranged to establish connection with one or other of *two* fixed contacts; the switch is then termed a *two-way* one. We may have a *double-pole two-way* switch by suitably combining two single-pole two-way switches. Very frequently, the moving element or contact of a switch is arranged to travel over a large number of fixed contacts; such switches are termed *multiple-way* or *multiple-contact* switches, and we have already come across them in considering *motor-starting* switches (§ 103). The special type of motor-starting and speed regulating multiple-way switch used in connection with tramway, railway and crane motors, is known as a *controller*. Multiple-way switches are frequently used for altering the number of secondary cells connected across the mains, in cases where no automatic reversible booster (§ 155) is employed, and are then known as *battery switches*. In generating stations, the field coils of the large shunt-wound dynamos are frequently arranged to be connected across the 'bus bars.

The common bars between which all the generators are connected in parallel. They form the terminals of the entire generating plant.

and since a sudden break in the field circuit of a large shunt-wound machine would be attended with great danger to the insulation (§ 103), it is usual in such cases to provide a switch of special design, so constructed that the field flux disappears gradually; a switch intended for this purpose is known as a *field-breaking* or *field* switch. In some cases, a *reversing switch* is required for the purpose of reversing the current in a certain part of the circuit.

A switch which is arranged to control simultaneously a number of other switches is spoken of as a *master switch*.

The switches so far enumerated are intended to be operated by hand, and hence may be termed *hand switches*. There is, however, a very large and important class of switches designed to operate automatically under certain conditions, and known as *automatic switches* or *cut-outs*. The most important of these are:—(1) the *maximum current cut-out*, which opens the circuit when the current rises above a certain value; (2) the *minimum current cut-out*, which breaks the circuit if the current falls below a certain limit; and (3) the *reverse-current cut-out*, which operates only if a reversal of current takes place in that part of the circuit in which the cut-out is placed.

§ 193. Principles of Switch Design.

The design of the connections and switch-gear required to perform a definite set of operations may become an extremely tedious task in cases where the operations are complicated, if no better method be used than that of trial and failure. It is therefore extremely important not to trust to any haphazard methods, but to proceed according to definite and systematic rules.

Diagrams should first be drawn showing the connections of the different parts of the circuit in the various cases under consideration.

These diagrams should then be compared with each other. It will in general be found that in addition to the *necessary* connections in each diagram, a number of *superfluous* connections may

An excellent account of the principles of switch-gear design, with a large number of illustrative examples, will be found in R. Edler's *Entwurf von Schaltungen und Schaltapparaten* (Hanover: Dr. Max Jänecke, 1905).

be introduced, such superfluous connections corresponding to *necessary* connections in one or more of the other diagrams. By introducing all the permissible superfluous connections into each diagram, we reduce the differences which originally existed among them. It is evident that since each superfluous connection occurs in each diagram (in *one* of them it is a *necessary* connection), all such connections may be made *permanent*, and will require no switches. Switches will only be required for effecting the changes corresponding to the outstanding differences among the diagrams, and these differences will themselves indicate the simplest arrangement of switch-gear for the purpose in hand.

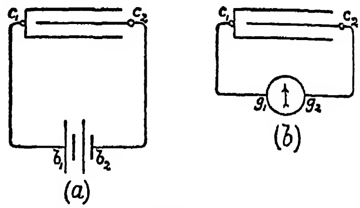

FIG. 162.—Diagrams of connections.

We shall illustrate the use of the above general principles by the aid of a simple example. Let a switch be required which in one position connects a condenser across a battery, thereby charging it, and in the other position disconnects the battery, and connects a galvanometer across the condenser terminals, thereby discharging it. We first draw the two diagrams, Fig. 162 (*a*) and (*b*), showing the *necessary* connections in each case. In order to bring out clearly the important part played by the *superfluous* connections in simplifying the switching arrangements, we shall in the first place purposely neglect the second rule of switch design explained above, and shall omit all superfluous connections.

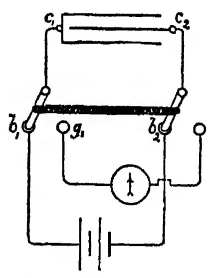

FIG. 163.—Coupled two-way switches.

Comparing the diagrams (*a*) and (*b*) of Fig. 162, we notice that while in (*a*) the point c_1 is in connection with b_1, in diagram (*b*) it is in connection with g_1. Now the passage from b_1 to g_1 may obviously be effected by a simple two-way switch. Similarly, in the case of the point c_2 the passage from b_2 in (*a*) to g_2 in (*b*) may be effected by means of a simple two-way switch. Hence two coupled two-way switches, forming a double-pole two-way switch, as shown in Fig. 163, may be made to effect the necessary change of connections.

But although the arrangement shown in Fig. 163 serves the required purpose, it is by no means the *simplest* possible. This is due to our not having inserted all permissible *superfluous* connections into our diagrams before proceeding to the design of the switching arrangements. On once more comparing the

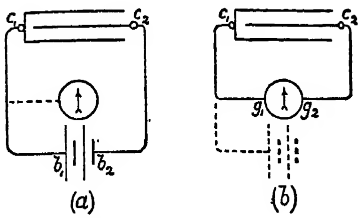

FIG. 164.—Diagrams showing superfluous connections.

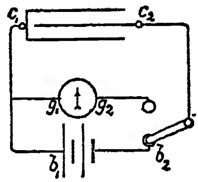

FIG. 165.—Simplified switching diagram.

diagrams (*a*) and (*b*) of Fig. 162, we notice that in passing from (*a*) to (*b*) it is only necessary to disconnect the battery on one side. Similarly, in passing from (*b*) to (*a*) it is only necessary to disconnect the galvanometer on one side. We are thus led to introduce the superfluous connections indicated by the dotted lines in Fig. 164 (*a*) and (*b*). Since these connections may be

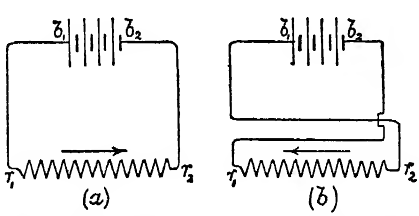

FIG. 166.—Change of connections for current reversal.

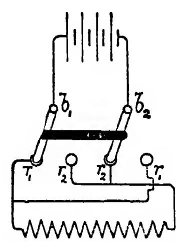

FIG. 167.—Reversing switch.

made permanent, we see that a *single* two-way switch may now be made to serve our purpose, as shown in Fig. 165—a considerable simplification on Fig. 163, where two such switches were employed.

As another example, we shall consider the case of a reversing switch. Let in Fig. 166 the resistance $r_1\ r_2$ be connected across a battery, and let a switch be required for producing a reversal

of current in $r_1 r_2$. The connections corresponding to the two positions of the switch are shown in Fig. 166 (*a*) and (*b*). It will be noticed that in this case *no superfluous connections are admissible*, and hence the switch must be a double-pole two-way switch as shown in Fig. 167, no further simplification being possible.

§ 194. Mechanical Construction of Switches.

The simpler forms of switches may be divided into (1) plug switches; (2) knife switches; and (3) laminated contact switches.

In Fig. 168 are shown three examples of *plug switches*. The first, (*a*), is widely used for ordinary testing work, in resistance boxes, &c., and is generally suitable where small currents only have to be dealt with. It consists of two massive contact-blocks which may be bridged by a conical plug; to secure a good fit, the plug should be carefully ground into its seating. (*b*) and (*c*) are two varieties of large switchboard screw plug switches. Such switches are used for connecting certain bars mounted on the front of the board to others mounted at the back, the plug passing through the switchboard itself. Efficient contact with the bar at the back is secured by screwing the plug into it; while contact with the front bar is obtained either by means of a wide collar bearing against the face of the bar, as in (*b*), or by making the front portion of the plug conical, and fitting it into a corresponding conical hole in the front bar, as in (*c*). It is evident that with plug switches of this type contact can be made or broken only comparatively slowly, and they are only used when the current at the time of making or breaking is either zero or negligibly small.

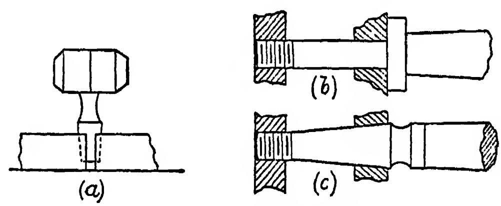

FIG. 168.—Plug switches.

Knife switches are used very largely. They are strong and simple in construction, and suitable for a wide range of currents. A knife switch consists essentially of a movable blade provided

with a handle, by means of which it may be forced between two contact plates or clips. A double-pole switch of this type is shown in Fig. 169. In order to secure good contact with the clips, these are slit so as to form several plates capable of moving more or less independently of each other. The blade and contact clips are made of hard rolled copper. The contacts are carefully ground in. It will be noticed that in the type of switch shown in Fig. 169 the hinges or joints of the knife-blade also serve as electrical contacts, and for the purpose of securing contact with the blade over a sufficient area large washers are placed outside the contact plates as shown. It has been supposed that the electrical contact at the joint is liable to be imperfect, and hence a "double-break" design of knife switch, shown in Fig. 170, has come into use. As a matter of fact, however, the prejudice entertained against switches of the type shown in Fig. 169, with a "single" break, has turned out to be unfounded, for experiments have shown that it is easier to

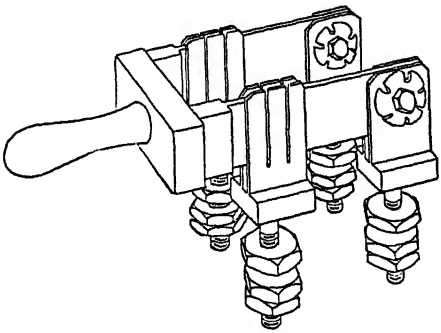

FIG. 169.—Double-pole knife switch (The British Thomson-Houston Co., Ltd.).

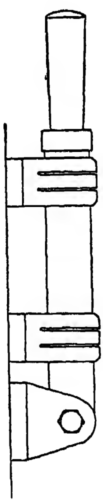

FIG. 170.—Double-break knife switch.

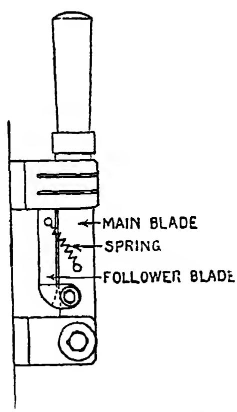

FIG. 171.—Quick-break knife switch.

The grinding should be done with powdered glass and oil; emery is unsuitable, as it becomes embedded in the metal and is difficult to remove.

obtain good contact at a properly ground-in joint than at contact clips.

A difficulty which always occurs with switches intended to deal with large currents is that of arcing at the contacts, resulting in their fusion and the gradual destruction of the switch. In order to prevent fusion of the contacts, the duration of the arc should be reduced as much as possible by a *quick break*. The knife switches just described may be broken quickly by hand; but if the switch blade be moved slowly, excessive arcing may take place. It is therefore desirable in some cases to adopt a design which will always ensure a quick break. A simple form of *quick-break* knife switch is shown in Fig. 171. The blade of the switch consists of two parts, the main blade and a smaller or "flier" or "follower" blade, hinged to the main blade and connected to it by a couple of springs, one on each side of the blade. When the main blade is withdrawn, the friction of the contact clips is sufficient to prevent the follower blade from coming away with the main blade, and the springs are gradually stretched. When a certain position has been reached, however, the inner edge of the main blade comes against the inner edge (shown by the dotted line in Fig. 171) of the tapered lower end of the follower blade, close to the hinge, and the two blades are now constrained to move together as a single piece. At the instant when the follower blade leaves the contact clips, the springs attached to it, being now free to act, cause it to snap away suddenly, giving a very quick break.

Single-blade knife switches are made for currents up to a few hundred amperes. For still larger currents, several blades may be arranged in parallel. The switches then become somewhat heavy to operate by hand, owing to excessive friction. The current-density at the contacts of knife switches should not exceed 50 amperes per square inch of contact area.

In the *laminated contact* type of switch, one example of which is shown in Fig. 172, the movable contact takes the form of a number of hard copper or brass strips clamped together, the edges of the strips being arranged to bear on the fixed contact blocks. Since the strips or laminations are to some extent free to move independently of each other, efficient contact is made by each of them, and as a result it is found that the current-density at the contact may be as high as 400 or even 500 amperes per

square inch of contact area. The motion of the laminated relatively to the fixed contacts is either parallel to the contact surfaces—as in the switch shown in Fig. 172—the edges of the laminations sliding along the fixed contact surfaces when the switch is operated, or at right angles to them, in which case the

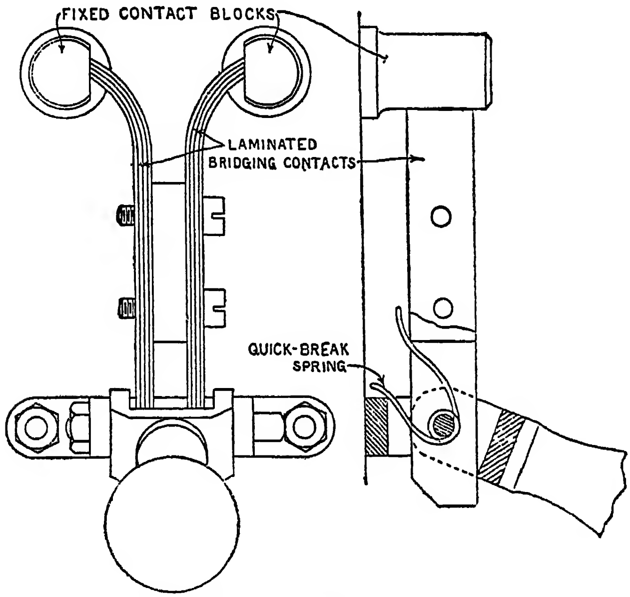

FIG. 172.—Laminated contact switch.

laminated contact which forms a bridge across the two fixed contact blocks is simply pressed against them—as in the automatic switch illustrated in Fig. 177.

§ 195. Conditions Determining Resistivity of Switch Contact.

The main factors which determine the resistivity of a switch contact are: (1) the mechanical stress intensity between the surfaces; (2) the nature of the two metals in contact; (3) the physical and chemical conditions of the surfaces; (4) the medium in which the contact is immersed; (5) temperature. To a

minor extent, current density exerts some effect on the contact resistivity.

The general nature of the relation connecting the contact resistivity with the stress intensity is indicated by Fig. 173. As will be seen, the resistivity decreases with increase of pressure, rapidly at first, then more and more slowly, reaching a practically constant value beyond a certain pressure. This pressure varies from about 20 to about 30 lbs. per square inch in different cases.

The only two metals which need be considered in connection with the main contacts of switches are brass and copper. For two surfaces having the same finish and fitted with equal care, a brass-to-brass contact is found to have a lower resistivity than a copper-to-copper contact. The best results are obtained with a copper-to-brass contact, and for this reason this combination has been adopted in a number of modern switches.

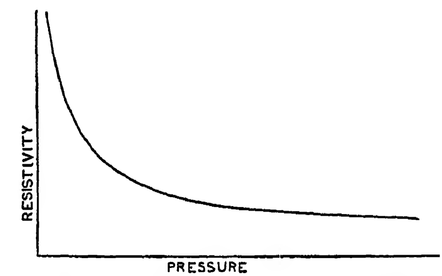

FIG. 173.—Relation connecting contact resistivity with pressure.

The physical and chemical conditions of the contact surfaces exert a very marked effect on the contact resistivity. As regards physical condition, surfaces prepared by the use of a rotating grinding disc, by the application of a sand-blast, and by careful scraping, all yield different results, the first method of treating the surface yielding the poorest and the last the best result. As regards chemical condition, the degree of oxidation of the surface is of very great importance, a badly oxidised surface giving a high contact resistivity. If two surfaces be carefully fitted together, and the contact resistivity be determined immediately afterwards, it will be found to have a much lower value than after the surfaces have been allowed to oxidise for even a few hours by exposure to the air.

The medium surrounding the contact has a surprising effect

[1] By contact resistivity is meant the resistance of unit area of contact surface. See papers by W. Browning in *Journal of the Institution of Electrical Engineers*, vol. xxxvii., p. 372 (1906), and vol. xxxix., p. 678 (1907); also an article by G. J. Meyer in *Elektrotechnische Zeitschrift*, vol. xxx., p. 244 (1909).

on the contact resistivity. The only two media of practical interest are air and oil or fat. The application of a thin layer of oil or vaseline to a switch contact reduces the resistivity to a fraction of its original value. A similar effect is produced by complete immersion (as in oil switches) of the contact in oil. It is probable that the effect of the vaseline or oil is an indirect one—the contact surfaces being cleaned and the layer of air in their immediate neighbourhood removed. For this reason, it is advisable to have switch contacts lightly vaselined.

Rise of temperature (provided it is not allowed to produce rapid oxidation of the surfaces) tends to lower the contact resistivity.

The resistivity of an ordinary good switch in the dry condition is of the order of 10^{-4} ohm/square inch, but may be reduced to much less than this amount by the application of a little vaseline.

§ 196. Regulating Switches for Secondary Batteries.

When no booster is used in connection with a secondary battery, some means of varying the number of cells in circuit must be provided, since the p.d. drops from a value of slightly above 2 volts per cell to 1·8 volts per cell as the discharge proceeds. The number of cells may be varied by means of a multiple-way switch known as an accumulator or battery *regulating switch* (Fig. 174). Such a switch consists essentially of a number of fixed contact blocks to which the consecutive end cells of the battery are connected, and over which one end of a laminated contact bridge may be made to travel, the other end of the bridge moving along a continuous contact bar which represents the terminal bar of the battery. From the point of view of mechanical construction, there are two distinct patterns of such regulating switches: in one, the contacts are arranged along a straight line, while in the other they are grouped around a circle. In the first or *rectangular pattern*, the motion of the laminated bridging contact is rectilinear, and is generally effected by means of a long screw; while in the second or *circular pattern*,

the laminated contact is given a motion of rotation by means of a suitable handle or handles.

In order to prevent any break in the circuit while changing the number of cells, the connections must be so arranged that during the change momentary contact is established between the terminal bar and two neighbouring cells. If this were done by simply allowing the laminated contact to bridge across two neighbouring contact blocks, it is evident that a short circuit would occur of the cell whose terminals are in connection with the blocks. In order to prevent this, and at the same time maintain continuity of contact with the battery, special arrangements must be adopted. Two methods commonly in use are shown in Fig. 174. In the first, (*a*), corresponding to each regulating cell there are two contact blocks, and the cell terminals are in connection with alternate blocks. Between each block which is in direct connection with the battery and its neighbour on one side is connected a safety resistance (generally a spiral of three or four turns) which is high enough to prevent the flow of an excessive current when it is bridged across a cell. It is evident that in passing from any cell to

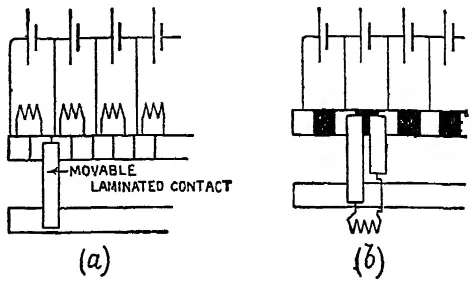

FIG. 174.—Types of battery switch.

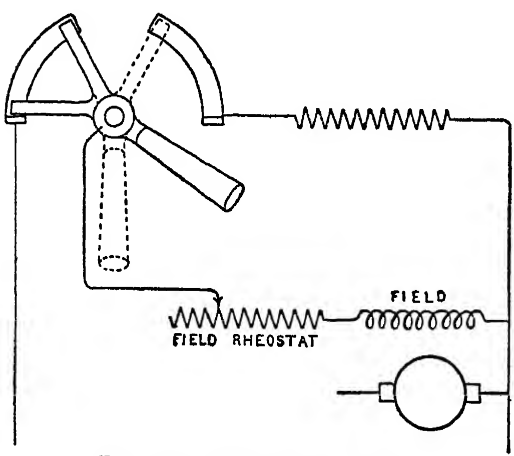

FIG. 175.—Field breaking switch.

a neighbouring one continuity of contact is maintained through this resistance.

An alternative arrangement is shown in Fig. 174 (*b*). Here the number of metal contact blocks provided for regulation is equal to the number of regulating cells. The travelling contact or brush consists of two component parts, connected through a safety resistance but otherwise insulated from each other. Between the metal contact blocks in connection with the cells are interposed blocks of insulating material, of width greater than either component of the travelling contact.

Comparing (*a*) and (*b*), we see that (*a*) requires as many safety resistances as there are regulating cells, and is on that score more expensive than (*b*), in which there is only one safety resistance. On the other hand, (*a*) has the advantage over (*b*) that the brush moves over what is practically a continuous metal surface (with the exception of the narrow strips of insulating material between the blocks), whereas in (*b*) the metal surface is interrupted by wide blocks of insulating material, and for this reason the arrangement is mechanically less satisfactory.

§ 197. Field Breaking Switches.

A field-breaking switch must be so designed that when the field is disconnected from the source of e.m.f., the magnetic flux through the field core is made to disappear slowly, thereby preventing the induction of any dangerously high e.m.f. in the field coils during its disappearance. One type of field switch is shown in Fig. 175. The contact blade of the switch is forked, so that at the instant when the field is about to be disconnected —*i.e.*, when the switch is brought into the position shown by the

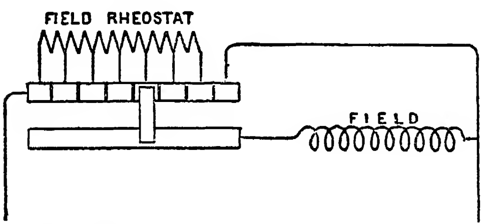

FIG. 176.—Combined field rheostat and field breaking switch.

In order to prevent any possibility of the brush leaving its metal contact as it begins to move over an insulating block, the surfaces of the insulating blocks would be kept somewhat below the surfaces of the metal blocks—an arrangement which is liable to interfere with the smoothness of the motion.

dotted lines—a non-inductive resistance (equal approximately to the resistance of the field) is connected across it. When the disconnection takes place, the field flux begins to decrease, but the current induced by this decrease, which circulates in the field coils and the non-inductive resistance connected in series with them, tends to maintain the flux, and thus the latter disappears more or less gradually.

Another arrangement for disconnecting the field is shown in Fig. 176. The field rheostat is here provided with a supplementary contact, so arranged that just before the break takes place the field is short-circuited (the rheostat being at the same time momentarily bridged across the mains), and the short-circuit is maintained after the break has taken place. As before, the field flux can only disappear slowly, owing to the retarding effect of the self-induced current circulating in the field coils.

§ 198. Methods of Suppressing Arcing at Switch Contacts.

In order to prevent the destruction of switch contacts by the arcing which is liable to take place between them, especially when a heavy current is interrupted, various expedients may be used. It is practically impossible to prevent the *formation* of an arc, and hence the only method of dealing with the difficulty is to reduce the *duration* of the arc. If the duration be so short that the metal surfaces do not get time to reach the temperature of fusion, there will be no damage to the switch contacts, the incipient arc being extinguished so promptly that it is justifiable to speak of the *absence* of arcing.

The following are the methods employed in practice for the suppression of arcing: (1) a quick and long break; (2) the use of supplementary carbon contacts, connected in parallel with the main contacts; (3) the use of auxiliary metal contacts, the arc between which is blown out by a magnetic field provided for the purpose; (4) the use of a movable plate of refractory non-conducting material (such as slate) which, as the break takes place, is made to move rapidly across the arc, thereby breaking it; (5) the immersion of the switch contacts in some medium of great dielectric strength, such as oil, or air under great pressure.

Of the above, (1), (2), and (3) are those most commonly used in connection with continuous current switches; (4) is used occasionally in special designs, while (5) is mainly of interest in connection with high-voltage alternate current switches. We have already (Figs. 171 and 172.) had occasion to consider (1). As regards (2), this is a device for transferring the arcing from the main metal contacts to the auxiliary carbon ones, where the arcing does no harm, as no fusion of the carbon takes place; the carbon blocks, it is true, are gradually destroyed, but are easily renewable. Examples of methods (2) and (3) are described below.

§ 199. Automatic Switches. Maximum Current Circuit-Breakers.

For the purpose of automatically breaking the circuit when the current rises above or falls below a pre-determined value, or undergoes reversal, *automatic switches* or *circuit-breakers* are employed.

A *maximum current* or *overload* circuit-breaker, switch, or cut-out is used to prevent the damage (such as destruction of the insulation of cables, machines, or other apparatus, with the attendant risk of fire) which would result from a heavy overload or an accidental short-circuit on that part of the system which the circuit-breaker is intended to protect.* A well-known form of circuit-breaker of this type is shown in Fig. 177. When the switch is closed the linkwork actuating it is held in position by a trigger against the tension of a powerful spring tending to open the switch. This trigger (whose lower surface is faced with steel) is pivoted about the same axis as the soft-iron plate which forms the armature of the *tripping electromagnet*, but is free to move independently of it within certain limits. The weight of the armature is supported by an adjustable spring, by means of which the position of the armature may be varied, and so the point of cut-off controlled. A small flat steel

Overload circuit-breakers should never be inserted between the 'bus bars and the generators, but only in the *feeder* circuits. If they were included in the machine circuits (*i.e.*, between 'bus bars and generators), then, assuming an accidental short-circuit to take place on the system during the hours of heavy load, and one of the circuit-breakers to open, the extra load would be thrown on the remaining machines, whose circuit-breakers would open one after another, causing a complete shut-down of the station. An over-load circuit breaker on a feeder would, on the other hand, merely isolate the faulty section of the supply network.

spring projects from the trigger and presses against the armature. Passing through a hole in the trigger is a screwed shank, fixed to the armature and carrying a nut immediately above the trigger. When the current exceeds a certain value, the tripping coil magnetises its core powerfully enough to cause it to pull down the armature against the tension of the supporting spring. During the initial stages of the motion, the trigger remains in

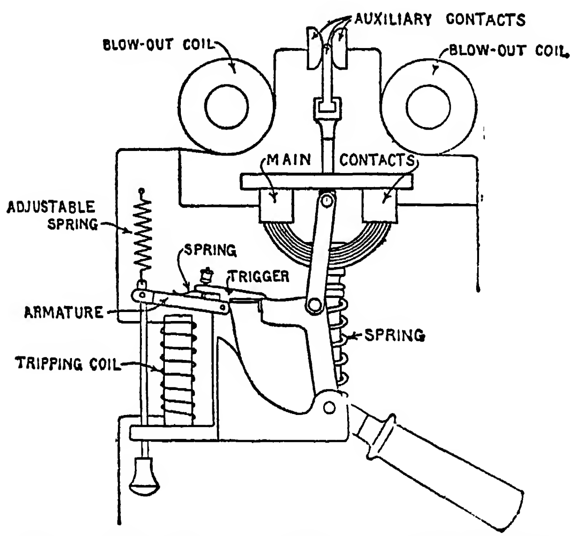

FIG. 177.—Automatic circuit-breaker (The British Thomson-Houston Co., Ltd.).

position, the flat spring at one end of it following the motion of the armature. But when the armature has moved down a sufficient amount, and acquired a certain velocity, the nut on the small screwed shank suddenly strikes the trigger, knocking it off the projection on the switch handle lever and allowing the large spring which is coiled around the main rod supporting the switch contacts to open the switch. The main laminated contacts are the first to leave their fixed contact blocks, and as soon as this happens the entire current flows through the blow-out magnet coils and the auxiliary copper contacts; immediately afterwards, the movable copper

tongue attached to the upper part of the switch rod slips away from the contacts on either side of it (these are pressed against it by springs), starting an arc between them. Now an arc forms a flexible conductor, and will tend to move across any magnetic field whose direction is at right angles to it (§ 2). Such a field is provided by the two blow-out magnets, and the direction of the field relatively to the arc is such as to cause the arc to travel upwards, and in so doing it lengthens until it breaks. The break takes place very rapidly, and is generally accompanied by a noise like an explosion.

The switch may be tripped by hand, by means of the small rod attached to the end of the armature and ending in a small handle; it is closed by means of the large handle. All this is clearly shown in Fig. 177.

Another type of maximum current circuit-breaker, constructed by Messrs. Crompton & Co., Ltd., of Chelmsford, and known as the "C.B.H." circuit-breaker, is shown in Figs. 178—180. It embodies a number of important features which distinguish it from some of the older types of circuit-breaker. One of these is that the speed of closing the contacts is entirely beyond the control of the operator. A quick "make" is desirable for the same reason as a quick "break," viz., to prevent any possible fusion of the contacts. If the conditions of the circuit are such that a very large current will flow on "making" the circuit, then in closing the contacts *slowly* the initial contact area will be small, the contact resistance considerable, and the momentary local heating so great as to partially fuse the edges of the laminated contact before the resistance has been sufficiently reduced by the proper closing of the switch. Another important feature of this circuit-breaker is the fact that it is impossible to maintain the switch forcibly closed by means of the handle if the current exceeds the limit for which the switch has been set, the construction being such that the switch will open quite independently of the position of the handle. In addition to these more recent features of switch design, the "C.B.H." circuit-breaker possesses the older ones of a quick break, and arcing contacts with a magnetic blow-out.

Fig. 178 gives a general perspective view of the switch in two positions, closed and open. In Fig. 179 are shown the more important details of the construction by means of which the quick

make and quick break are obtained. The main contact brush is carried by a vertical spindle free to move inside a guide tube, the lower part of which is fitted with a lug. A collar containing a roller catch is free to slide along the guide tube (when not locked to it by the catch), and is attached to the main lever by means of two links. On raising the handle of the main lever, the roller catch engages the lug on the guide tube, raising this latter, and with it the spindle carrying the main contact brush, and compressing the "quick break" spring. When a certain

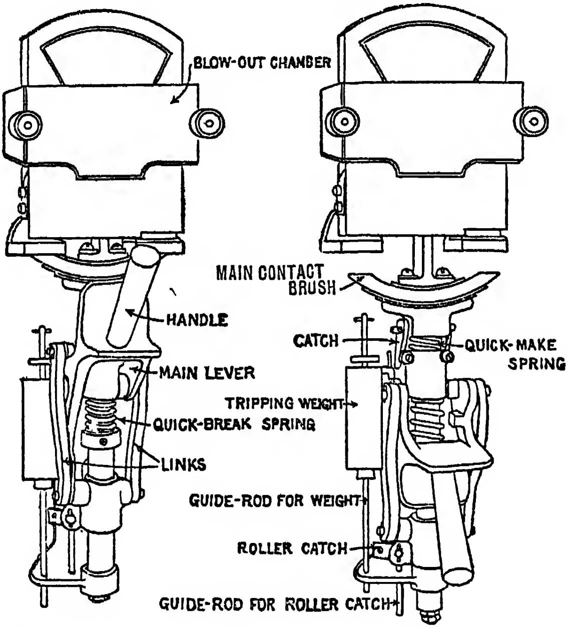

FIG. 178.—" C.B.H." overload circuit-breaker (Crompton & Co., Ltd.).

position has been reached, two small catches suspended from the main contact spindle engage hardened stops fixed to the frame of the circuit-breaker, thereby momentarily arresting the travel of the spindle, and causing a compression of the "quick make" spring. Just before the handle reaches its highest or "on" position, two projections on the switch lever knock the catches off their stops, allowing the quick make spring to act, and to close the switch rapidly. At the same time, the lower end, or

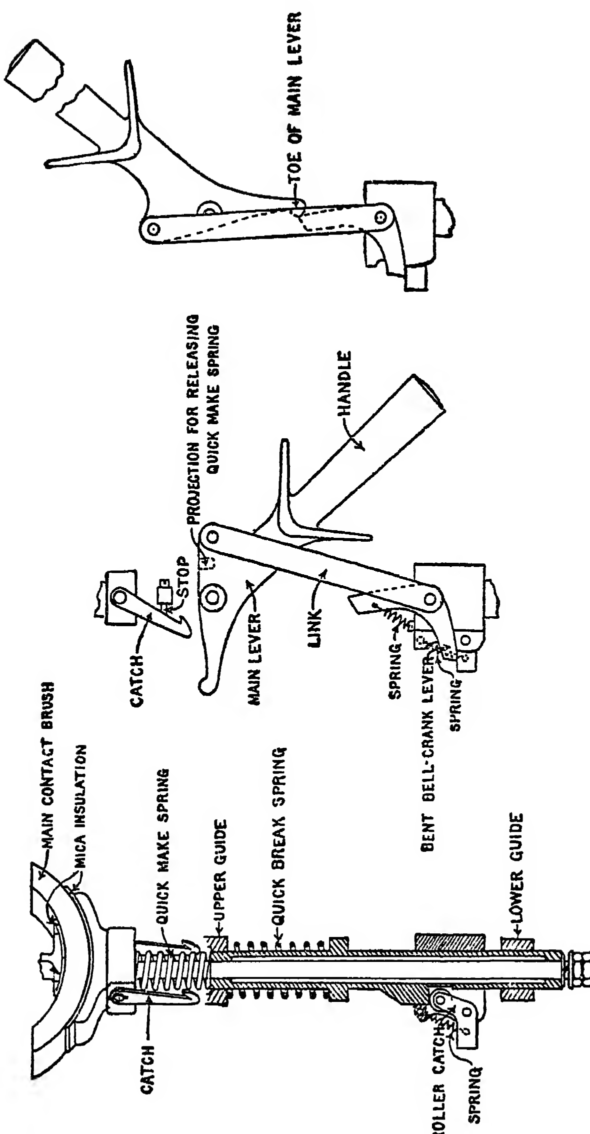

FIG. 179.—Details of "C.B.H." circuit-breaker.

toe, of the main lever brushes past the bent bell-crank lever shown by the side of the roller catch, and the small lever is pulled back into its original position by a spring; the toe of the

main lever now rests against it. The main lever is maintained in its "on" position by the action of the quick break spring, the links connecting the lever to the collar containing the roller catch being slightly over the "dead centre."

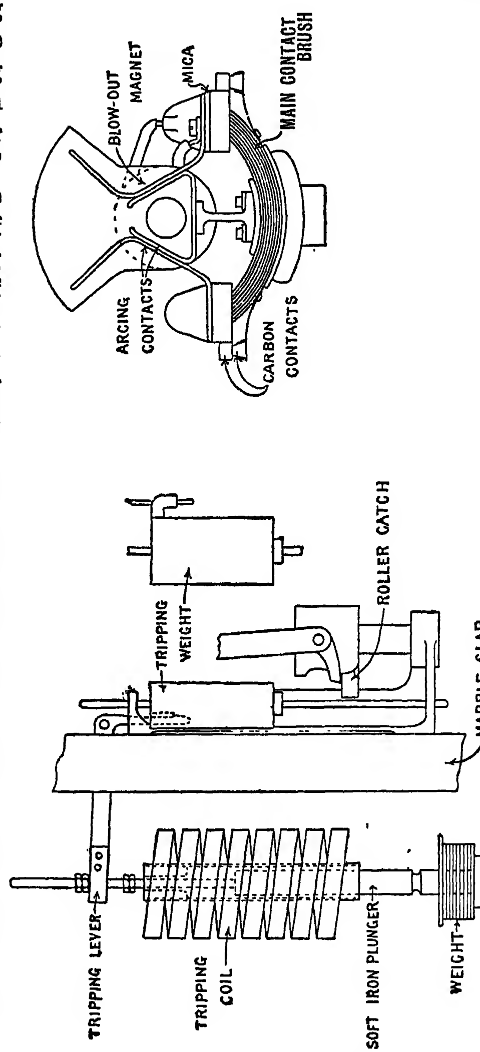

FIG. 180.—Details of "C.B.H." circuit-breaker

In Fig. 180 are shown details of the tripping mechanism and of the switch contacts. Some distance above the roller catch is suspended a tripping weight, which consists of a rectangular block of metal provided with a guide-rod which passes through it and which moves in two guide-holes. When the switch is closed the tripping weight is supported in position by a catch which forms one end of a bell-crank tripping lever, the other end of which is weighted with the plunger of the tripping coil. When the current exceeds the maximum value for which the switch has been adjusted, the tripping coil sucks in its plunger, and the latter delivers a blow against the end of the tripping lever, releasing the tripping weight, which in falling strikes against the

roller catch and thereby releases the quick break spring which opens the circuit-breaker. The switch is fitted with a double set of auxiliary contacts—a set of carbon contacts, and arcing contacts of brass strip which are placed in the field of the blow-out magnet. The main contacts part company first, then the carbon contacts —at which stage the current is diverted into the blow-out coil— and finally the arcing contacts, the arc being promptly blown out by the intense magnetic field in which it is placed.

As the tripping weight strikes the roller catch, the bent bell crank lever which supports the main lever in its "on" position is momentarily pulled away from the toe of the main lever by the spring attached to it, the other end of the small lever, which rests against the roller catch lever, following the downward motion of this latter as it is struck by the falling weight. The toe of the main lever being thereby momentarily deprived of its support, the lever falls back, and in so doing its toe engages the projecting portion of the tripping weight, and, lifting it, restores it to its initial position.

The switch is readily tripped by hand, by pulling down the main lever. This causes the toe of the lever to move the bent bell-crank lever, whose other end presses down the roller catch, thereby allowing the switch to open.

§ 200. Retarding or Time-limit Devices. Thermal Release Circuit-breaker.

The action of the circuit-breakers described is practically instantaneous. Now in some cases it is desirable to have a circuit-breaker whose action depends not only on the value of the overload, but also on its duration. A severe *momentary* overload would in many instances be permissible, and the instant response of a circuit-breaker to such a temporary overload of very short duration might be inconvenient. In such cases, some special device may be adopted to control the action of the circuit-breaker in such a manner as to make it dependent on the product of current × time for any value of the current which exceeds the normal limit. The circuit-breaker will then operate slowly if the overload is slight, and rapidly if it is severe. A device for accomplishing this purpose is known as a *retarding* or *time-element*

or *time-limit device*, and it may take various forms. In some designs the plunger of the tripping electro-magnet is connected to a piston which moves in a cylinder filled with some viscous liquid. The piston is made to fit the cylinder exactly, so that very little liquid can pass between the piston and the cylinder. The piston is perforated by one or more apertures of suitable size, and when the tripping coil sucks in its plunger, the oil or other liquid in the cylinder has to flow from one side of the piston to the other through the perforations, and a considerable resistance is offered to the motion. The plunger can therefore only move more or less slowly, and if the duration of the overload has not been sufficient, it will fall back without having opened the circuit-breaker. It is evident that the motion of the plunger will be the more rapid the heavier the overload.

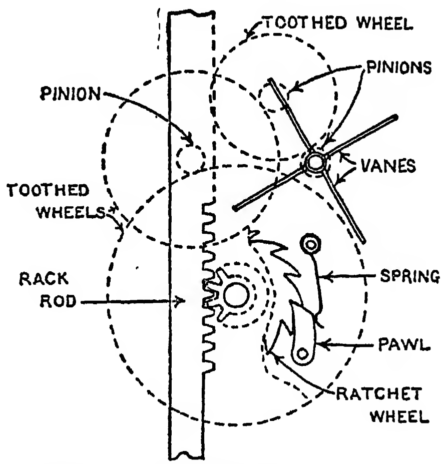

FIG. 181.—Air-vane time-limit attachment.

Another time-limit device, of a simpler kind, consists of a thin rod ending in a flat disc and attached to the plunger of the tripping magnet. The disc rests at the bottom of a glass vessel, and is just covered by a thin layer of glycerine. When the coil sucks up the plunger, some time must elapse before the disc can be detached from the glycerine, and a retarding effect is thus obtained.

A third type of time-element device consists of a fuse placed as a shunt across the tripping coil of the circuit-breaker. Owing to its appreciable capacity for heat, a fuse necessarily has a time-element (§ 202). When the fuse melts, the entire current passes through the tripping coil, opening the circuit-breaker.

A fourth example of an inverse time-element device is afforded by the air-vane time-limit attachment. This may be used in connection with any type of circuit-breaker for producing the necessary

retardation of the moving part. The device is shown in Fig. 181. The moving element of the switch is attached to a rack rod which gears with a pinion mounted on a spindle, the spindle carrying a toothed wheel which forms the first element of a clockwork train ending in a spindle provided with air-vanes. The air-friction caused by the vanes furnishes the necessary retarding effect. A quick-return motion is provided by means of a ratchet-wheel and pawl.

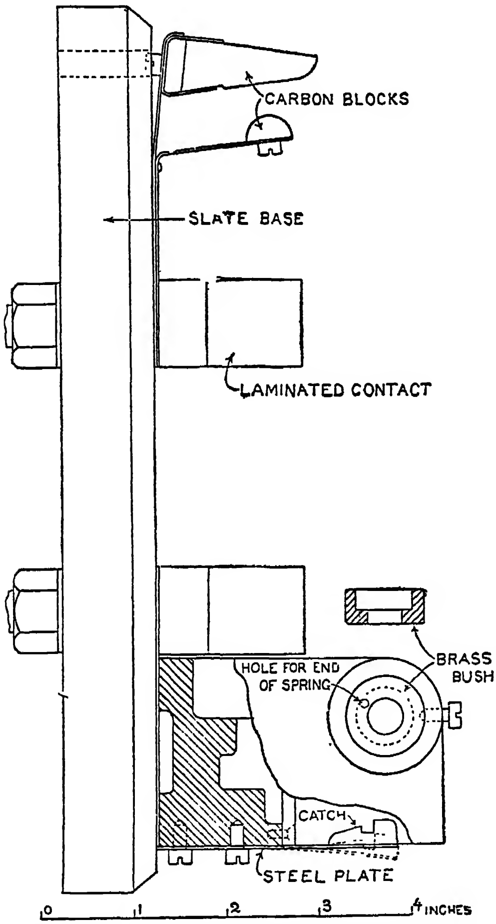

FIG. 182.—Thermal time-element circuit-breaker.

A fifth type of time-limit device is represented by the *thermal release* circuit-breaker. One example of such a circuit-breaker, designed by Messrs. Morris & Lister, Ltd., is illustrated in Figs. 182—186. The general arrangement of the fixed contacts and other fixed parts of the circuit-breaker is shown in Fig. 182. The main contacts are of laminated copper, the construction being shown clearly in Fig. 184 (*a*). Auxiliary arcing contacts of carbon are provided. Attached to the lower portion of the slate base on which the fixed contacts are mounted is a -shaped casting which supports the movable members of the switch. The yoke of the is shown in section in Fig. 182. Its limbs or

sides consist of two thick plates into which are fitted the brass bushes which form the bearings for the spindle passing through the movable members of the switch. The bottom of the [is closed by a tempered steel spring-plate, fixed by means of three screws, which carries a block with a rectangular groove forming

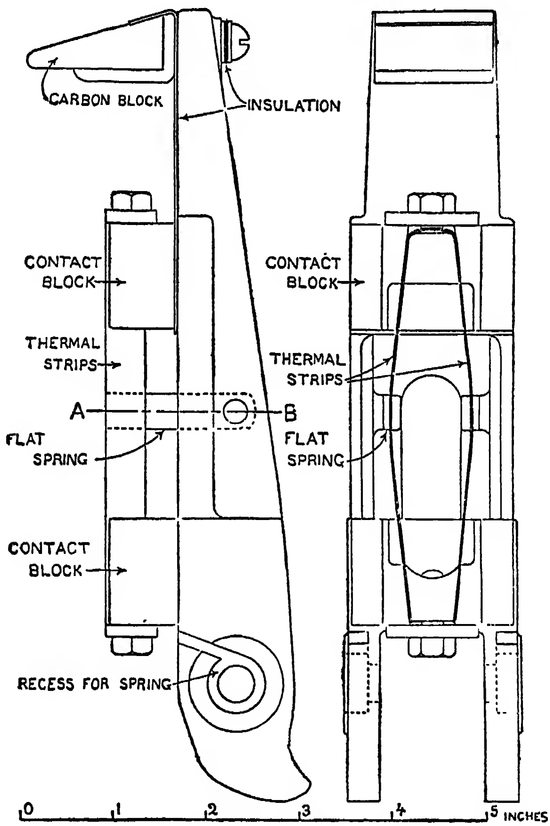

FIG. 183.—Circuit-breaker arm of thermal release circuit-breaker.

a catch. The movable members of the switch are three in number: the circuit-breaker arm, the locking or detent arm, and the free or loose handle. These have been omitted from Fig. 182, but are shown in detail in the other illustrations. The circuit-breaker arm (Fig. 183) consists of a forked casting on which are mounted the movable contacts of the switch and the

thermal release device. The main movable contacts consist of heavy blocks of brass, and when the switch is closed these are pressed against the fixed laminated copper contacts, as shown clearly in Fig. 184 (*a*). The switch spindle whose axis forms the common axis of rotation for all the movable members passes loosely through the circuit-breaker arm, and the outer portions of the lower or forked end of this arm are recessed to receive a portion of a spiral spring, the straight end of the spring fitting into the straight rectangular groove clearly shown in Fig. 183. The remainder of each spiral spring fits into a recess in the brass bush shown in Fig. 182. The outer ends of the springs are bent at right angles, and fit into holes in the brass bushes. These bushes are fixed so that the springs tend to keep the arm in the "off" position. The upper contact-block (together with the auxiliary carbon contact connected to it by a wide strip of brass) is insulated from the arm. The two contact-blocks are connected by means of the thermal strips. These are arranged so as to form an elongated lozenge-shaped figure, and are clamped to the brass blocks by means of wedge-shaped pieces which fit into corresponding openings in the blocks, and which are held in place by set-screws as shown. The central portions

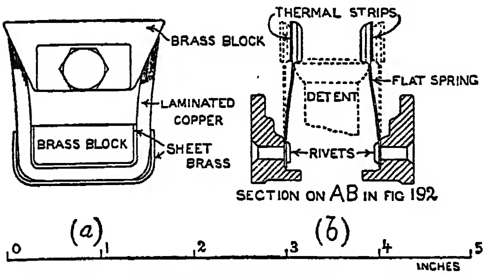

FIG. 184.—Details of contact and thermal release.

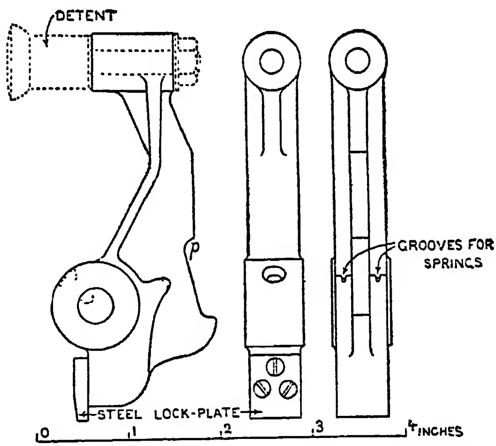

FIG. 185.—Locking arm of thermal release circuit-breaker.

of the thermal strips are forced outwards by two flat springs bearing against them. These springs are shown in detail in Fig. 184 (*b*). When owing to an overload excessive heating of the strips takes place, they expand, and are pushed further apart by the flat springs, which come into the positions indicated by dotted lines in Fig. 184 (*b*). Each flat spring has riveted to its free end a steel plate, and a detent piece, shown dotted in Fig. 184 (*b*), bears against these plates and keeps the circuit-breaker arm pressed against the fixed contacts. When, however, expansion of the strips and consequent separation of the ends of the flat springs takes place, these ends are finally able to slip round the edge of the detent, and the spiral springs controlling the circuit-breaker arm throw it into the "off" position. The detent is fixed to the end of the second movable member of the switch—the locking arm, shown in Fig. 185. This arm is rigidly attached to the spindle. Its lower end carries a steel lock-plate

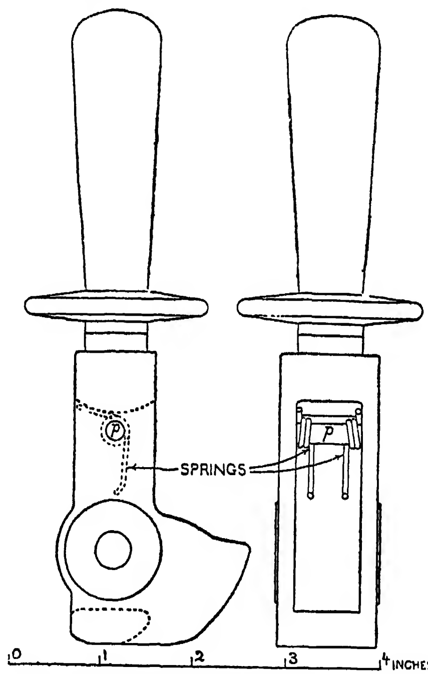

FIG. 186.—Free handle of thermal time-element circuit-breaker.

or tongue which, when the switch is closed, drops into the groove of the catch shown at the bottom of Fig. 182, and so maintains the switch closed. The closing or opening of the switch by the operator is effected by means of the third movable member—the free or loose handle, shown in Fig. 186. This consists of a handle of insulating material attached to a box-shaped casting whose lower end is provided with toe-pieces. The free handle casting fits the spindle loosely, but two spiral springs resist its separation from the locking arm, the ends of the springs resting

in the shallow grooves in the locking arm, shown in Fig. 185. In the act of closing the switch, the handle is thrown forward, and the pin marked p in Fig. 186 which carries the spiral springs engages the locking arm at the depression marked p in Fig. 185, and carries both locking and (through the medium of the detent and flat springs) circuit-breaker arms forward. The lock-plate attached to the bottom of the locking arm glides over the catch, depressing the steel plate (Fig. 182) which supports it, until, when the switch has been closed, the lock-plate reaches the groove in the catch, when the latter springs back into its normal position, thereby locking the switch.

In order to open the switch, the free handle is pulled back. This causes its toe-pieces to press against the steel spring-plate supporting the catch, the plate being deflected until the catch releases the lock-plate on the locking arm and allows the switch to open.

§ 201. Minimum and Reverse Current Circuit-breakers.

Circuit-breakers designed to operate when the current falls below a certain value are chiefly used in connection with dynamos charging secondary cells. A simple form of circuit-breaker of this type is shown in Fig. 187. The curved switch lever is weighted, and tends to open the switch; it is prevented from doing so by the pull which is exerted on a soft-iron armature by a magnet whose coil carries the main current, the armature being attached to the lever If the current falls below a certain value, the pull on the armature is no longer sufficient to balance the couple due to gravity, the lever drops down and strikes the switch blade, thus opening the switch. The fall of the switch lever is broken by suitable buffer springs. Arrangements are frequently provided for closing a local circuit and ringing a bell when the circuit-breaker opens.

Reverse-current circuit-breakers are intended to protect dynamos running in parallel, and are inserted between the machines and the 'bus bars. Every type of reverse-current

If one of the paralleled machines should lose its field (owing to a break in the shunt, for example), and hence also its e.m.f., an enormous current would be sent into its armature in the reverse direction by the other machines, and the insulation would be destroyed in a very short time. Further, owing to the heavy load suddenly

switch is provided with two coils, a series coil carrying the main current and a shunt coil connected across the mains (the shunt coil may, if necessary, be joined in series with an extra resistance). There are two distinct types of reverse-current switch. In one, the series and shunt coils are wound around a common core which supports an armature and maintains the switch closed so long as the current through the series coil has its normal direction, the magnetic effects of the coils being then additive (the shunt coil being, however, the stronger of the two). Should the main current undergo a reversal, the series coil will oppose the shunt coil, and if the reverse current is large enough, the magnetism of the core will be wiped out, and the armature will drop. The release of the armature is arranged to trip the switch. In the second type of reverse-current switch, the shunt coil, which is not provided with any core, is made movable, and is placed in the field of the series magnet, which is fixed. A reversal of the field will cause the coil to move so as to trip the switch.

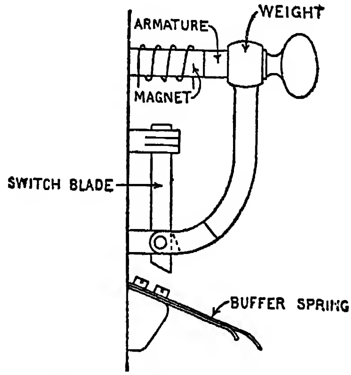

FIG. 187.—Minimum current automatic switch.

The second type is greatly to be preferred to the first, as it is bound to operate, whereas with a very sudden reversal the first type may fail to do so, the reverse current producing a reversal of magnetism in the magnet core so rapidly that the latter is able to retain the armature, which has no time to move appreciably before the reversal has taken place.

An example of the second type of reverse-current circuit-breaker is shown diagrammatically in Fig. 188. This diagram refers to a combined maximum and reverse-current circuit-breaker, but may be converted into one for a simple reverse-current switch by fixing the armature marked "pivoted armature" and omitting the rod marked "rod suspended from lever." The

thrown on the other machines, there would be a sudden drop of voltage over the entire system. It is against accidents of this kind that a reverse-current circuit-breaker is intended to afford protection.

shunt coil is suspended from an axle which carries a horizontal arm, one side of which supports a counterweight, and the other a screwed shank on which rests a curved arm projecting from the tripping lever (this lever moves in a plane normal to that of the drawing). Under normal conditions, the couple acting on the shunt coil has a clockwise direction. But if the current undergoes a reversal, the shunt coil is driven in a counter-clockwise direction, tilting up the tripping lever and thus opening the switch.

By making the armature below the series magnet movable, fitting it with a plate (marked "tripping plate" in Fig. 188)

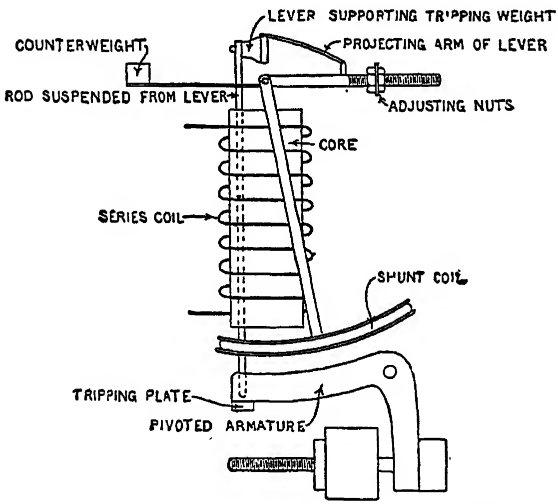

FIG. 188.—Diagram of maximum and reverse-current circuit-breaker (Crompton & Co., Ltd.).

which projects sideways, and suspending a rod from the tripping lever so that when the armature is attracted by the magnet the tripping plate strikes the rod and tilts the tripping lever, the switch is converted from a simple reverse-current into a combined reverse-current and overload one.

§ 202. Fuses.

The earliest form of device intended to protect a circuit against the effects of an abnormally large current was the *fuse*. Although fuses have very generally been abandoned for the protection of main circuits conveying large currents, they are still widely used

for the protection of branch circuits conveying relatively small currents, the first cost of a circuit-breaker in such cases being prohibitive.

A fuse consists of a short length of wire or strip of metal mounted between suitable terminals. The cross-section of the fuse is so chosen that when the current exceeds the limit of safety the fuse melts, thereby breaking the circuit.

The main advantages of a fuse as compared with an automatic switch or circuit-breaker are its low first cost and the fact that it possesses, by the very nature of the case, a *time-limit*. On the other hand, it has numerous defects, among which the more important are its unreliability—apparently similar fuses similarly placed fusing with different currents—the impossibility of immediately replacing it, and the cost of renewals; and the fact that in the case of fuses of the *open* type—especially those intended for large currents—the fuse itself becomes a source of danger, the explosive violence with which it melts when a sudden short-circuit takes place causing the molten metal to be scattered in all directions.

When a fuse has reached a steady temperature, it is evident that the rate at which heat is being generated in it by the current must equal the rate at which heat is being lost by conduction to the terminals, by convection and radiation. Since the loss by radiation depends on the nature of the surface, it is not difficult to see that a change in the surface—such as slight tarnishing or deposition of dirt—will affect the value of the fusing current. This value also depends, for a given shape and cross-section of fuse, of given material, on the length of fuse (decreasing as the length increases, since the rate of heat conduction from the middle portion of the fuse to the terminals is decreased on account of the smaller temperature gradient), and on the mass of the terminals.

Besides the open type of fuse, in which the fuse wire or strip is freely exposed, and whose disadvantage—the scattering of molten metal—has already been noticed, there are two other kinds—the *protected type*, in which the fuse is enclosed in a porcelain tube open at both ends; and the *totally enclosed* type, in which the fuse is contained in a sealed tube (of porcelain or of pasteboard rendered non-inflammable by a special process). In this latter, the tube is generally filled with some highly

refractory non-conducting powder (sand, fireclay, emery, French chalk, &c.) which is intended to prevent the formation of an arc; and in some cases the middle portion of the tube is not filled up, an air space being thus formed which is intended to facilitate the breaking of the fuse at this point, by depriving it of all mechanical support.

Fuses may be required to possess either a very short or a considerable time lag. In the case of a lighting circuit, for example, it is desirable that a fuse should act promptly, as no appreciable overload can possibly arise so long as the conditions of the circuit remain normal. But the case is entirely different if the load consists of a motor, which may be subjected to a heavy overload for a short period without any evil results, and without the existence of any abnormal conditions; in this case it would obviously be an advantage to use a slow-acting fuse, or one having a large time-element. Now the time-element of a fuse depends on the material of which the fuse is made. If a quick-acting fuse is desired, silver and copper are the most suitable materials, and are about equally good, copper being the cheaper, but more liable to oxidation (copper fuse wires are for this reason generally tinned or silvered). The best material for a slow-acting fuse is zinc: it possesses a large time-element and has a relatively smaller mass than a fuse of any other metal giving about the same time-element.

CHAPTER XVIII.

§ 203. Conductors. Copper—§ 204. Aluminium and Steel. Composite conductors—§ 205. Tungsten—§ 206. High Resistivity Alloys—§ 207. Carbon—§ 208. Insulators, India-rubber—§ 209. Gutta-percha—§ 210. Mica—§ 211. Porcelain, Marble and Slate—§ 212. Paraffin Wax, Ozokerite, and Bitumen—§ 213. Fibrous Insulators—§ 214. Shellac—§ 215. Bakelite.

§ 203. Conductors. Copper.

Of the materials commonly used as electrical conductors, by far the most important is copper. The greatest copper-producing country is the United States, which supplies over 50 per cent. of the total. Next in order of importance come Mexico, Spain, Portugal, and Japan. The price of copper (in the form of bars) has been subject to violent fluctuations, varying from a little over £40 per ton to as much as £110 per ton. The fluctuations in the price of electrolytic copper since 1915 are shown in Fig. 189.

The density of copper is, on the average, about 8·9. A cubic foot of copper weighs 555 lbs.; a cubic inch, ·32 lb.

The temperature coefficient of linear expansion of copper is about 17×10^{-6} per degree C. Its specific heat is about ·094, and its thermal conductivity ·86 (gramme-calories per sq. cm. per sec. per degree C./cm.).

The resistivity of copper depends very largely on its purity and on the physical treatment which it has undergone. The copper ordinarily used for electrical conductors is of a very high degree of purity. It is used in two forms, viz., as soft or annealed copper, which has a resistivity of about 1·56 microhm per cm. cube (·616 microhm per inch cube) at 0° C., and a tensile strength of about 28,000 lbs. per square inch; and as hard-drawn copper, having a resistivity of about 1·6 microhm per cm. cube at 0° C., and a tensile strength of nearly 65,000 lbs. per square inch. Soft copper is, on account of its higher conductivity, preferred in all cases where mechanical strength is not of great import-

ance. Hard-drawn copper is used in the case of overhead trolley wires and power transmission lines, telegraph and telephone lines.

In specifications relating to the resistivity of copper conductors, reference is still in some cases made to an old standard known as *Matthiessen's standard.* For soft or annealed copper, this is represented by copper of such resistivity that 100 inches of a wire of it weighing 100 grains have a resistance of ·1502 ohm at 60° F. (or 1 metre weighing 1 gramme a resistance of ·1508 ohm; or 1 foot weighing 1 grain a resistance of ·2162 ohm); this is equivalent to a resistivity of 1·694 microhm per cm. cube at 60° F. (or 1·589 at 0° C.). In the case of hard-drawn copper, Matthiessen's standard is represented by copper of which a wire 1 metre long and weighing 1 gramme has a resistance of ·15386 ohm. at 60° F. The *percentage conductivity* of any given sample of copper is 100 times the ratio of its conductivity to the conductivity of Matthiessen's standard.

The great strength of hard-drawn copper wire resides mainly in its surface layers or skin; hence in handling such wire great care should be taken to prevent any damage to the *surface,* such as scratches or abrasions.

The tensile strength of hard-drawn copper wire may, according to A. P. Trotter, be determined by means of the formula

$$T = 30 - 20\,D,$$

where $D =$ diameter of wire, in inches, and $T =$ tensile strength, in tons/sq. in. This formula holds good for wires less than $\frac{1}{2}$ inch in diameter.

The value of Young's modulus of elasticity for hard-drawn copper is 16×10^6 lbs./sq. inch.

The temperature coefficient of resistance of copper varies with its density and hardness between the limits of about ·00415 (hard-drawn) and ·00428 (annealed) per degree C., from and at 0° C.

Copper melts at a temperature of about 1,080° C.

In the case of conductors of large cross-section, in order to secure sufficient flexibility the conductor is made up of strands. The number of strands is generally so chosen that the cylindrical form of the conductor is roughly preserved. Hence the typical

The correctness of this view has been challenged. See *Journal of the Institute of Metals,* vol. 6, p. 186 (1911).

stranded conductor or cable consists of a single wire at the centre, surrounded by a number of cylindrical layers, each layer containing as many wires as can be got into the space available. This mode of construction leads to perfectly definite numbers of strands. These numbers are, in ascending order of magnitude,

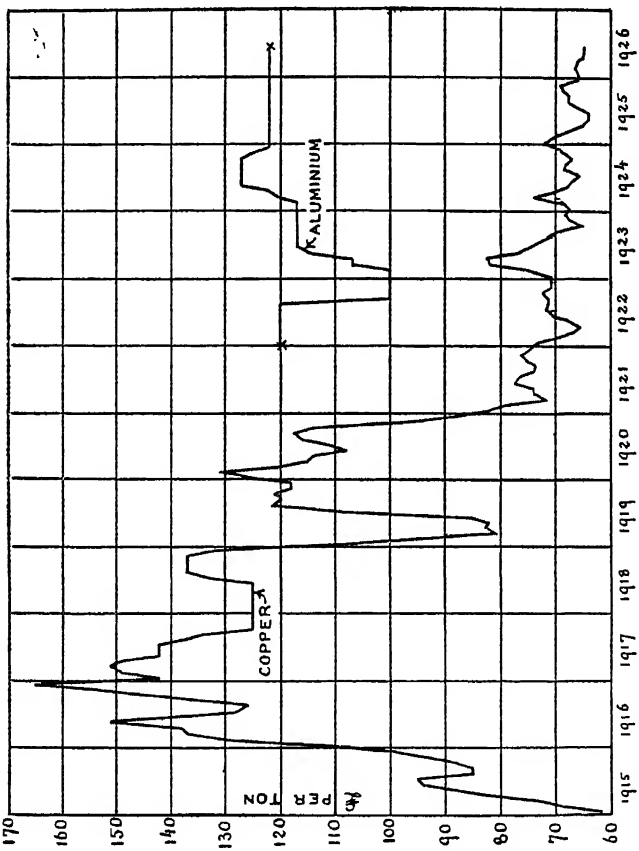

FIG. 189.—Fluctuations in the prices of electrolytic coppe: and aluminium.

7, 19, 37, 61, 91, 127. In the above standard form of cable the central conductor is the only straight one; the wires in the successive cylindrical layers surrounding the axial wire being twisted round this wire and following helical paths. When there are several layers surrounding the central wire, the helical paths of the wires in the successive layers are alternately right- and left-handed, in

order to prevent the wires of any one layer from sliding down into the spaces between the wires of the layer below, and so destroying the symmetrical outline of the cable. The *lay* of the strands in any layer is defined to be the distance measured along the axis of the cable within which each strand of that layer completes a turn. More frequently, the term *lay ratio* is used in this connection, and is defined as the ratio of the lay to the mean diameter of the layer. A common value for the lay ratio in the smaller sizes of cables is 20; if the cable is of large size, the lay ratio decreases from the innermost to the outermost layers.

Three-strand cables are fairly frequently in use; the three strands being arranged at the corners of an equilateral triangle, and there being no axial wire.

Occasionally, exceptional forms of construction for cables are adopted. Thus, instead of the usual axial wire, a stranded central core of 3 or 4 wires is used, surrounded by cylindrical layers as in the standard form.

Multiple-stranded conductors are used in cases where extreme flexibility is required. In these, each component strand is itself a stranded conductor.

Owing to the fact that the area of contact between the neighbouring strands of a cable is very small (being no more than a geometrical line in the ideal case), there is no appreciable flow of current from one strand to another, the current flowing in helical paths corresponding to the strands. Now since the length of each helical strand exceeds that of the axial conductor, the resistance of a cable will be greater than that of an equal number of straight wires laid side by side. The usual allowance made for this is an increase of 2 per cent. in the resistance.

The copper wires used in winding armatures and magnet coils, instrument coils, &c., are generally provided with a double covering or lapping of cotton. This increases the diameter of the wire by about 10 mils. (·010 inch) for all sizes less than No. 18, by about 12 mils. for all sizes between Nos. 17 and 13, and by about 14 mils. for all sizes larger than No. 13 S.W.G. (standard wire gauge). The heavy copper conductors used in large armatures are frequently insulated with a thin paper lapping and a double braiding of cotton, the thickness of the covering being

A certain amount of confusion exists in connection with this terminology, the term *lay* being frequently used to denote *lay ratio*.

17 mils., so that the insulation increases each transverse dimension of the conductor by about ·034 inch.

Where, as in the case of fine wire used in winding instrument coils, it is important to reduce the thickness of the insulating covering to the smallest possible amount, *enamel-insulated* wire is used, the thin insulating coating consisting of an enamel prepared by blending a varnish with a special resin. Cellulose acetate has been used for the same purpose.

In cases where the very highest tensile strength is required—as in very long spans across rivers or over valleys—alloys of copper with tin, known as *bronzes*, are employed, containing a small percentage of other elements. The best known bronzes for electrical purposes are *phosphor-bronze*,* which contains minute quantities of phosphorus, and *silicon-bronze*, which contains silicon. Such bronzes have tensile strengths varying from 65,000 to 110,000 lbs. per square inch, and their conductivity is only slightly inferior to that of copper.

§ 204. Aluminium and Steel. Composite Conductors.

Aluminium has to some extent been used as a substitute for copper in the case of overhead power transmission lines. It is produced by the electrolysis of fused alumina, and aluminium works have been established in Scotland, France, Switzerland, and the United States. The fluctuations in the price of aluminium during the last few years are shown in Fig. 198.

The density of aluminium is only about 2·6, and its resistivity at 0° C. about 2·7 microhm per cm. cube. Hence the weight of an aluminium conductor of given length and resistance will be only about half that of a copper one.

The melting point of aluminium is about 660° C. Its tensile strength is about 20,000 lbs. per square inch.

The temperature coefficient of linear expansion of aluminium is about $23{\cdot}4 \times 10^{-6}$ per degree C. Its specific heat is about ·212, and its thermal conductivity ·50 (gramme-calories per sq. cm. per sec. per degree C./cm.).

* Phosphor-bronze wire contains at least 94 per cent. of copper, and from ·11 to 4 per cent. of phosphorus; the remainder being tin.

The temperature coefficient of resistance of aluminium varies between the limits of about ·0032 and ·004 per degree C., from and at 0° C.

Aluminium oxidises very readily, but the continuous thin coating of oxide formed protects the metal from further oxidation when exposed to the atmosphere. Impurities greatly lower the resistance of aluminium to atmospheric influences. The coating of oxide being non-conducting, advantage has been taken of this fact in winding the magnet coils of machines with *bare* aluminium wire (or, more correctly, with aluminium wire provided with a thin insulating coating of aluminium oxide).

Steel is being largely used as a conductor in connection with electric tramways and railways. The ordinary running rails generally serve as the return conductor; in the third-rail system an insulated rail, supported on porcelain insulators, serves as the positive conductor. It is usual to express the resistivity of steel conductor rails by stating the resistance, in microhms, at 60° F. (15·6° C.), of a rail of the material in question having a length of 1 yard and weighing 100 lbs. The value of this resistance ranges from 15 to 20 microhms. The temperature coefficient of steel conductor rails is about ·47 per cent. per degree C. (·26 per cent. per degree F.).

Composite conductors have to some extent been used for overhead lines, where mechanical strength is as important as electrical conductivity. In the case of a stranded conductor, since the axial conductor is normally subjected to the greatest stress, cables with a central wire of *soft* copper and an outer layer or layers of *hard-drawn* copper have sometimes been used. The soft copper core will stretch without breaking while the helices of the surrounding strands are being pulled out under an abnormal stress. In such a conductor, the stress is more or less evenly distributed among the outer strands, and the plasticity of the central wire prevents its breaking. A precisely opposite principle is used in cables consisting of a central strand or strands of *steel*, with outer strands of *aluminium*. The stress is here taken by the central steel wires, while the plastic outside aluminium wires provide the necessary conductivity. Another form of composite conductor consists of *copper-clad* steel strands; these are obtained by starting with a steel billet on which the required thickness of copper has been deposited, and rolling and drawing the billet to the size required. The copper coating serves

the double purpose of protecting the steel from corrosion and furnishing the necessary conductivity, while the stress is taken practically by the steel.

§ 205. Tungsten.

The great importance of the metal *tungsten* * (or wolfram) in the electrical industry is due to its use in the form of incandescent lamp filaments.

Tungsten does not occur free in nature, but its ores have a very wide distribution, and supplies are obtainable from all parts of the world. The chief ores are sheelite (calcium tungstate) and wolframite (iron-manganese tungstate).

When first obtained in the metallic state, tungsten was found to be extremely brittle. The earliest tungsten incandescent lamps were provided with *squirted* filaments. These were prepared by mixing finely powdered metallic tungsten with an organic binding medium, squirting the paste so obtained into filaments, drying these, heating them *in vacuo* to carbonise the binding material, and then heating them in an atmosphere of hydrogen and nitrogen in order to drive off the carbon in the form of cyanogen and hydrocarbons. Such squirted filaments were found to be very brittle, and their brittleness stood in the way of a more general adoption of the lamp.

In 1910, however, it was discovered that the extremely brittle cast tungsten could be made *ductile* by raising it to incandescence and hammering it while at this high temperature in an atmosphere of hydrogen (to prevent oxidation). The metal could then be drawn into wire. Filaments of *drawn* tungsten wire are extremely strong and capable of withstanding rough handling. All modern tungsten lamps have drawn-wire filaments.

Ductile tungsten has the appearance of steel. Its density is about 20. It is the strongest metal known, having a tensile strength of about 500,000 lbs./sq. in.† The modulus of elasticity is about 50×10^6 lbs./sq. in. It may be hardened by hammer-

The word *tungsten* is of Swedish origin and means "heavy stone."

† The tensile strength progressively increases with the number of drawings, having the value of 260,000 lbs./sq. in. for a wire 18 mils. in diameter, and rising to 590,000 lbs./sq. in. for a wire 1·14 mils. in dia.

ing, rolling, or drawing, but not by heating and quenching. The hardness may be entirely removed by raising the metal to a white heat. Tungsten cannot be welded.

Tungsten is the most refractory metal known: it melts at about 3,300°C. Its coefficient of linear expansion with heat is $4{\cdot}3 \times 10^{-6}$ per degree C. Its specific heat is ·036, and thermal conductivity ·35 (gramme-calories per sq. cm. per sec. per degree C./cm.).

The resistivity of tungsten is from 4·4 (annealed) to 5·5 (hard-drawn) microhms per cm. cube at 0° C. The temperature coefficient of resistance is about ·0051 per degree C.

At ordinary temperatures, tungsten is not attacked by either dilute or concentrated acids when used singly, but dissolves rapidly in a mixture of hydrofluoric and nitric acids. It dissolves slowly in an aqueous solution of sodium or potassium carbonate.

Besides its use as a material for incandescent lamp filaments, tungsten has received a number of other applications. It is very suitable for electrical contacts, such as those in induction coils, relays, keys, &c., as it is much harder and wears longer than platinum, and as no welding of the contacts can take place. It has also been found a very satisfactory material for the targets or anti-cathodes of X-ray tubes.

§ 206. High Resistivity Alloys.

By using suitable proportions of various metals, it is possible to obtain alloys having a very high resistivity and a very low temperature coefficient of resistance. Such alloys are suitable for the construction of resistance coils, and are very largely used for this purpose. Among the older alloys of this type may be mentioned *platinum-silver* and *German silver* (copper, zinc, nickel). Depending on the proportions of the ingredients, German silver may have a resistivity of from 18 to 40 microhms per cm. cube (*i.e.*, from about 12 to about 27 times that of copper), and a temperature coefficient of from $7{\cdot}6 \times 10^{-4}$ to $2{\cdot}1 \times 10^{-4}$, the lower temperature coefficient being associated with the higher resistivity. German silver is the cheapest of the high-resistivity alloys, but is not very permanent, and, especially if used at a fairly high temperature, is liable to become exceedingly brittle in course of time.

A favourite material for standard resistance coils, ammeter shunts, &c., is the alloy known as *manganin* (84 per cent. of copper, 12 of manganese, and 4 of nickel). Its resistivity is about 41 microhms per cm. cube, and its temperature coefficient is for all ordinary purposes negligible, being only about 10^{-5} per degree C.

Instead of manganin, alloys of *copper* and *nickel* are largely used. They have a resistivity slightly higher than manganin, an extremely low temperature coefficient, and are considerably cheaper than manganin. The usual proportions are about 60 per cent. of copper to 40 of nickel. These alloys are known under various trade names, such as *eureka*, *constantan*, *ferry*.

Nickel-steel alloys have an extremely high resistivity (of the order of 84 microhms per cm. cube), with a fairly low temperature coefficient (about 8×10^{-4} per degree C.), but are liable to rust in a damp atmosphere.

A special class of alloys are those used in electric heating and cooking appliances. An essential requirement in such cases is that the alloy should be capable of being used at a very high temperature without undergoing oxidation. The most successful alloy of this class, and one which at the present time is used almost exclusively in all heating and cooking appliances, is *nickel-chromium*, known under the trade names of *nichrome* and *chronic*. It is made in several grades. Its resistivity has the exceptionally high value of about 93 microhms per cm. cube. The temperature coefficient is about $4{\cdot}2 \times 10^{-4}$ per degree C. One grade of this alloy may be permanently run at a temperature of 1,000° C. without undergoing oxidation. It is a useful material for the heating coils of small electric furnaces.

§ 207. Carbon.

Carbon is largely used in arc lamps, continuously variable resistances, dynamo brushes, and incandescent lamps. Continuously variable carbon resistances consist of blocks of gas retort carbon. Arc lamp carbons are prepared by grinding gas retort carbon to a fine powder, mixing it intimately with gas tar to form a dough, passing this latter through a nozzle by hydraulic

pressure, cutting the rod so produced into lengths of about a yard, heating in a furnace so as to carbonise the whole, and finally cutting the carbon rod into the required lengths and pointing these. The same general method is used in the manufacture of carbon brushes, but a larger variety of materials is employed in the original mixture, which, besides gas carbon, contains anthracite, graphite, soot, petroleum coke, and remains of arc lamp carbons. After having been made into a dough with tar, the mixture is moulded to the required shape, and carbonised. The blocks are then ground to the exact size required, and in most cases the part of the brush which is intended to fit into the holder is coppered, nickelled or silvered electrolytically, so as to enable good contact to be obtained with the body of the brush. The various grades (as regards hardness) of brush are obtained by varying the composition of the original mixture, and the temperature of the furnace in which carbonisation takes place.

Arc lamp carbons should be of uniform diameter and as nearly straight as possible. A bent carbon causes the crater to climb up its side, altering the distribution of the luminous flux and causing unsteady burning. The quantity of incombustible matter deposited in the form of *ash* should not exceed from 1 to $1\frac{1}{2}$ per cent. of the weight of carbon consumed.

§ 208. Insulators. India-rubber.

Insulators may be divided into two large classes, viz., hygroscopic and non-hygroscopic ones. Among the most important insulators belonging to the latter class are india-rubber and gutta-percha.

India-rubber was first introduced into Europe by the French explorer, La Condamine, who, in 1736, exhibited and described the properties of this material to the French Academy of Sciences.

Rubber is obtained by making incisions in the bark of certain tropical trees, and collecting the milky juice or latex which exudes from the middle layers of the bark. On applying heat to the latex, or treating it with an acid or an alkali, coagulation sets in, and the coagulated product forms the raw rubber of commerce. The most important rubber-yielding trees are *Hevœa brasiliensis* (this furnishes Para rubber, so called from the province of Para in

In making *cored* carbons, a die is used containing a central pin which causes the mixture as it is squeezed out to assume the form of a thick-walled hollow cylinder. The axial hole is then filled up with a softer material forming the "core."

Brazil), *Castilloa elastica* (a native of Central America), various species of *Manihot* (South America), *Ficus Elastica* (Assam) and *Funtumia elastica* (Africa). At one time the world's entire supply of rubber was derived from trees growing wild in forests, but at present the greater part of the supply is obtained from cultivated trees, there being extensive plantations of rubber trees in India, Ceylon, the Malay Archipelago and other places. The rubber obtained from cultivated trees is known as *plantation rubber*, to distinguish it from the *wild rubber* of forest trees.

The fluctuations in the price of Para rubber during the years 1915–1926 are shown in Fig. 190.

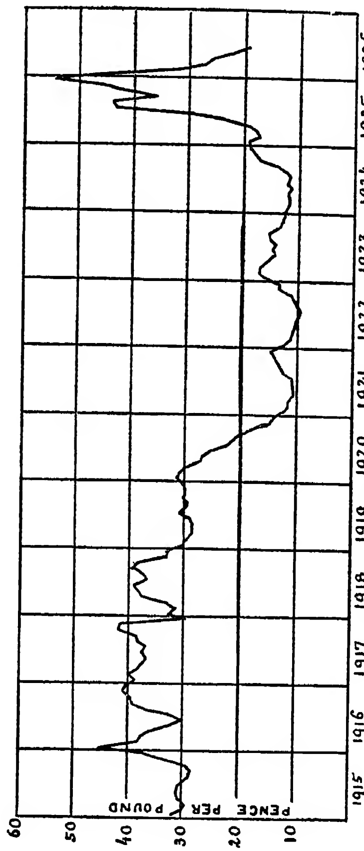

FIG. 190.—Fluctuations in the price of Para rubber.

India-rubber has a composition represented by the empirical formula $C_4 H_7$. Its density varies from ·92 to ·96. It has a resistivity of about 11×10^{15} ohms per cm. cube at 24° C.

It is totally insoluble in water, and is unacted on by alkalies, alcohol or dilute acids, but is attacked by strong nitric and sulphuric acids and chlorine. It is also slowly attacked by oils, and by Portland cement. The best solvents for rubber are benzene, carbon bisulphide, turpentine, and coal-tar naphtha.

When freely exposed to moist air, especially in the presence of light, rubber undergoes oxidation, and becomes very brittle. A rise of temperature has the effect of rendering it more plastic. At 120° C. it begins to melt, undergoing decomposition into a viscous substance which does not readily harden again.

Owing to its perishable nature, pure rubber is practically useless in cases where it is exposed to the air. The durability of rubber may, however, be greatly increased by the process of *vulcanisation*, invented independently by Hancock in England and Goodyear in America (1843). This process consists essentially in mixing the pure rubber with about $\frac{1}{20}$th of its weight of *sulphur* and with various other ingredients (calcium carbonate, French chalk, zinc oxide, litharge, &c.), and subjecting the mixture to a temperature of from 120° to 150° C. The compound so formed is mechanically much stronger, more durable, and able to withstand much higher temperatures than pure rubber. It is also very much cheaper.

The insulation of vulcanised india-rubber covered cables generally consists of three distinct layers. Immediately surrounding the copper conductor is a double lapping of *pure* rubber tape; then comes a layer of *separator* rubber, which consists of a mixture of rubber and zinc oxide; and finally a thick layer of vulcanised rubber. The separator rubber is intended to prevent the sulphur contained in the outer layer from working its way through the pure rubber, thus reaching the conductor and attacking it. As a mechanical protection, a layer of vulcanised tape (cotton tape impregnated with vulcanised rubber) is laid spirally around the insulated conductor, and in addition an external braiding of hemp or jute saturated with a bituminous compound is generally provided.

If rubber is mixed with about half its own weight of sulphur, and the mixture subjected to a temperature of 75° C. for several hours under considerable pressure (4 to 5 atmospheres), a material is obtained which is known as *ebonite*, *vulcanite*, or *hard rubber*. This has a higher resistivity than pure rubber, and has none of the plasticity of the latter; it is capable of being sawn or turned, and takes a very high polish. Ebonite is used to a very large extent in the construction of electrical instruments. When exposed to light it undergoes oxidation in moist air, a layer of acid being gradually formed over its surface. For this reason it should not be unnecessarily exposed to light, and should be kept in a dry atmosphere.

§ 209. Gutta-percha.

Gutta-percha is the coagulated latex of certain trees which are to be found only within a limited region of the Malay Archipelago. In some respects it resembles india-rubber, but is easily distinguished from it by the important property which it possesses of becoming quite soft at a temperature of about 65° C., when it may be easily moulded into any shape required. In common with india-rubber, it is rapidly attacked when exposed to the air in the presence of light, undergoing oxidation and becoming extremely brittle; a similar effect is produced by allowing it to get wet and dry alternately. But when constantly immersed in water, and screened from light, it appears to be quite permanent, and exhibits no signs of decay. It is this feature which renders the material so valuable in connection with its most important application in electrical engineering—the insulation of *submarine cables*. Although rubber has to a small extent been used for the same purpose, its durability is a very doubtful factor. The wholesale price of *raw* gutta-percha has during the last few years fluctuated from about 6*s*. to about 9*s*. per lb.

Chatterton's compound, which is used in jointing submarine cables, consists of 3 parts by weight of gutta-percha, 1 of Stockholm tar, and 1 of resin.

Gutta-percha has a density of about ·98, and a resistivity of about 2×10^9 ohm per cm. cube. Its composition is represented by the empirical formula C_5H_8.

§ 210. Mica.

Mica is a general term applied to a group of minerals characterised by the ease with which they may be split up into thin plates.

Chemically, mica consists in most cases of a double silicate of aluminium and potassium, with various other admixtures; the potassium is in some specimens replaced by sodium or magnesium. Those varieties which contain potassium are either transparent or of a faint greenish, yellowish, reddish, or grey colour; if magnesium is present the colours are much more intense; while the presence of iron in large quantities imparts to

the mica a colour ranging from grey to black. Mica has a density of about 3.

Mica is an extremely important insulator. It is non-hygroscopic, perfectly fire-proof, possesses great dielectric strength and high resistivity. Its great disadvantage is its mechanical weakness. The price of mica is determined almost entirely by its quality and the size of the plates, raw mica costing from about 1*s.* to 1*s.* 6*d.* per lb., while large sheets cut to definite size may cost as much as 15*s.* per lb. Mica is mined in India, Canada, the United States, and some other countries.

One of the most important uses of mica is in the construction of commutators, where it forms the dielectric between the segments. It is important to select for this purpose a brand of mica which will wear at about the same rate as the copper of the segments; otherwise projecting ridges of mica will be formed which will cause sparking. Canadian "amber" mica is found to answer this purpose best.

Mica forms the main ingredient of various insulating materials, such as *micanite*, which is prepared by cementing together small flakes or splittings of mica; micanite cloth, and micanised paper.

§ 211. Porcelain, Marble and Slate.

Porcelain is an artificial product, prepared by raising to a high temperature a very intimate mixture of kaolin or china clay (a hydrated aluminium silicate), finely ground felspar, flint, and some other ingredients. The insulators supporting overhead wires are mostly made of porcelain, and among other uses of it may be mentioned switch bases and covers, lampholders, ceiling roses, &c.

The best grades of porcelain are non-hygroscopic, and will insulate well even if deprived of their glaze. The glaze is in this case merely intended to give the insulator a smooth surface which is easily cleaned and which will not collect dirt or dust readily. Inferior grades of porcelain are, on the other hand, porous and strongly hygroscopic, and their insulating power depends entirely on the continuity of the coating of glaze. A rough test of the

Where such trouble is experienced, a radical cure may be effected by grooving out the mica between the segments, so as to depress the tops of the mica plates slightly below the level of the segments

quality of porcelain may be obtained by breaking off a piece and applying the broken surface to the tongue; if the tongue shows a tendency to adhere to the porcelain, the porcelain is of inferior quality.

The mechanical strength of the highest grade of porcelain in *compression* is as much as 28 tons/sq. in. Its tensile strength (which is somewhat difficult to determine) is only about $\frac{1}{3}$rd of this amount.

The electric (or dielectric) strength of porcelain is about 4×10^5 volts/cm., which is about $10\frac{1}{2}$ times that of air. The dielectric constant is about 9.

Marble and *slate* are valuable as incombustible and mechanically fairly strong insulators. They are largely used in the construction of switchboards, switches, resistance frames, &c.

§ 212. Paraffin Wax, Ozokerite, and Bitumen.

Paraffin wax is obtained by the distillation of lignite or peat. It is a mixture of several hydrocarbons of the paraffin series, $C_n H_{2n+2}$. Its melting point is below 100° C., so that it could not possibly be used as an insulator where any considerable rise of temperature is likely to occur. If raised above 100° C. paraffin begins to decompose, and its resistivity is greatly decreased. For this reason it should always be melted in a water-bath. Paraffin wax is used for improving the insulating qualities of various materials, such as wood, by soaking them in it.

Ozokerite, or earth wax, is a natural product found in Galicia and near Baku, on the Caspian Sea. Like paraffin wax, it is a mixture of hydrocarbons of the paraffin series.

Bitumen, asphalt, or earth pitch, is found in Trinidad (also in India), where it forms lakes of large extent. Chemically, it consists of a mixture of hydrocarbons which have become modified by the absorption of oxygen from the air. It is used as a filling for the troughs in which underground cables are laid. Chemically, bitumen is extremely inert, and this, in addition to its insulating properties, is one of its most valuable features. Unlike rubber, it is not attacked by rats or mice. *Vulcanised bitumen* is used as a covering for cables, and is a cheaper insulator than vulcanised rubber.

§ 213. Fibrous Insulators.

Besides non-hygroscopic materials like vulcanised rubber and vulcanised bitumen, various fibrous materials have been used for the insulating covering of cables. The most important of these is *paper*. This is wound round the cable in a spiral, and when a sufficient number of layers have been wound on to give the necessary thickness of insulating covering, the cable is dried and then soaked in an insulating compound. Since all fibrous materials are hygroscopic, cables of this type require a continuous waterproof covering in order to exclude moisture, and such a covering is provided in the form of a *lead sheathing*, which is put on over the paper insulation.

Cotton and silk are two fibrous materials largely employed for insulating the conductors and wires used in the construction of dynamos, instruments, &c.

Press-board or *fuller-board* is a special kind of paper. *Vulcanised fibre* is also a form of vegetable fibre specially treated and consolidated under great pressure.

Asbestos is a term applied to certain minerals which are capable of being split up into very thin flexible fibres; these may be spun into thread. Asbestos ("unquenchable") owes its name to its heat-resisting property, as it is capable of withstanding temperatures of 1,000° C. and more. Originally, asbestos was mined in Italy, but the bulk of it now comes from the eastern townships of Quebec in Canada. The mineral there mined is known as *chrysotile*, and occurs in veins of closely packed transverse fibres. Chemically, asbestos is a hydrated silicate of magnesium ($Mg_3 H_4 Si_2 O_9$).

An insulating covering of asbestos enables copper conductors to be worked at much higher temperatures than would be possible with the ordinary cotton covering. Hence asbestos insulation is used in cases where the conductors are liable to be overheated—as in the field coils of railway motors; to some extent it is used for insulating armature conductors. The main disadvantages of asbestos insulation are its extreme mechanical weakness and hygroscopic nature. Asbestos is used as an ingredient in certain compound types of insulating materials which are employed in making various moulded articles, and which would be too brittle without the strengthening effect of the asbestos fibres embedded in them.

Vulcabeston is prepared by mixing rubber with asbestos, and subjecting the mixture to a high temperature. It is only slightly hygroscopic, and is capable of withstanding a temperature as high as 300° C. without any change. It is obtainable in sheets from 1 to 3 mm. thick, and is easily worked.

§214. Shellac.

Shellac is of great importance as an insulator, forming an ingredient of many insulating varnishes, and a useful insulating cement.* It is imported from India, Ceylon, Burmah, Siam, and the Malay Archipelago. It is obtained from certain trees infested by an insect which punctures the bark and secretes a substance known as *lac;* this it forms into a regular structure like a honeycomb, depositing its eggs in the cells. The insect never leaves the twig which it has punctured, and dies after having formed the incrusting mass containing the eggs. This mass consists partly of the fluid secreted by the insect, partly of the sap which has exuded from the tree. The twigs are collected, broken up into short lengths (known as *stick lac*), and the incrustation detached by passing a heavy roller over them. A red dye, known as lac dye, having been extracted from the incrustation by treating it with hot water, the remaining portion (known as *seed lac*) is subjected to a roasting process and formed into the shellac of commerce. Shellac readily dissolves in alcohol, forming the well-known *shellac varnish;* this also contains various other ingredients.

§ 215. Bakelite.

Bakelite (named after Dr. Baekeland) is a synthetic resin which has come into fairly extensive use as an insulating material. It is prepared by the action of phenol on formaldehyde in the presence of a catalytic agent (ammonia). The process is carried out in three stages, and the corresponding products are known as bakelites A, B and C. A is a liquid of great penetrating power and hence suitable for impregnating fibrous insulating materials; prolonged heating

* Shellac also forms an ingredient of sealing-wax, of the lacquers used for coating metal work, and of Indian ink.

at from 60° to 70° C. converts it into a solid which is hard and brittle (but which may be softened by the application of heat), and which is known as Bakelite B. The B form is also prepared by allowing the chemical reaction to take place beyond the A stage. Heating B at about 160° C. changes it into the final or C form, and the aim in all cases is to reach this final stage.

Bakelite C is a hard solid (harder than ebonite) unaffected by most chemicals and capable of withstanding temperatures as high as 300° C. without softening or deterioration. It is, however, extremely brittle, and this is its main defect. In colour it varies from that of a colourless solid like glass to a dark brown.

When arcing takes place over a surface of bakelite, the carbonisation which results produces a permanently conducting path. This effect is known as "tracking," and renders the use of bakelite unsatisfactory in cases where arcing is liable to occur.

ANSWERS TO EXAMPLES.

Chapter I.—1. 12 ohms; 2. 3·2 pence; 3. 6·7 h.p.; 4. 3,132 amperes; 5. ·00324 lb. weight; 6. 6·8 microcoulombs; 7. 3·08 lbs.; 8. $26{\cdot}86 \times 10^6$ joules; 9. 832; 10. 3·72 gramme-cm.; 11. ·853; 15°.

Chapter II.—1. 63 and 78·8, 1813; 2. 14·8; 3, 1·14 microcoulombs; 4, 2,230 C. G. S. lines / sq. cm.; 5. 17·8 watts.

Chapter III.—1. 6,840 ampere-turns; 2. 4,580 ampere-turns; 3. ·01 ohm; 4. 216·05 lbs.; 5. 214.

Chapter VII.—1. 480 volts; 2. $4{\cdot}6 \times 10^6$; 3. 15,800; 4. 500, 508, 512, 520, 524, 532, 536, 544, 548.

Chapter VIII.—1. 1·8 volts; 2. ·00183 sec.; 3. ·000988 sec.; 4. 1,260 watts; 5. 1,530; 6. 4,320.

Chapter IX.—1. 59·9 lb. ft.; 2. 456 volts; 3. 1,007 r.p.m.; 4. 232 lbs. weight.

Chapter X.—1. About 15 ins.; 2. from 7 to 9 ins.; 3. $18{\cdot}1 \times 10^{-6}$; 4. about £300, about 90 cwt.

Chapter XI.—1. 535; 2. 1,930 ampere-turns; 3. 7,330; 4. 1,485; 5. (*a*) 1,192, (*b*) 953.

Chapter XII.—1, 97·14 per cent.; 2. 3·2 b.h.p.; 3. 84·3 per cent.; 4. 87 per cent.; 5. 93 per cent.

Chapter XIV.—1. (*a*) 162·8 ampere-hours, (*b*) 91·2 per cent., (*c*) 77·2 per cent.; 2. (*a*) 85·2 ampere-hours, (*b*) 91 per cent., (*c*) 68·5 per cent.; 3. (*a*) 2,020 lbs., (*b*) £277 8*s*.; 4. (*a*) about 37½ tons, (*b*) about £3,500; (*c*) about 1,200 sq. ft.

Chapter XVI.—1. 1,697 c.p.; 2. 16 c.p.; 3. ·237 candle-foot.

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